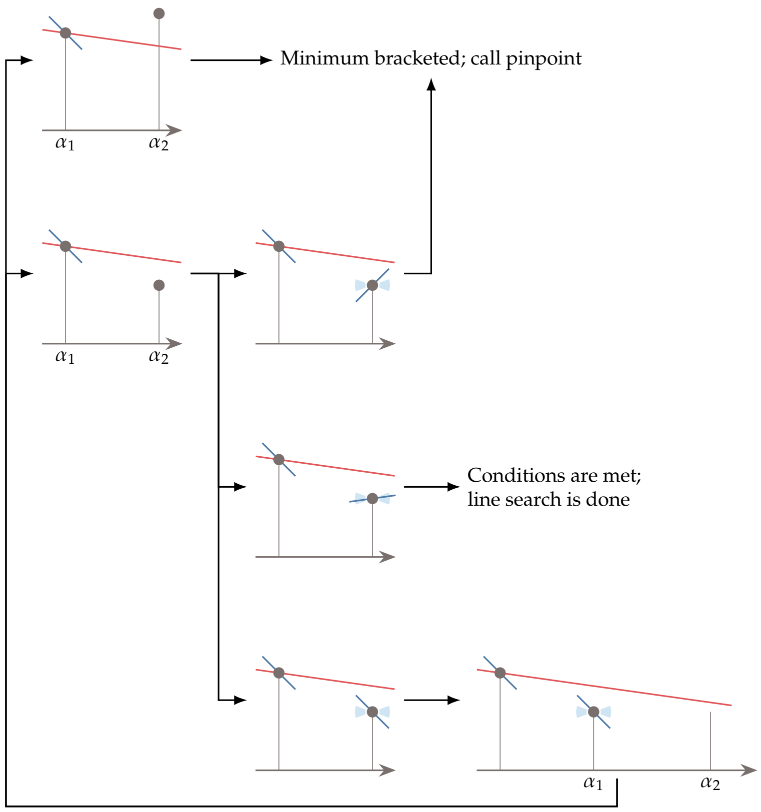

Fig. 4.28 Visual representation of the bracketing algorithm. The sufficient decrease line is drawn as if α_1 were the starting point for the line search, which is the case for the first line search iteration but not necessarily the case for later iterations.

and illustrated in Fig. 4.29. The intervals generated by this algorithm, bounded by α_{low} and α_{high}, always have the following properties:

1. The interval has one or more points that satisfy the strong Wolfe conditions.
2. Among all the points generated so far that satisfy the sufficient decrease condition, α_{low} has the lowest function value.
3. The slope at α_{low} decreases toward α_{high}.

The first step of pinpointing is to find a new point within the given interval. Various techniques can be used to find such a point. The simplest one is to select the midpoint of the interval (bisection), but this method is limited to a linear convergence rate. It is more efficient to perform interpolation and select the point that minimizes the interpolation function, which can be done analytically (see Section 4.3.3). Using this approach, we can achieve quadratic convergence.

Once we have a new point within the interval, four scenarios are possible, as shown in Fig. 4.29. The first scenario is that $\phi\left(\alpha_p\right)$ is above robustness. One of these criteria is to ensure that the new point in the pinpoint algorithm is not so close to an endpoint as to cause the interpolation to be ill-conditioned. A fallback option in case the interpolation fails could be a simpler algorithm, such as bisection. Another criterion is to ensure that the loop does not continue indefinitely in case finite-precision arithmetic leads to indistinguishable function value changes. A limit on the number of iterations might be necessary.

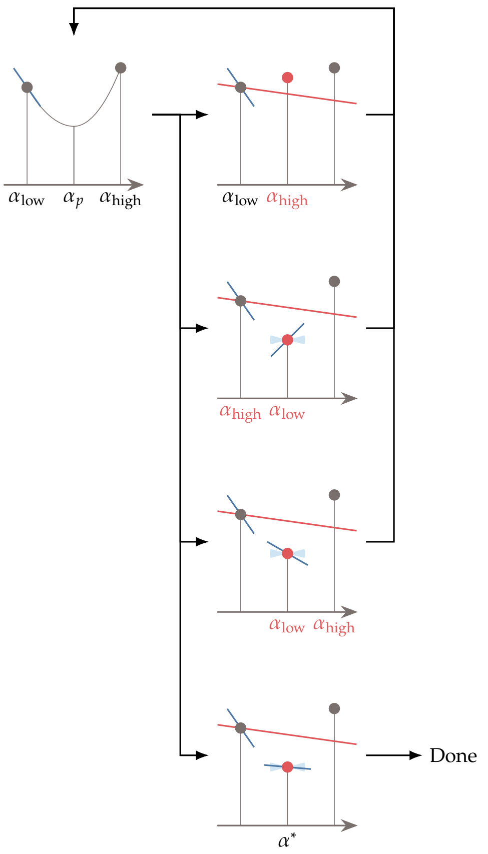

Fig. 4.29 Visual representation of the pinpointing algorithm. The labels in red indicate the new interval endpoints.

Example 4.9 Line search with bracketing and pinpointing

Let us perform the same line search as in Alg. 4.2 but using bracketing and pinpointing instead of backtracking. In this example, we use quadratic interpolation, the pinpointing phase uses a step size increase factor of $\sigma = 2$, and the sufficient curvature factor is $\mu_2 = 0.9$. Bracketing is achieved in the first iteration by using a large initial step of $\alpha_{\text{init}} = 1.2$ (Fig. 4.30, left). Then pinpointing finds an improved point through interpolation. The small initial step of $\alpha_{\text{init}} = 0.05$ (Fig. 4.30, right) does not satisfy the strong Wolfe conditions,