Problem Response Theory and its Application for Tutoring

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Problem solving is an important component of education. To make problem solving activities attractive, it is important to confront students with problems of suitable difficulty – neither too easy, nor too difficult. Since students vary in their skills, it is also crucial to make problem recommendations individually adaptive. We present a novel problem response theory, which predicts how much time will a student need to solve a given problem. Our theory is an analogy of the item response theory, but instead of probability of a correct answer we model problem solving time. We introduce a problem solving tutor, which uses the theory to make adaptive predictions and to recommend students problems of suitable difficulty.

1. PROBLEM RESPONSE THEORY

People enjoy the learning process most when facing problems of a challenging difficulty – neither boring, nor frustrating [Csikszentmihalyi 1975]. Our main aim in this paper is to predict a difficulty of problems, more specifically to predict time it will take a person to solve a problem. We aim to do the prediction based on previous data about problem solving activity of this and other persons (as opposed to prediction based on analysis of a problem structure). To this end we propose a "problem response theory", which models a relation between a problem solving ability and a time to solve a problem. The theory is a variation of the standard item response theory [Baker 2001].

Item response theory is used mainly in testing. Main assumption is that a given test measures one latent ability θ, and models give a relation between this ability θ and the probability P that a test item is correctly answered. This relation is expressed by an item response function. The most common model is a 3 parameter logistic model, which has the following parameters: b is a basic difficulty of an item, a is a discrimination factor, and c is a pseudo-guessing parameter (see Fig. 1).

There are many extensions of the basic model, particularly models which take into account response times [Van Der Linden 2009]. But none of these models is directly applicable to the problem solving setting. Therefore, we propose a problem response theory, which models relation between problem solving ability and time to solve a problem.

Similarly to item response theory, we assume that a problem solving performance depends on one latent problem solving ability θ. We are interested in problem response function $f(\theta)$, which for a given ability θ gives an estimate of a time to solve a problem. More specifically, the function gives a probabilistic density of times (see Fig. 1).

To obtain a specific model we make the following two assumptions, which are grounded on data about human problem solving from our previous experiments [Jarušek and Pelánek 2011; Pelánek 2011]. Firstly, the distribution of solving times $f(\theta)$ for persons with a fixed ability θ is a log-normal distribution. Secondly, the mean and variance of the distribution $f(\theta)$ are exponentially dependent on θ.

Our basic model is a 3 parameter model in which the intuitive meaning of the parameters is the following (we intentionally use notation analogical to item response theory): a is a discrimination factor, b is a basic difficulty of a problem, and c is a randomness factor. The problem response function, i.e., the probability density that a person with ability θ will solve a problem with logarithm of time $\ln t$, is given by a normal

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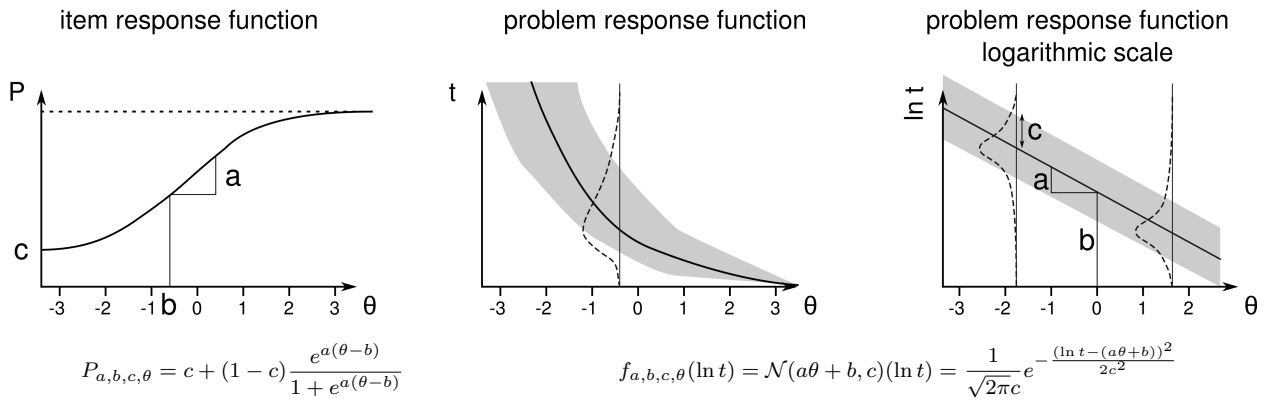

$$P_{a,b,c,\theta} = c + (1-c)\frac{e^{a(\theta-b)}}{1+e^{a(\theta-b)}} \qquad f_{a,b,c,\theta}(\ln t) = \mathcal{N}(a\theta+b,c)(\ln t) = \frac{1}{\sqrt{2\pi}c} e^{-\frac{(\ln t-(a\theta+b))^2}{2c^2}}$$

Fig. 1. Intuitive illustration of item response function, general problem response function, and a specific problem response function under our assumptions. Dashed lines illustrate distributions for certain skill θ; solid lines denotes the median of a time distribution, grey areas depict the area into which most attempts should fall.

distribution with a mean $b + a\theta$ and a variance c^2. This model and intuition behind its parameters are illustrated in Fig. 1.

2. PROBLEM SOLVING TUTOR

Intelligent tutoring systems [Anderson et al. 1985] are computer programs used to make learning process more adaptive and student oriented. We apply our theory in development of a "Problem solving tutor" – a web portal for practicing problem solving skills, which is available at `tutor.fi.muni.cz`. The tutor contains large set of problems of different types (math and programming problems, logic puzzles).

Problem parameters a, b, c and user skills θ are estimated using an iterative computation: problem parameters are computed using estimates of user skills; user skills are improved using estimates of problem parameters (both direction are computed by maximum likelihood estimation); and this process continues until requested precision is reached. Based on these estimates the system predicts problem solving times and recommends a suitable problem to solve. The collected data problem solving data are continuously used to further improve parameters estimates and problem recommendations.

Preliminary evaluation shows that predictions based on the problem response theory bring significant improvement over a baseline prediction algorithm (using mean times). Detailed evaluation will be presented in a future paper.

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