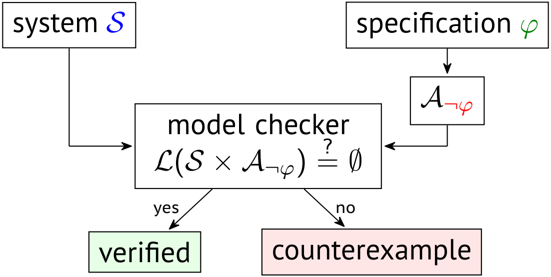
LTL3TELA

Translator of LTL to small ω-automata

Juraj Major, František Blahoudek, Jan Strejček

LTL to ω-automata

- translate an LTL formula to an equivalent ω-automaton
- usage: LTL model checking



- other uses: controller synthesis for reactive systems, ...
- deterministic or nondeterministic ω-automata needed

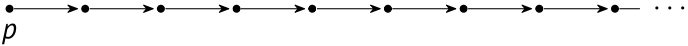
Formulae of Linear Temporal Logic

- $\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \mathsf{X}\varphi \mid \varphi \,\mathsf{U}\, \varphi \mid \varphi \,\mathsf{R}\, \varphi$
  where $p \in AP$ is an *atomic proposition*

Formulae of Linear Temporal Logic

- $\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \mathsf{X}\varphi \mid \varphi \,\mathsf{U}\, \varphi \mid \varphi \,\mathsf{R}\, \varphi$
  where $p \in AP$ is an *atomic proposition*

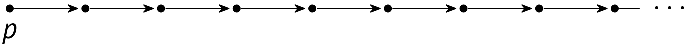
p



Formulae of Linear Temporal Logic

- $\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \mathsf{X}\varphi \mid \varphi \,\mathsf{U}\, \varphi \mid \varphi \,\mathsf{R}\, \varphi$
  where $p \in AP$ is an *atomic proposition*

p



$\neg p \wedge \mathsf{X} q$

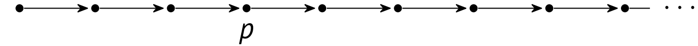
Formulae of Linear Temporal Logic

- $\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \mathsf{X}\varphi \mid \varphi\,\mathsf{U}\,\varphi \mid \varphi\,\mathsf{R}\,\varphi$
  where $p \in AP$ is an *atomic proposition*

p

$\neg p \wedge \mathsf{X}q$

$\mathsf{F}p$



Formulae of Linear Temporal Logic

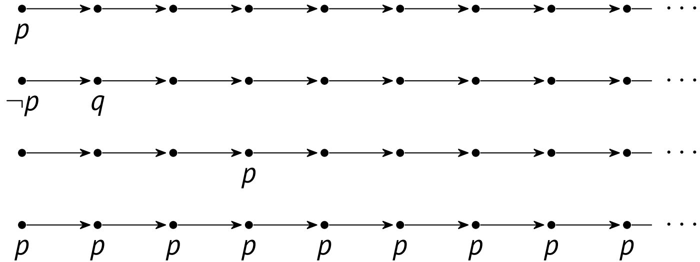
- $\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \mathsf{X}\varphi \mid \varphi\,\mathsf{U}\,\varphi \mid \varphi\,\mathsf{R}\,\varphi$
  where $p \in AP$ is an *atomic proposition*

p

$\neg p \wedge \mathsf{X}q$

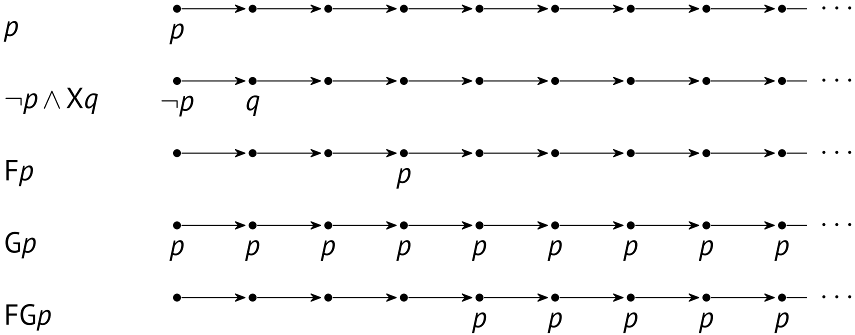
$\mathsf{F}p$

$\mathsf{G}p$



Formulae of Linear Temporal Logic

- $\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \mathsf{X}\varphi \mid \varphi\,\mathsf{U}\,\varphi \mid \varphi\,\mathsf{R}\,\varphi$
  where $p \in AP$ is an *atomic proposition*



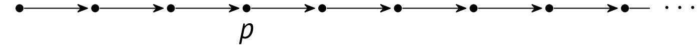
Formulae of Linear Temporal Logic

- $\varphi ::= \mathsf{tt} \mid \mathsf{ff} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \mathsf{X}\varphi \mid \varphi\,\mathsf{U}\,\varphi \mid \varphi\,\mathsf{R}\,\varphi$
  where $p \in AP$ is an *atomic proposition*

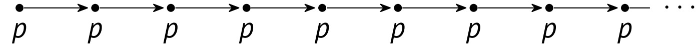
p

$\neg p \wedge \mathsf{X}q$

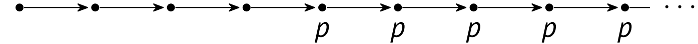
$\mathsf{F}p$



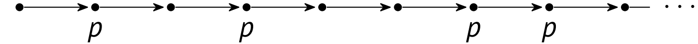
$\mathsf{G}p$



$\mathsf{FG}p$



$\mathsf{GF}p$

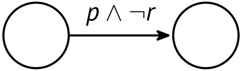


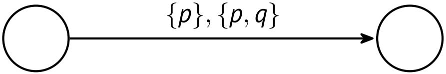
The alphabet

- alphabet $\Sigma = 2^{AP}$ for some finite set AP of atomic propositions
- e.g. $\Sigma = 2^{\{p,q\}} = \{\emptyset, \{p\}, \{q\}, \{p, q\}\}$
- words are infinite sequences of subsets of AP

The alphabet

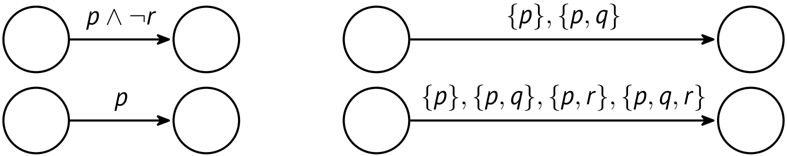
- alphabet $\Sigma = 2^{AP}$ for some finite set AP of atomic propositions
- e.g. $\Sigma = 2^{\{p,q\}} = \{\emptyset, \{p\}, \{q\}, \{p,q\}\}$
- words are infinite sequences of subsets of AP
- multiple letters denoted by the corresponding logical formula
  - an example for $AP = \{p, q, r\}$





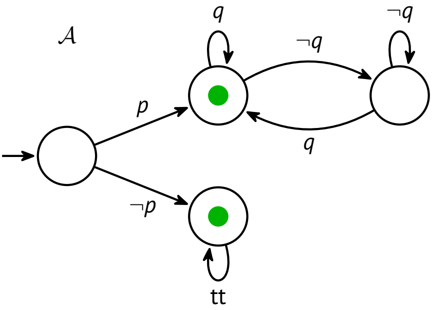
The alphabet

- alphabet $\Sigma = 2^{AP}$ for some finite set AP of atomic propositions
- e.g. $\Sigma = 2^{\{p,q\}} = \{\emptyset, \{p\}, \{q\}, \{p,q\}\}$
- words are infinite sequences of subsets of AP
- multiple letters denoted by the corresponding logical formula
  - an example for $AP = \{p, q, r\}$



Büchi automata

- accept if some accepting state is visited infinitely many times

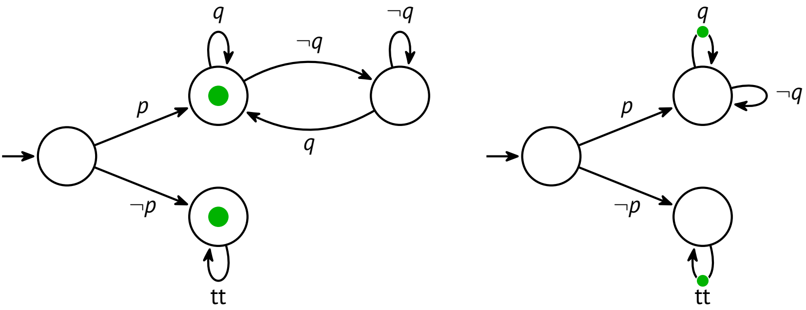


- if p holds in the first letter, then q has to hold infinitely many times
- so $\mathcal{L}(\mathcal{A})$ corresponds to $p \implies \mathsf{G}\mathsf{F}q$

Beyond classical Büchi automata

Transition-based acceptance

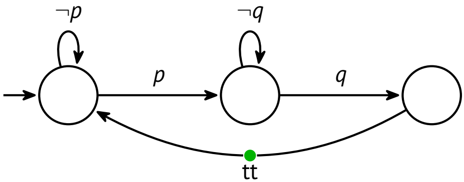
- still $p \implies \mathsf{GF}q$, but now one state smaller

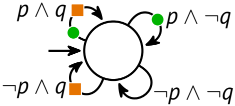


Beyond classical Büchi automata

Generalized Büchi

- visit each accepting set infinitely often
- ω-automaton for $\mathsf{GF}p \wedge \mathsf{GF}q$

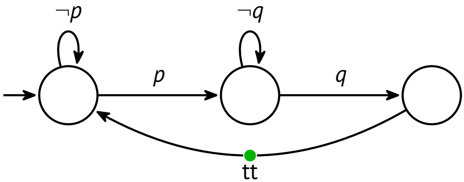


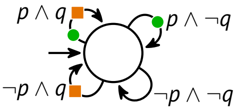


Beyond classical Büchi automata

Generalized Büchi

- visit each accepting set infinitely often
- ω-automaton for G F$p \wedge$ G Fq





- many other acceptance conditions: co-Büchi, Rabin, Streett, …

Emerson-Lei automata

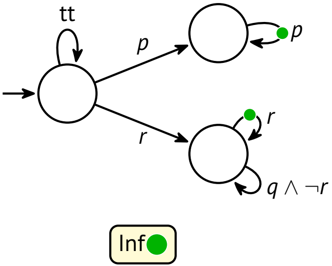
- acceptance formula built of positive boolean combinations of terms Inf● and Fin●
- can express

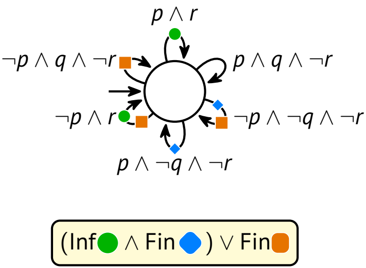
Büchi	Inf●
generalized Büchi	Inf❶ $\land$ Inf❷ $\land$ Inf❸ $\land \ldots$
co-Büchi	Fin●
Rabin	(Fin❶ $\land$ Inf$\boxed{1'}$) $\lor$ (Fin❷ $\land$ Inf$\boxed{2'}$) $\lor \ldots$

- recently reinvented as Hanoi Omega-Automata Format

LTL3TELA

- a new translator of LTL to transition-based Emerson-Lei automata (TELA)
- smaller deterministic or nondeterministic TELA
- example: an automaton for $\mathsf{F}(\mathsf{G}p \vee \mathsf{G}(q\,\mathsf{U}\,r))$ built by Spot (on the left) and LTL3TELA (on the right)





LTL3TELA

The algorithm

- split the formula by logical operators
- translate each subformula several times
  - using novel translation procedure via alternating automata
  - using Spot library
  - apply reductions on both automata
  - determinize if needed
  - try the above for the negation, dualize if deterministic
  - choose the smallest automaton
- combine the separate automata using products

Experimental evaluation

- tested on 1000 randomly generated formulae and 397 formulae from literature and scalable patterns
- compared to state-of-the-art translators
  - deterministic automata: `ltl2tgba` (Spot), Delag, Rabinizer 4
  - nondeterministic automata: `ltl2tgba`, `ltl3ba`

Experimental evaluation

Deterministic automata

	random[1]		literature	
tool	states	acc. marks	states	acc. marks
`ltl3tela -D1`	**5934**	**1268**	**2536**	**454**
`ltl2tgba -DG`	6799	1575	3905	652
Delag	7176	3089	8661	1196
Rabinizer 4	7581	2786	2969	1133

[1] 353 usable formulae

Experimental evaluation

Deterministic automata – cross-comparison

		ltl3tela -D1	ltl2tgba -DG	Delag	Rabinizer
random	`ltl3tela -D1`	–	215	264	232
	`ltl2tgba -DG`	0	–	230	192
	Delag	87	208	–	105
	Rabinizer	86	207	158	–
literature	`ltl3tela -D1`	–	25	72	64
	`ltl2tgba -DG`	0	–	70	67
	Delag	13	24	–	27
	Rabinizer	17	30	54	–

Experimental evaluation

Nondeterministic automata

	random[2]		literature	
tool	states	acc. marks	states	acc. marks
`ltl3tela`	**5109**	1135	**2378**	544
`ltl2tgba -G`	5391	1041	2398	642
`ltl2tgba`	5413	**1034**	2651	**502**
`ltl3ba -H2`	6103	1616	4654	822

[2] 368 usable formulae

Experimental evaluation

Nondeterministic automata – cross-comparison

		ltl3tela	ltl2tgba -G	ltl2tgba	ltl3ba -H2
random	ltl3tela	–	180	213	455
	ltl2tgba -G	0	–	40	350
	ltl2tgba	1	15	–	319
	ltl3ba	1	1	13	–
literature	ltl3tela	–	13	73	160
	ltl2tgba -G	0	–	62	172
	ltl2tgba	0	0	–	151
	ltl3ba	0	0	0	–

More about the topic

Juraj Major, František Blahoudek, Jan Strejček, Miriama Sasaráková, and Tatiana Zbončáková. *LTL3TELA: Small Deterministic or Nondeterministic Automata from LTL*. To appear in ATVA 2019 Proceedings.

František Blahoudek, Juraj Major, and Jan Strejček. *LTL to Smaller Self-Loop Alternating Automata*. To appear in ICTAC 2019 Proceedings.

URL: `https://github.com/jurajmajor/ltl3tela`.