Refining the Undecidability Border of Weak Bisimilarity

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Abstract

Weak bisimilarity is one of the most studied behavioural equivalences. This equivalence is undecidable for *pushdown processes* (PDA), *process algebras* (PA), and *multiset automata* (MSA, also known as *parallel pushdown processes*, PPDA). Its decidability is an open question for *basic process algebras* (BPA) and *basic parallel processes* (BPP). We move the undecidability border towards these classes by showing that the equivalence remains undecidable for weakly extended versions of BPA and BPP.

Key words: weak bisimulation, infinite-state systems, decidability

1 Introduction

Equivalence checking is one of the main streams in verification of concurrent systems. It aims at demonstrating some semantic equivalence between two systems, one of which is usually considered as representing the specification, the other its implementation or refinement. The semantic equivalences are designed to correspond to the system behaviours at the desired level of abstraction; the most prominent ones being strong and weak bisimulations.

Current software systems often exhibit an evolving structure and/or operate on unbounded data types. Hence automatic verification of such systems usually requires modeling them as infinite-state ones. Various specification formalisms have been developed with their respective advantages and limitations. Petri nets (PN), pushdown processes (PDA), and process algebras like BPA, BPP, or PA all serve to exemplify this. Here we employ the classes

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of infinite-state systems defined by term rewrite systems and called *Process Rewrite Systems* (PRS) as introduced by Mayr [13]. PRS subsume a variety of the formalisms studied in the context of formal verification (e.g. all the models mentioned above). The relevance of various subclasses of PRS for modelling and analysing programs is shown, for example, in [5]; for automatic verification we refer to surveys [2,22].

The relative expressive power of various process classes has been studied, especially with respect to strong bisimulation; see [3,17], also [13] showing the hierarchy of PRS classes. Adding a finite-state control unit to the PRS rewriting mechanism results in so-called state-extended PRS (sePRS) classes, see for example [8]. We have extended the PRS hierarchy by sePRS classes and refined this extended hierarchy by introducing restricted state extensions of two types: PRS equipped with a weak finite-state unit (wPRS, inspired by weak automata [18]) [11,10] and PRS with finite constraint unit (fcPRS) [23].

Research on the expressive power of process classes has been accompanied by exploring algorithmic boundaries of various verification problems. In this paper we focus on the equivalence checking problem taking weak bisimilarity as the notion of behavioral equivalence.

The state of the art: Regarding sequential systems, i.e. those without parallel composition, the weak bisimilarity problem is undecidable for PDA even for the normed case [19]. However, it is conjectured [14] that weak bisimilarity is decidable for BPA; the best known lower bound is *EXPTIME*-hardness [14].

Considering parallel systems, even strong bisimilarity is undecidable for MSA [17] using the technique as introduced in [6]. However, it is conjectured [7] that the weak bisimilarity problem is decidable for BPP; the best known lower bound is *PSPACE*-hardness [20].

For the simplest systems combining both parallel and sequential operators, called PA processes [1], the weak bisimilarity problem is undecidable [21]. It is an open question for the normed PA; the best known lower bound is *EXPTIME*-hardness [14].

Our contribution: We move the undecidability border of the weak bisimilarity problem towards the classes of BPA and BPP, where the problem is conjectured to be decidable. We show that the problem remains undecidable for the weakly extended versions of both BPA (wPBA) and BPP (wBPP). In fact, the result is not new for wBPA: Mayr [14] has shown that adding a finite-state unit of the minimal non-trivial size 2 to the BPA process already makes weak bisimilarity undecidable. By inspection of his proof, we note that the result is valid for wBPA as well - see Sections 2 and 3 for the definition of wBPA and more detailed discussion.

2 Preliminaries

We recall the definitions of labelled transition system and weak bisimilarity. Then we define the syntax of process rewrite systems and (weak) finite-state unit extensions of PRS. Their semantics is given in terms of labelled transition systems.

Let $Act = \{a, b, \ldots\}$ be a set of *actions* such that Act contains a distinguished *silent action* τ. A *labelled transition system* is a pair $(S, \longrightarrow)$, where S is a set of *states* and $\longrightarrow \subseteq S \times Act \times S$ is a *transition relation*. We write $s_1 \xrightarrow{a} s_2$ instead of $(s_1, a, s_2) \in \longrightarrow$. The transition relation is extended to finite words over Act in the standard way. Further, we extend the relation to language $L \subseteq Act^*$ such that $s_1 \xrightarrow{L} s_2$ if $s_1 \xrightarrow{w} s_2$ for some $w \in L$. Moreover, we write $s_1 \longrightarrow^* s_2$ instead of $s_1 \xrightarrow{Act^*} s_2$. The *weak transition relation* $\Longrightarrow \subseteq S \times Act \times S$ is defined as $\stackrel{\tau}{\Longrightarrow} = \xrightarrow{\tau^*}$ and $\stackrel{a}{\Longrightarrow} = \xrightarrow{\tau^* a \tau^*}$ for all $a \neq \tau$.

A binary relation R on states of a labelled transition system is a *weak bisimulation* iff whenever $(s_1, s_2) \in R$ then for any $a \in Act$:

- if $s_1 \xrightarrow{a} s_1'$ then $s_2 \stackrel{a}{\Longrightarrow} s_2'$ for some s_2' such that $(s_1', s_2') \in R$ and
- if $s_2 \xrightarrow{a} s_2'$ then $s_1 \stackrel{a}{\Longrightarrow} s_1'$ for some s_1' such that $(s_1', s_2') \in R$.

States s_1 and s_2 are *weakly bisimilar*, written $s_1 \approx s_2$, iff $(s_1, s_2) \in R$ for some weak bisimulation R.

We use a characterization of weak bisimilarity in terms of a *bisimulation game*. This is a two-player game between an *attacker* and a *defender* played in rounds on pairs of states of a considered labelled transition system. In a round starting at a pair of states (s_1, s_2), the attacker first chooses $i \in \{1, 2\}$, an action $a \in Act$, and a state s_i' such that $s_i \xrightarrow{a} s_i'$. The defender then has to choose a state s_{3-i}' such that $s_{3-i} \stackrel{a}{\Longrightarrow} s_{3-i}'$. The states s_1', s_2' form a pair of starting states for the next round. A *play* is a maximal sequence of pairs of states chosen by players in the given way. The defender is the winner of every infinite play. A finite game is lost by the player who cannot make any choice satisfying the given conditions. It can be shown that two states s_1, s_2 of a labelled transition system are not weakly bisimilar if and only if the attacker has a winning strategy for the bisimulation game starting in these states.

Let $Const = \{X, \ldots\}$ be a set of *process constants*. The set of *process terms* (ranged over by $t, \ldots$) is defined by the abstract syntax $t ::= \varepsilon \mid X \mid t.t \mid t\|t$, where ε is the *empty term*, $X \in Const$ is a process constant; and '.' and '$\|$' mean *sequential* and *parallel composition* respectively. We always work with equivalence classes of terms modulo commutativity and associativity of '$\|$', associativity of '.', and neutrality of ε, i.e. $\varepsilon.t = t = t.\varepsilon$ and $t\|\varepsilon = t$. We distinguish four *classes of process terms* as:

1 – terms consisting of a single process constant only, in particular $\varepsilon \notin 1$,

S – *sequential* terms - terms without parallel composition, e.g. $X.Y.Z$,

P – *parallel* terms - terms without sequential composition, e.g. $X \| Y \| Z$,

G – *general* terms - terms with arbitrarily nested sequential and parallel compositions, e.g. $(X.(Y \| Z)) \| W$.

Let α, β be classes of process terms $\alpha, \beta \in \{1, S, P, G\}$ such that $\alpha \subseteq \beta$. An *(α, β)-PRS (process rewrite system)* Δ is a finite set of *rewrite rules* of the form $t_1 \xrightarrow{a} t_2$, where $t_1 \in \alpha \smallsetminus \{\varepsilon\}$, $t_2 \in \beta$ are process terms and $a \in Act$ is an action. Given a PRS Δ, let $Const(\Delta)$ and $Act(\Delta)$ be the respective (finite) sets of all constants and all actions which occur in the rewrite rules of Δ.

An (α, β)-PRS Δ determines a labelled transition system where states are process terms $t \in \beta$ over $Const(\Delta)$. The transition relation $\longrightarrow$ is the least relation satisfying the following inference rules (recall that '$\|$' is commutative):

$$\frac{(t_1 \xrightarrow{a} t_2) \in \Delta}{t_1 \xrightarrow{a} t_2} \qquad \frac{t_1 \xrightarrow{a} t_2}{t_1 \| t_1' \xrightarrow{a} t_2 \| t_1'} \qquad \frac{t_1 \xrightarrow{a} t_2}{t_1.t_1' \xrightarrow{a} t_2.t_1'}$$

The formalism of process rewrite systems can be extended to include a finite-state control unit in the following way. Let $M = \{m, n, \ldots\}$ be a set of *control states*. Let $\alpha, \beta \in \{1, S, P, G\}, \alpha \subseteq \beta$ be the classes of process terms. An *(α, β)-sePRS (state extended process rewrite system)* Δ is a finite set of *rewrite rules* of the form $(m, t_1) \xrightarrow{a} (n, t_2)$, where $t_1 \in \alpha \smallsetminus \{\varepsilon\}$, $t_2 \in \beta$, $m, n \in M$, and $a \in Act$. $M(\Delta)$ denotes the finite set of control states which occur in Δ.

An (α, β)-sePRS Δ determines a labelled transition system where states are the pairs of the form (m, t) such that $m \in M(\Delta)$ and $t \in \beta$ is a process term over $Const(\Delta)$. The transition relation $\longrightarrow$ is the least relation satisfying the following inference rules:

$$\frac{((m, t_1) \xrightarrow{a} (n, t_2)) \in \Delta}{(m, t_1) \xrightarrow{a} (n, t_2)} \qquad \frac{(m, t_1) \xrightarrow{a} (n, t_2)}{(m, t_1 \| t_1') \xrightarrow{a} (n, t_2 \| t_1')} \qquad \frac{(m, t_1) \xrightarrow{a} (n, t_2)}{(m, t_1.t_1') \xrightarrow{a} (n, t_2.t_1')}$$

To shorten our notation we write mt in lieu of (m, t).

An (α, β)-sePRS Δ is called a *process rewrite system with weak finite-state control unit* or just a *weakly extended process rewrite system*, written (α, β)-wPRS, if there exists a partial order $\leq$ on $M(\Delta)$ such that every rule $(m, t_1) \xrightarrow{a} (n, t_2)$ of Δ satisfies $m \leq n$.

Some classes of (α, β)-PRS correspond to widely known models as finite-state systems (FS), basic process algebras (BPA), basic parallel processes (BPP), process algebras (PA), pushdown processes (PDA, see [4] for justification), and Petri nets (PN). The other (α, β)-PRS classes were introduced and named as PAD, PAN, and PRS by Mayr [13]. The correspondence between (α, β)-PRS classes and the acronyms is given in Figure 1. Instead of (α, β)-sePRS or (α, β)-wPRS we use the prefixes 'se-' and 'w-' in connection with the acronym for the corresponding (α, β)-PRS class. For example, we use wBPA and wBPP rather than $(1, S)$-wPRS and $(1, P)$-wPRS, respectively.

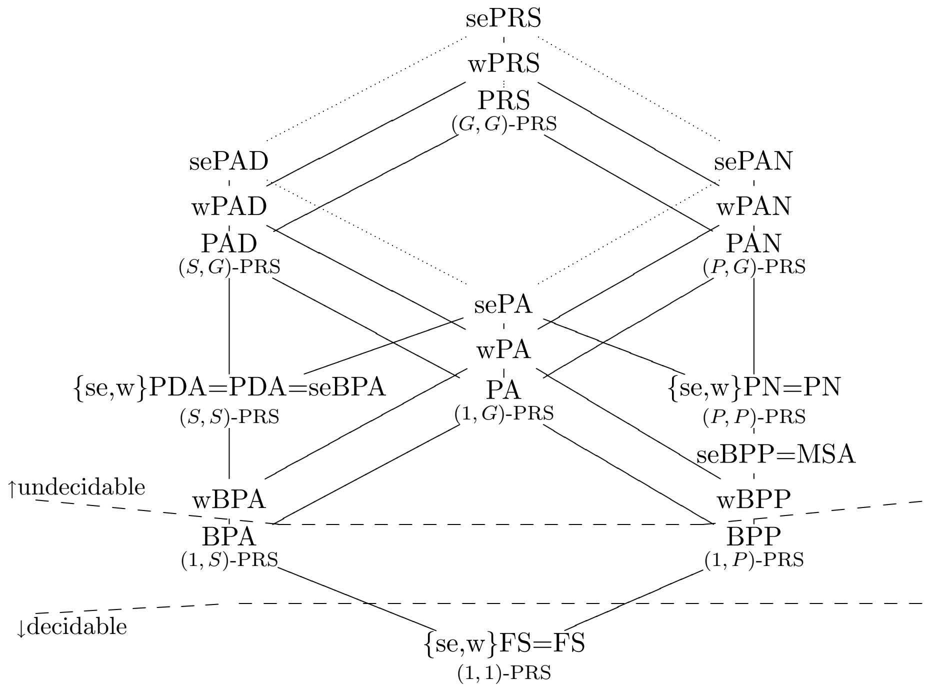


Fig. 1. The hierarchy with (un)decidability boundaries of $\approx$.

Finally, we note that seBPP are also known as multiset automata (MSA) or parallel pushdown processes (PPDA).

Figure 1 depicts relations between the expressive power of the considered classes. The expressive power of a class is measured by the set of labelled transition systems that are definable (up to strong bisimulation equivalence) by the class. A solid line between two classes means that the upper class is strictly more expressive than the lower one. A dotted line means that the upper class is at least as expressive as the lower class (and the strictness is just our conjecture). Details can be found in [11,10].

3 Undecidability of weak bisimilarity

In this section we show that weak bisimilarity is undecidable for the classes wBPA and wBPP. More precisely, we study the following two problems.

Problem:	*Weak bisimilarity problem for wBPA (or wBPP respectively)*
Instance:	A wBPA (or wBPP) system Δ and two of its states $mt, m't'$
Question:	Are the two states weakly bisimilar?

3.1 wBPA

In [14] Mayr studied the question of how many control states are needed in PDA to make weak bisimilarity undecidable.

Theorem 3.1 ([14], Theorem 29) *Weak bisimilarity is undecidable for pushdown automata with only 2 control states.*

The proof is done by a reduction of Post's correspondence problem to the weak bisimilarity problem for PDA. The constructed PDA has only two control states, p and q. Quick inspection of the construction shows that the resulting pushdown automata are in fact wBPA systems as there is no transition rule changing q to p and each rule has only one process constant on the left hand side. Hence Mayr's theorem can be reformulated as follows.

Theorem 3.2 *Weak bisimilarity is undecidable for wBPA systems.*

3.2 wBPP

We show that the non-halting problem for Minsky 2-counter machines can be reduced to the weak bisimilarity problem for wBPP. First, let us recall the notions of Minsky 2-counter machine and the non-halting problem.

A *Minsky 2-counter machine*, or a *machine* for short, is a finite sequence

$$N = l_1 \colon i_1,\ l_2 \colon i_2,\ \ldots,\ l_{n-1} \colon i_{n-1},\ l_n \colon \texttt{halt}$$

where $n \geq 1$, $l_1, l_2, \ldots, l_n$ are *labels*, and each i_j is an instruction for

- *increment*: $\texttt{c}_k\texttt{:=c}_k\texttt{+1; goto}\ l_r$, or
- *test-and-decrement*: $\texttt{if c}_k\texttt{>0 then c}_k\texttt{:=c}_k\texttt{-1; goto}\ l_r\ \texttt{else goto}\ l_s$

where $k \in \{1, 2\}$ and $1 \leq r, s \leq n$.

The semantics of a machine N is given by a labelled transition system the states of which are *configurations* of the form (l_j, v_1, v_2) where l_j is a label of an instruction to be executed and v_1, v_2 are nonnegative integers representing current values of counters $\texttt{c}_1$ and $\texttt{c}_2$, respectively. The transition relation is the smallest relation satisfying the following conditions: if i_j is an instruction of the form

- $\texttt{c}_1\texttt{:=c}_1\texttt{+1; goto}\ l_r$, then $(l_j, v_1, v_2) \xrightarrow{inc} (l_r, v_1 + 1, v_2)$ for all $v_1, v_2 \geq 0$;
- $\texttt{if c}_1\texttt{>0 then c}_1\texttt{:=c}_1\texttt{-1; goto}\ l_r\ \texttt{else goto}\ l_s$, then $(l_j, v_1 + 1, v_2) \xrightarrow{dec} (l_s, v_1, v_2)$ and $(l_j, 0, v_2) \xrightarrow{zero} (l_r, 0, v_2)$ for all $v_1, v_2 \geq 0$;

and the analogous condition for instructions manipulating $\texttt{c}_2$. We say that the (computation of) machine N *halts* if there are numbers $v_1, v_2 \geq 0$ such that $(l_1, 0, 0) \longrightarrow^* (l_n, v_1, v_2)$. Let us note that the system is deterministic, i.e. for every configuration there is at most one transition leading from the configuration.

The *non-halting problem* is to decide whether a given machine N does not halt. The problem is undecidable [16].

Let us fix a machine $N = l_1 : i_1,\ l_2 : i_2,\ \ldots,\ l_{n-1} : i_{n-1},\ l_n : \texttt{halt}$. We construct a wBPP system Δ such that its states $simL_1$ and $simL_1'$ are weakly bisimilar if and only if N does not halt. Roughly speaking, we create a set of wBPP rules allowing us to simulate the computation of N by two separate sets of terms. If the instruction `halt` is reached in the computation of N, the corresponding term from one set can perform the action *halt*, while the corresponding term from the other set can perform the action *halt*$'$. Therefore, the starting terms are weakly bisimilar if and only if the machine does not halt.

The wBPP system Δ we are going to construct uses five control states, namely $sim, check_1, check'_1, check_2, check'_2$. We associate each label l_j and each counter $\mathtt{c}_k$ with process constants L_j, L'_j and X_k, Y_k respectively. A parallel composition of x copies of X_k and y copies of Y_k, written $X_k^x \| Y_k^y$, represents the fact that the counter $\mathtt{c}_k$ has the value $x - y$. Hence, terms $simL_j \| X_1^{x_1} \| Y_1^{y_1} \| X_2^{x_2} \| Y_2^{y_2}$ and $simL'_j \| X_1^{x_1} \| Y_1^{y_1} \| X_2^{x_2} \| Y_2^{y_2}$ are associated with a configuration $(l_j, x_1 - y_1, x_2 - y_2)$ of the machine N. Some rules contain auxiliary process constants. In what follows, β stands for a term of the form $\beta = X_1^{x_1} \| Y_1^{y_1} \| X_2^{x_2} \| Y_2^{y_2}$. The control states $check_k, check'_k$ for $k \in \{1, 2\}$ are intended for testing emptiness of the counter $\mathtt{c}_k$. The only rules applicable in these control states are:

$$check_1 X_1 \xrightarrow{chk_1} check_1 \varepsilon \qquad\qquad check_2 X_2 \xrightarrow{chk_2} check_2 \varepsilon$$
$$check'_1 Y_1 \xrightarrow{chk_1} check'_1 \varepsilon \qquad\qquad check'_2 Y_2 \xrightarrow{chk_2} check'_2 \varepsilon$$

One can readily confirm that $check_k \beta \approx check'_k \beta$ if and only if the value of $\mathtt{c}_k$ represented by β equals zero.

In what follows we construct a set of wBPP rules for each instruction of the machine N. At the same time we argue that the only chance for the attacker to win is to simulate the machine without cheating as every cheating can be punished by the defender's victory. This attacker's strategy is winning if and only if the machine halts.

Halt: l_n: `halt`

Halt instruction is translated into the following two rules:

$$simL_n \xrightarrow{halt} sim\varepsilon \qquad\qquad simL'_n \xrightarrow{halt'} sim\varepsilon$$

Clearly, *the states* $simL_n \| \beta$ *and* $simL'_n \| \beta$ *are not weakly bisimilar.*

Increment: l_j: $\mathtt{c}_k$`:=`$\mathtt{c}_k$`+1; goto` l_r

For each such instruction of the machine N we add the following rules to Δ:

$$simL_j \xrightarrow{inc} simL_r \| X_k \qquad\qquad simL'_j \xrightarrow{inc} simL'_r \| X_k$$

Obviously, *every round of the bisimulation game starting at states* $simL_j \| \beta$ *and* $simL'_j \| \beta$ *ends up in states* $simL_r \| X_k \| \beta$ *and* $simL'_r \| X_k \| \beta$.

Test-and-decrement: l_j: `if` $\mathtt{c}_k$`>0 then` $\mathtt{c}_k$`:=`$\mathtt{c}_k$`-1; goto` l_r `else goto` l_s

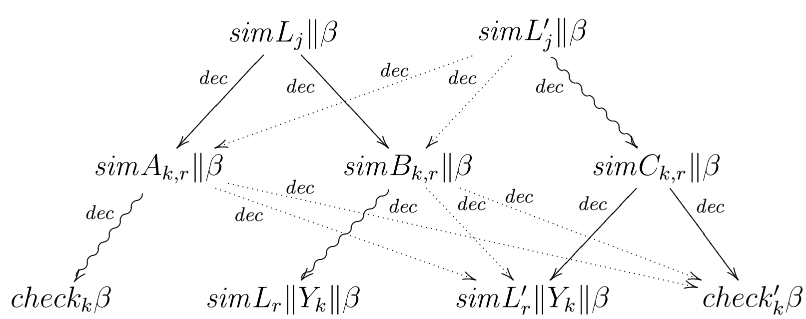
For any such instruction of the machine N we add two sets of rules to Δ, one for the $\mathtt{c}_k > 0$ case and the other for the $\mathtt{c}_k = 0$ case. The wBPP formalism has no power to rewrite a process constant corresponding to a label l_j and to check whether $\mathtt{c}_k > 0$ at the same time. Therefore, in the bisimulation game it is the attacker who has to decide whether $\mathtt{c}_k > 0$ holds or not, i.e. whether he will play an action *dec* or an action *zero*. We show that whenever the attacker tries to cheat, the defender can win the game.

At this point our construction of wBPP rules uses a variant of the technique called *defender's choice* [9]. In a round starting at the pair of states s_1, s_2, the attacker is forced to choose one specific transition (indicated by a wavy arrow henceforth). If he chooses a different transition, say $s_k \xrightarrow{a} s$ where $k \in \{1, 2\}$, then there exists a transition $s_{3-k} \xrightarrow{a} s$ that enables the defender to reach the same state and win the play. The name of this technique refers to the fact that after the attacker chooses the specific transition, the defender can choose an arbitrary transition with the same label. These transitions are indicated by solid arrows. The dotted arrows stands for auxiliary transitions which compel the attacker to play the specific transition.

First, we discuss the following rules designed for the $\mathtt{c}_k > 0$ case:

$$
\begin{array}{lll}
simL_j \xrightarrow{dec} simA_{k,r} & simA_{k,r} \xrightarrow{dec} check_k\varepsilon & simB_{k,r} \xrightarrow{dec} simL_r\|Y_k \\
simL_j \xrightarrow{dec} simB_{k,r} & simA_{k,r} \xrightarrow{dec} simL'_r\|Y_k & simB_{k,r} \xrightarrow{dec} simL'_r\|Y_k \\
simL'_j \xrightarrow{dec} simA_{k,r} & simA_{k,r} \xrightarrow{dec} check'_k\varepsilon & simB_{k,r} \xrightarrow{dec} check'_k\varepsilon \\
simL'_j \xrightarrow{dec} simB_{k,r} & & simC_{k,r} \xrightarrow{dec} simL'_r\|Y_k \\
simL'_j \xrightarrow{dec} simC_{k,r} & & simC_{k,r} \xrightarrow{dec} check'_k\varepsilon
\end{array}
$$

The situation can be depicted as follows.



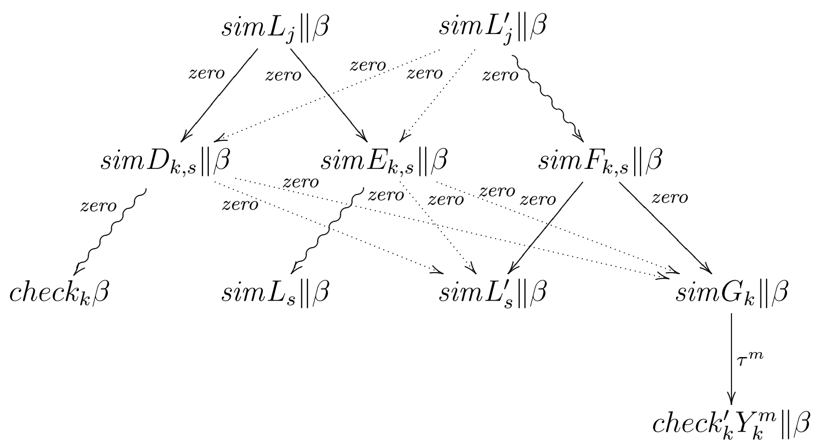
Let us assume that in a round starting at states $simL_j\|\beta$, $simL'_j\|\beta$ the attacker decides to perform the action *dec*. Due to the principle of defender's choice employed here, the attacker has to play the transition $simL'_j\|\beta \xrightarrow{dec} simC_{k,r}\|\beta$, while the defender can choose between the transitions leading from $simL_j\|\beta$ either to $simA_{k,r}\|\beta$ or to $simB_{k,r}\|\beta$. Thus, the round will finish either in states

$simA_{k,r}\|\beta, simC_{k,r}\|\beta$ or in states $simB_{k,r}\|\beta, simC_{k,r}\|\beta$. Using the defender's choice again, one can easily see that the next round ends up in $check_k\beta$ or $simL_r\|Y_k\|\beta$, and $simL'_r\|Y_k\|\beta$ or $check'_k\beta$. The exact combination is chosen by the defender. The defender will not choose any pair of states where one control state is *sim* and the other is not as such states are clearly not weakly bisimilar. Hence, the two considered rounds of the bisimulation game end up in a pair of states either $simL_r\|Y_k\|\beta, simL'_r\|Y_k\|\beta$ or $check_k\beta, check'_k\beta$. The latter pair is weakly bisimilar iff the value of c_k represented by β is zero, i.e. iff the attacker cheats when he decides to play an action *dec*. This means that *if the attacker cheats, the defender wins. If the attacker plays the action dec correctly, the only chance for either player to force a win is to finish these two rounds in states* $simL_r\|Y_k\|\beta, simL'_r\|Y_k\|\beta$ *corresponding to the correct simulation of an test-and-decrement instruction with a label* l_j.

Now, we focus on the following rules designed for the $\mathsf{c}_k = 0$ case:

$$
\begin{array}{lll}
simL_j \xrightarrow{zero} simD_{k,s} & simD_{k,s} \xrightarrow{zero} check_k\varepsilon & simE_{k,s} \xrightarrow{zero} simL_s \\
simL_j \xrightarrow{zero} simE_{k,s} & simD_{k,s} \xrightarrow{zero} simL'_s & simE_{k,s} \xrightarrow{zero} simL'_s \\
simL'_j \xrightarrow{zero} simD_{k,s} & simD_{k,s} \xrightarrow{zero} simG_k & simE_{k,s} \xrightarrow{zero} simG_k \\
simL'_j \xrightarrow{zero} simE_{k,s} & simF_{k,s} \xrightarrow{zero} simL'_s & simG_k \xrightarrow{\tau} simG_k\|Y_k \\
simL'_j \xrightarrow{zero} simF_{k,s} & simF_{k,s} \xrightarrow{zero} simG_k & simG_k \xrightarrow{\tau} check'_kY_k
\end{array}
$$

The situation can be depicted as follows.



Let us assume that the attacker decides to play the action *zero*. The defender's choice technique allows the defender to control the two rounds of the bisimulation game starting at states $simL_j\|\beta$ and $simL'_j\|\beta$. The two rounds end up in a pair of states $simL_s\|\beta, simL'_s\|\beta$ or in a pair of the form $check_k\beta, check'_kY_k^m\|\beta$ where $m \geq 1$; all the other choices of the defender lead to his loss. As in the previous case, the latter possibility is designed to punish any possible attacker's cheating. The attacker is cheating if he plays the action *zero* and the value of c_k represented by β, say v_k, is positive.[4] In such a case, the defender can control the two rounds to end up in states $check_k\beta, check'_kY_k^{v_k}\|\beta$ which are weakly bisimilar. If the attacker plays correctly, i.e. the value of c_k represented by β is zero, then the defender has to control the two discussed rounds to end up in states $simL_s\|\beta, simL'_s\|\beta$ as the states $check_k\beta, check'_kY_k^m\|\beta$ are not weakly bisimilar for any $m \geq 1$. To sum up, *the attacker's cheating can be punished by defender's victory. If the attacker plays correctly, the only chance for both players to win is to end up after the two rounds in states* $simL_s\|\beta, simL'_s\|\beta$ *corresponding to the correct simulation of the considered instruction.*

It has been argued that if each of the two players wants to win, then both players will correctly simulate the computation of the machine N. The computation is finite if and only if the machine halts. The states $simL_1$ and $simL'_1$ are not weakly bisimilar in this case. If the machine does not halt, the play is infinite and the defender wins. Hence, the two states are weakly bisimilar in this case. In other words, *the states* $simL_1$ *and* $simL'_1$ *of the constructed wBPP* Δ *are weakly bisimilar if and only if the Minsky 2-counter machine* N *does not halt.*

Theorem 3.3 *Weak bisimilarity is undecidable for wBPP systems.*

4 Conclusion

We have shown that the weak bisimilarity problem remains undecidable for weakly extended versions of BPP (wBPP) and BPA (wBPA) process classes.

We note that the result for wBPA is just our interpretation (in terms of weakly extended systems) of Mayr's proof showing that the problem is undecidable for PDA with two control states ([14], Theorem 29).

In terms of parallel systems, our technique used for wBPP is new. To mimic the computation of a Minsky 2-counter machine, one has to be able to maintain its state information – the label of a current instruction and the values of the counters c_1 and c_2. As the finite-state unit of wBPP is weak, it cannot be used to store even a part of such often changing information. Hence, contrary to the constructions in more expressive systems (PN [6] and MSA [17]) we have made the term part to manage it as follows. In a test-and-decrement instruction the process constant L_j has to be changed and moreover one of the counters has to be decreased at the same time. As two process constants cannot be rewritten by one wBPP rewrite rule, we introduce new process constants Y_1 and Y_2 to represent inverse elements to X_1 and X_2 respectively and we make a term $X_k^x\|Y_k^y$ to represent the counter c_k the value

[4] We do not have to consider the case when β represents a negative value of c_k as such a state is reachable in the game starting in states $simL_1, simL'_1$ only by unpunished cheating.

of which is $x - y$. We note that the weak state unit allows for controlling the correct order of the successive stages in the progress of a bisimulation game.

In fact, our results hold for even a bit more restricted classes fcBPA and fcBPP (see [23] for the definitions of fcBPA and fcBPP) and remain valid for the normed subclasses of fcBPP and fcBPA [12]. Hence, they hold for normed wBPP and normed wBPA as well. Due to the technical nature of the presentation we have demonstrated the results for (unnormed) wBPP and (unnormed) wBPA only.

We recall that the decidability of weak bisimilarity is an open question for BPA and BPP. We note that these problems are conjectured to be decidable (see [14] and [7] respectively) in which case our results would establish a fine undecidability border of weak bisimilarity.

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