

2005 - Exercises I.

1. Let $q > 1$. What is the relation ($\leq, \geq, =$) between:

- (a) $A_q(n, d)$ and $A_q(n/2, d/2)$
- (b) $A_q(n, d)$ and $A_q(n + 2, d + 1)$
- (c) $A_q(2n, 2d + 1)$ and $A_q(n, d)$
- (d) $A_q(n, d)$ and $A_{2q}(n, d)$
- (e) $A_q(n, d)$ and $q^{(n-d+1)}$

2. Determine

- (a) $A_q(5, 3)$ for $q=2,3,4$
- (b) $A_2(n, n - 2)$ for $n=3,4$
- (c) $A_2(10, d)$ for $d=1,2$
- (d) $A_2(n, 10)$ for $n=11,12$

3. You have captured the following message. Determine the name of the book hidden in this message. This message is a standard type of file you may find in your computer.

```
50 4b 03 04 14 00 01 00 00 00 c6 b0 32 33 c3 e4
c4 30 1d 00 00 00 11 00 00 00 0d 00 00 00 34 6e
75 6d 61 6c 70 68 61 2e 74 78 74 0f 2b f9 68 0c
42 d2 c3 76 f2 c1 a3 55 eb 9f 23 d6 da a0 3d 65
be 95 79 cc 0d 6a 33 8b 50 4b 01 02 14 00 14 00
01 00 00 00 c6 b0 32 33 c3 e4 c4 30 1d 00 00 00
11 00 00 00 0d 00 00 00 00 00 00 00 01 00 20 00
00 00 00 00 00 00 34 6e 75 6d 61 6c 70 68 61 2e
74 78 74 50 4b 05 06 00 00 00 00 01 00 01 00 3b
00 00 00 48 00 00 00 00 00
```

4. Let us consider the following channel:

- $0 \mapsto *$ with probability p
- $0 \mapsto 0$ with probability $1-p$
- $1 \mapsto *$ with probability p
- $1 \mapsto 1$ with probability $1-p$

Let us use the error correcting code $C = \{0000, 1111\}$ (which maps 0 to 0000 and 1 to 1111). What is the probability that C fails to repair a random single-character message?

5. Show equivalence of the following verification procedures of ISBN correctness:

$$\sum_{i=1}^{10} (11 - i) x_i \equiv 0 \pmod{11}$$

(this is given in the standard) and

$$\sum_{i=1}^{10} ix_i \equiv 0 \pmod{11}$$

(the one from lecture), for general ISBN code $x_1 x_2 \dots x_{10}$.

6. The following message was received from outer space:

```
001100000110001111111011001001100100110010111100  
0100100010010001001001100110
```

Why might it be conjectured that the message was sent by a race of human beings who have one arm twice as long as the other?