# **Convolutional network**



# **Convolutional networks – architecture**

#### Denote

- X a set of input neurons
- Y a set of output neurons
- Z a set of all neurons  $(X, Y \subseteq Z)$
- individual neurons denoted by indices i, j etc.
  - ξ<sub>j</sub> is the inner potential of the neuron j after the computation stops

▶ *y<sub>j</sub>* is the output of the neuron *j* after the computation stops

(define  $y_0 = 1$  is the value of the formal unit input)

▶ *w<sub>ji</sub>* is the weight of the connection **from** *i* **to** *j* 

(in particular,  $w_{j0}$  is the weight of the connection from the formal unit input, i.e.  $w_{j0} = -b_j$  where  $b_j$  is the bias of the neuron *j*)

- *j*← is a set of all *i* such that *j* is adjacent from *i* (i.e. there is an arc **to** *j* from *i*)
- *j*→ is a set of all *i* such that *j* is adjacent to *i* (i.e. there is an arc **from** *j* to *i*)
- [*ji*] is a set of all connections (i.e. pairs of neurons) sharing the weight w<sub>ji</sub>.

- Visualize weights
- Visualize most "important" inputs for a given class
- Visualize effect of input perturbations on the output
- Construct a local "interpretable" model

# Alex-net - filters of the first convolutional layer



64 filters of depth 3 (RGB).

Combined each filter RGB channels into one RGB image of size 11x11x3.

Assume a trained model giving a score for each class given an input image.

- Denote by y<sub>i</sub>(I) the value of the output neuron i on an input image I.
- Maximize

$$y_i(I) - \lambda \, \|I\|_2^2$$

over all images I.

- A maximum image computed using gradient descent.
- Gives the most "representative" image of the class *c*.

# Maximizing input - example



dumbbell

cup

dalmatian

## Image specific saliency maps

- Let us fix an output neuron *i* and an image  $I_0$ .
- Rank pixels in *l*<sub>0</sub> based on their influence on the value y<sub>i</sub>(*l*<sub>0</sub>).
- Approximate y<sub>i</sub> locally around I<sub>0</sub> with the linear part of the Taylor series:

$$y_i(I) \approx y_i(I_0) + w^T(I - I_0) = w^T I + (y_i(I_0) - w^T I_0)$$

where

$$w = \frac{\delta y_i}{\delta I}(I_0)$$

Heuristics: The magnitude of the derivative indicates which pixels need to be changed the least to affect the score most.

# Saliency maps - example





Quite noisy, the signal is spread and does not tell much about the perception of the owl.



Average several saliency maps of noisy copies of the input.

- Systematically cover parts of the input image.
- Observe the effect on the output value.
- Find regions with the largest effect.

### **Occlusion - example**



#### ['harmonica, mouth organ, harp, mouth harp']





Let us fix an image  $I_0$  to be explained.

Outline:

- Consider superpixels of  $I_0$  as interpretable components.
- Construct a linear model approximating the network aroung the image *l*<sub>0</sub> with weights corresponding to the superpixels.
- Select the superpixels with weights of large magnitude as the important ones.

# Superpixels as explainable components



**Original Image** 



Interpretable Components

Denote by  $P_1, \ldots, P_\ell$  all superpixels of  $I_0$ .

Consider binary vectors  $\vec{x} = (x_1, \dots, x_\ell) \in \{0, 1\}^\ell$ .

Each such vector  $\vec{x}$  determines a "subimage"  $I[\vec{x}]$  of  $I_0$  obtained by removing all  $P_k$  with  $x_k = 0$ .



### LIME

- Let us fix an output neuron *i*, we dnote by y<sub>i</sub>(I) the value of *i* for a given input image *I*.
- Consider the following training set:

$$\mathcal{T} = \{ (\vec{x}_1, y_i(I_0[\vec{x}_1])), \dots, (\vec{x}_p, y_i(I_0[\vec{x}_p])) \}$$

Here  $\vec{x}_h = (x_{h1}, \dots, x_{h\ell})$  are (some) binary vectors of  $\{0, 1\}^{\ell}$ . E.g. randomly selected.

 Train a linear model (ADALINE) with weights w<sub>1</sub>,..., w<sub>l</sub> on *T* minimizing the mean-squared error (+ a regularization term making the number of non-zero weights as small as possible).

Intuitively, the linear model approximates the networks on the

"subimages" of *I* obtained by removing unimportant superpixels.

Inspect the weights (magnitude and sign).

# **LIME - example**



Original Image P(tree frog) = 0.54





Explanation

# **LIME - example**



(a) Original Image

(b) Explaining *Electric guitar* (c) Explaining *Acoustic guitar* 

(d) Explaining Labrador

# LIME - example

