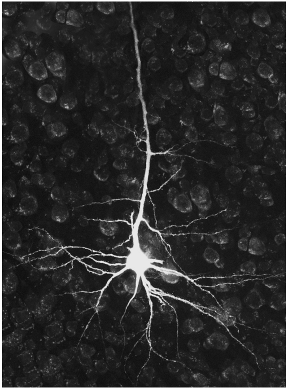
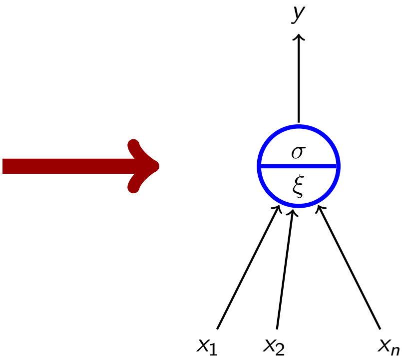
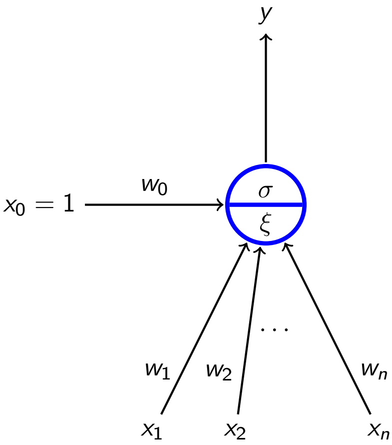
(Primitive) Mathematical Model of Neuron





Formal neuron



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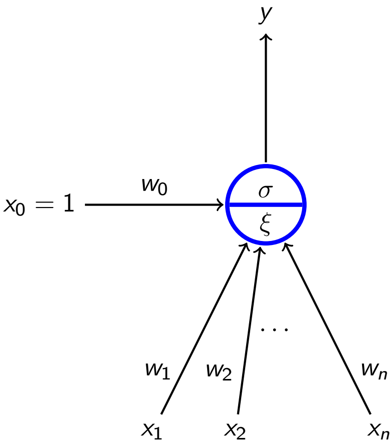

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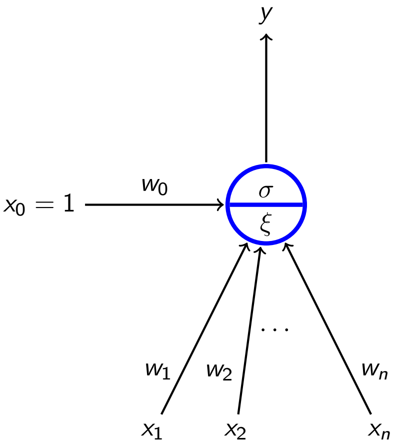

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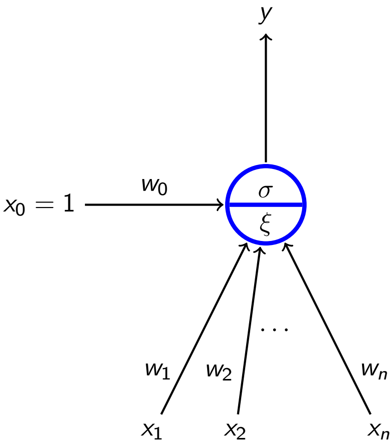

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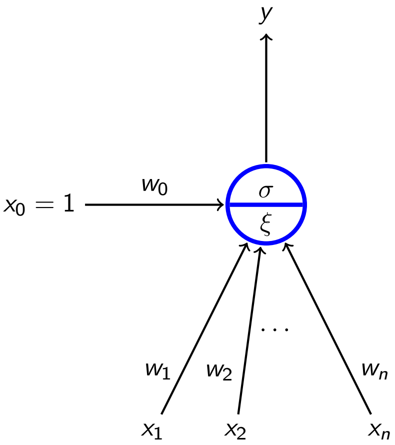

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- y *output* defined by $y = \sigma(\xi)$ where σ is an *activation function*.
  We consider several activation functions.

  e.g., *linear threshold function*

$$\sigma(\xi) = sgn(\xi) = \begin{cases} 1 & \xi \geq 0\,; \\ 0 & \xi < 0. \end{cases}$$

Formal Neuron vs Linear Models

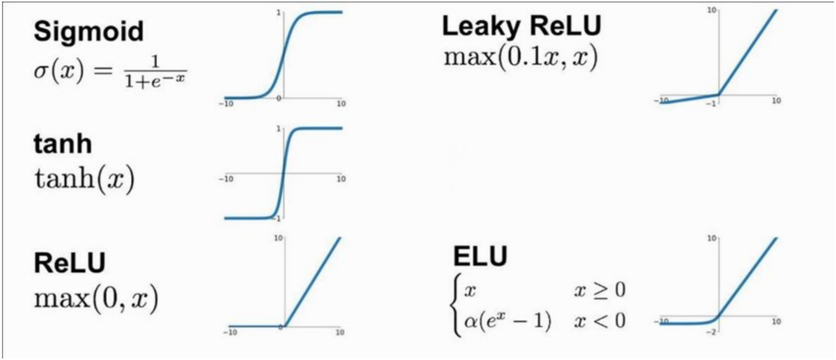
- If σ is a linear threshold function

$$\sigma(\xi) = \begin{cases} 1 & \xi \geq 0\,; \\ 0 & \xi < 0. \end{cases}$$

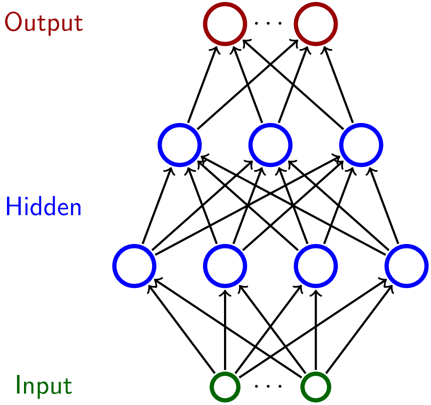
  we obtain a linear classifier.
- If σ is identity, i.e. $\sigma(\xi) = \xi$, we obtain a linear (affine) function.
- If $\sigma(\xi) = 1/(1 + e^{-\xi})$ we obtain the logistic regression.

Also, other activation functions are used in neural networks!

Sigmoid Functions



Multilayer Perceptron (MLP)



- Neurons are organized in *layers*
  (input layer, output layer, possibly several hidden layers)
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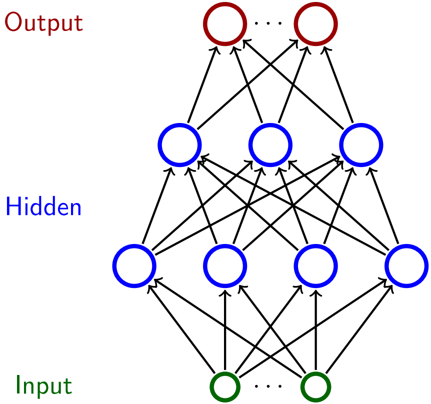

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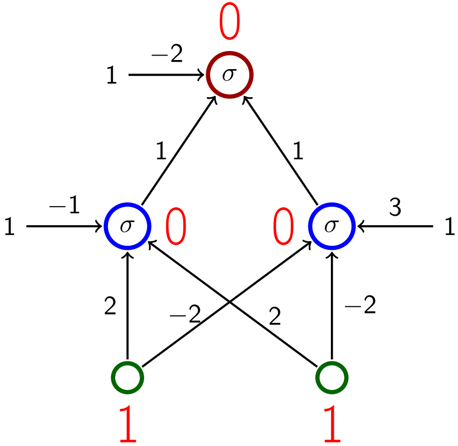
Intuition: The network computes a function as follows: Assign input values to the input neurons and 0 to the rest. Proceed upwards through the layers, one layer per step. In the ℓ-th step consider output values of neurons in $\ell-1$-th layer as inputs to neurons of the ℓ-th layer. Compute output values of neurons in the ℓ-th layer.

Example



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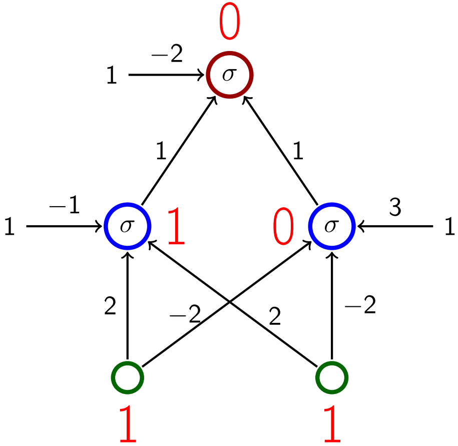

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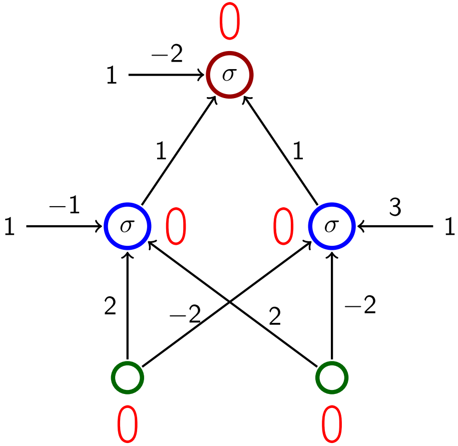

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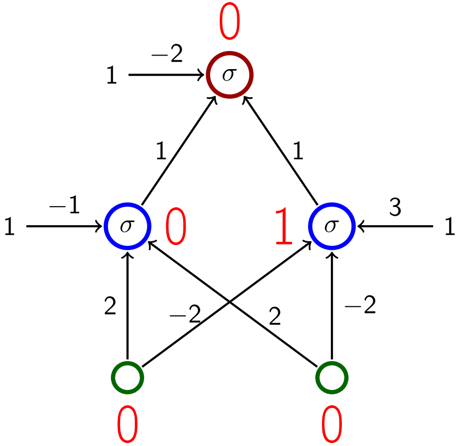

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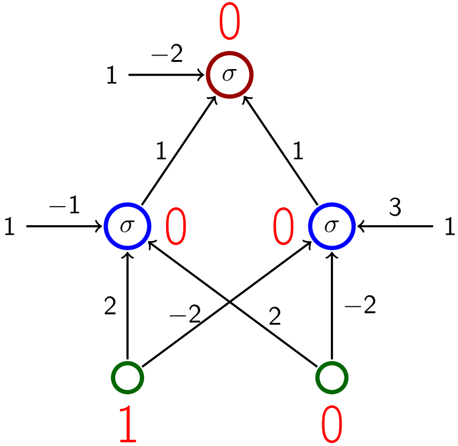

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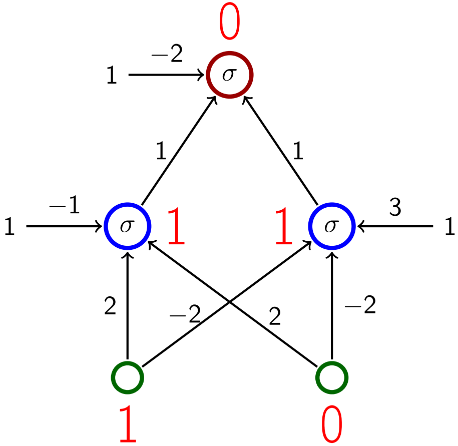

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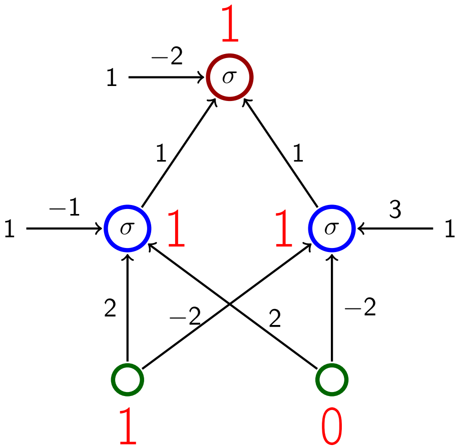

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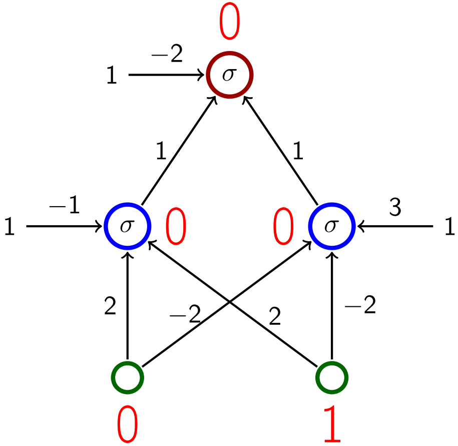

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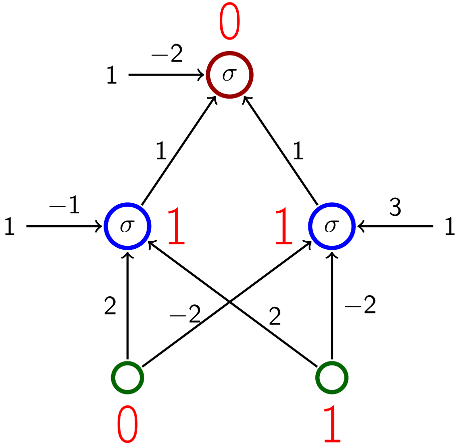

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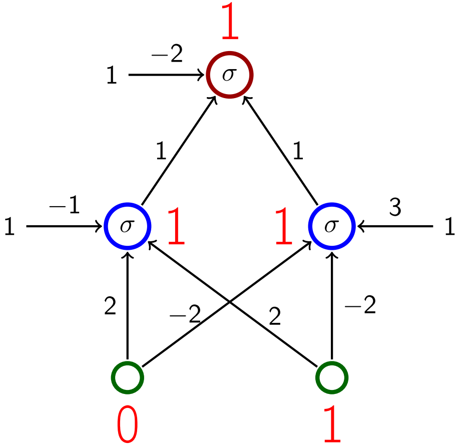

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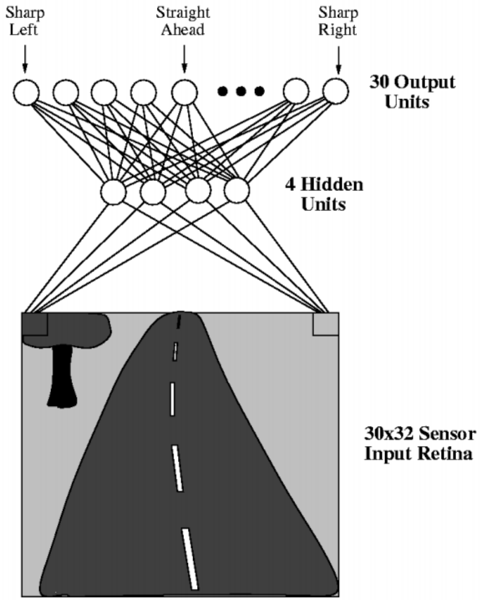
Classical Example – ALVINN

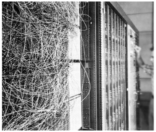


- One of the first autonomous car driving systems (in 90s)
- ALVINN drives a car
- The net has $30 \times 32 = 960$ input neurons (the input space is $\mathbb{R}^{960}$).
- The value of each input captures the shade of gray of the corresponding pixel.
- Output neurons indicate where to turn (to the center of gravity).

Source: `http://jmvidal.cse.sc.edu/talks/ann/alvin.html`

A Bit of History

- Perceptron (Rosenblatt et al, 1957)



- Single layer (i.e. no hidden layers), the activation function *linear threshold*

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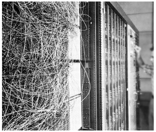

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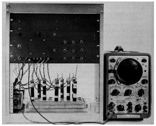
- Adaline (Widrow & Hof, 1960)



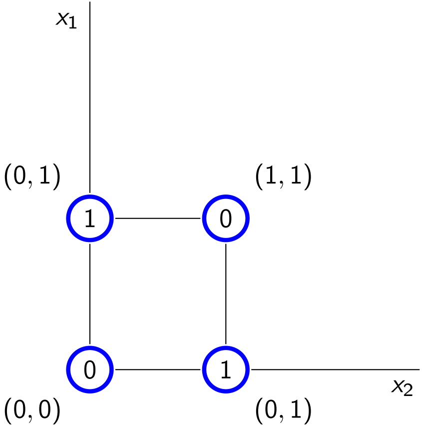
  - Single layer, the activation function *identity*

    (i.e., a bit more linear function)
  - Online version of the gradient descent
  - Used a new circuitry element called *memristor* which was able to "remember" history of current in form of resistance

In both cases, the expressive power is rather limited – can express only linear decision boundaries and linear (affine) functions.

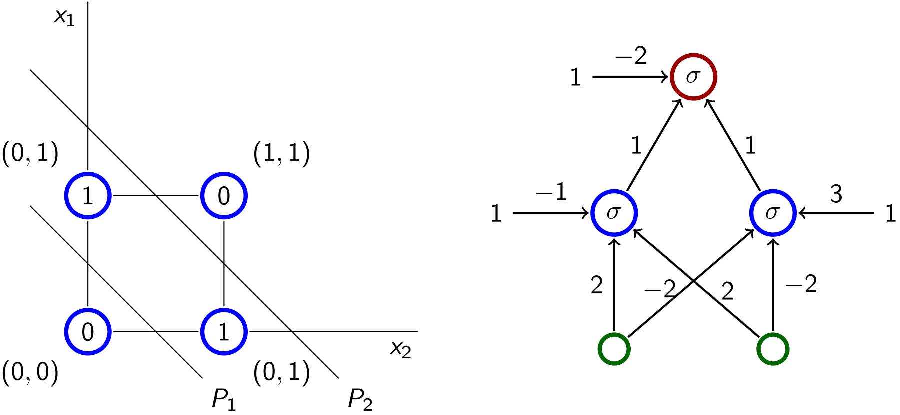
A Bit of History

One of the famous (counter)-examples: XOR



No perceptron can distinguish between ones and zeros.

XOR vs Multilayer Perceptron



(Here σ is a linear threshold function.)

$$P_1 : -1 + 2x_1 + 2x_2 = 0 \qquad P_2 : 3 - 2x_1 - 2x_2 = 0$$

The output neuron performs an intersection of half-spaces.

Boolean functions

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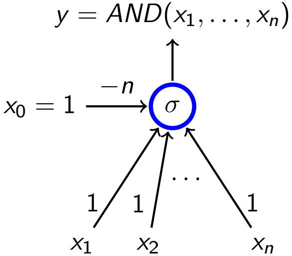

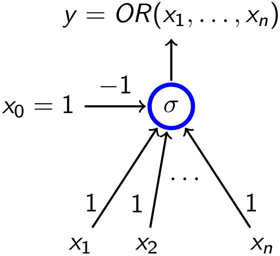

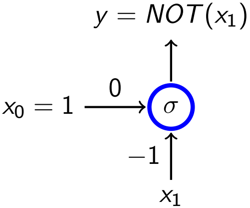

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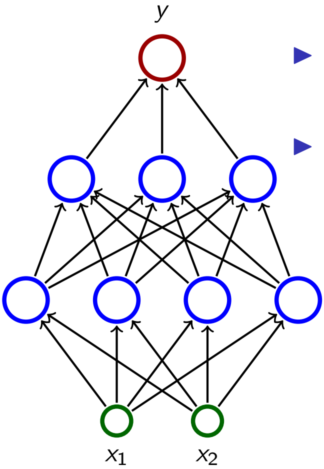

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- The network divides the input space in two subspaces according to the output (0 or 1).

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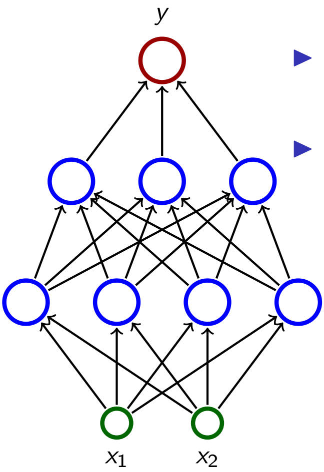

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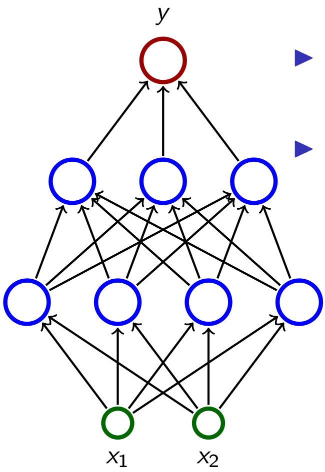

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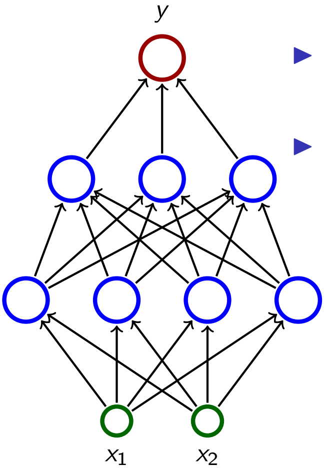

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  - The third layer may, e.g., make unions of some convex sets.

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... then an efficient way of using the gradient descent was published in 1986!

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$$\xi_j = \sum_{i \in j_{\leftarrow}} w_{ji} y_i$$

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- Fixing weights of all neurons, the network computes a function $F[\vec{w}] : \mathbb{R}^{|X|} \to \mathbb{R}^{|Y|}$ as follows: Assign values of a given vector $\vec{x} \in \mathbb{R}^{|X|}$ to the input neurons, evaluate the network, then $F[\vec{w}](\vec{x})$ is the vector of values of the output neurons.

  Here we implicitly assume a fixed orderings on input and output vectors.

MLP – Learning

- Given a set D of training examples:

$$D = \left\{ \left(\vec{x}_k, \vec{d}_k\right) \quad \middle| \quad k = 1, \ldots, p \right\}$$

Here $\vec{x}_k \in \mathbb{R}^{|X|}$ and $\vec{d}_k \in \mathbb{R}^{|Y|}$. We write d_{kj} to denote the value in $\vec{d}_k$ corresponding to the output neuron j.

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- **Error Function:** $E(\vec{w})$ where $\vec{w}$ is a vector of all weights in the network. The choice of E depends on the solved task (classification vs regression etc.).
Example (Squared error):

$$E(\vec{w}) = \sum_{k=1}^{p} E_k(\vec{w})$$

where

$$E_k(\vec{w}) = \frac{1}{2} \sum_{j \in Y} (y_j[\vec{w}](\vec{x}_k) - d_{kj})^2$$

MLP – Batch Gradient Descent

The algorithm computes a sequence of weights $\vec{w}^{(0)}, \vec{w}^{(1)}, \ldots$.

- weights $\vec{w}^{(0)}$ are initialized randomly close to 0

MLP – Batch Gradient Descent

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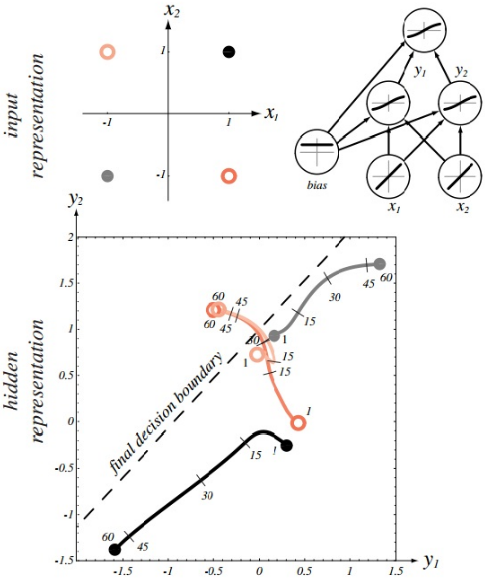
Note that $\frac{\partial E}{\partial w_{ji}}(\vec{w}^{(t)})$ is a component of ∇E, i.e. the weight change in the step $t+1$ can be written as follows: $\vec{w}^{(t+1)} = \vec{w}^{(t)} - \varepsilon(t) \cdot \nabla E(\vec{w}^{(t)})$.

Illustration of Gradient Descent – XOR



Source: Pattern Classification (2nd Edition); Richard O. Duda, Peter E. Hart, David G. Stork

Stochastic Gradient Descent (SGD)

Assume that $E(\vec{w}) = \sum_{k=1}^{p} E_k(\vec{w})$ where $E_k(\vec{w})$ is an error w.r.t. the single training example $(\vec{x}_k, \vec{d}_k)$.

- weights in $\vec{w}^{(0)}$ are randomly initialized to values close to 0
- in the step $t+1$ (here $t = 0, 1, 2 \ldots$), weights $\vec{w}^{(t+1)}$ are computed as follows:
  - Choose (randomly) a set of training examples $T \subseteq \{1, \ldots, p\}$
  - Compute

    $$\vec{w}^{(t+1)} = \vec{w}^{(t)} + \Delta\vec{w}^{(t)}$$

    where

    $$\Delta\vec{w}^{(t)} = -\varepsilon(t) \cdot \sum_{k \in T} \nabla E_k(\vec{w}^{(t)})$$

- $0 < \varepsilon(t) \leq 1$ is a *learning rate* in step $t+1$
- $\nabla E_k(\vec{w}^{(t)})$ is the gradient of the error of the example k

Note that the random choice of the minibatch is typically implemented by randomly shuffling all data and then choosing minibatches sequentially.

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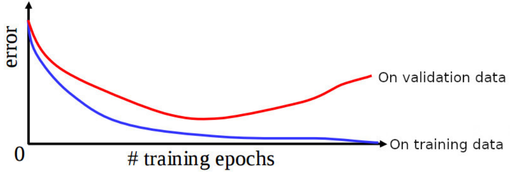
- Many epochs (thousands) may be required, hours or days of training for large networks.

There are many issues concerning learning efficiency (data normalization, selection of activation functions, weight initialization, learning rate, efficiency of the gradient descent itself etc.) – see PV021.

Overfitting

- Due to their expressive power, neural networks are quite sensitive to overfitting.



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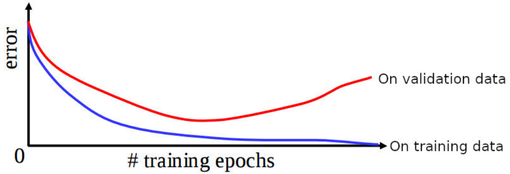

- Keep a hold-out validation set and test the error of the network on this set after every epoch. Stop training when additional epochs actually increase the validation error.
  The validation error can be measured by completely different means than the training error E.

Hidden Neurons Representations

Trained hidden neurons can be seen as newly constructed features.
E.g., in a two layer network used for classification, the hidden layer transforms the input so that important features become explicit (and hence the result may become linearly separable).

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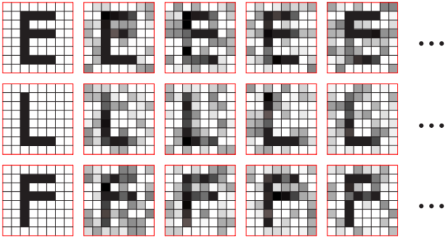
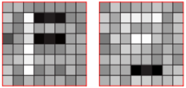
Consider a two-layer MLP, 64-2-3 for classification of letters (three output neurons, each corresponds to one of the letters).

sample training patterns





learned input-to-hidden weights

Optimal Architecture?

- For MLP: Too few hidden neurons prevent the network from adequately fitting the data. Too many hidden units can result in overfitting.

  (There are advanced methods that prevent overfitting even for rich models, such as regularization, where the error function penalizes overfitting – see PV021.)

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- There are (almost) infinitely many types of architectures of neural networks (convolutional, recurrent, transformers, adversarial, etc.) suitable for various tasks.
- **Transfer learning:** Start with a known solution to a related problem.

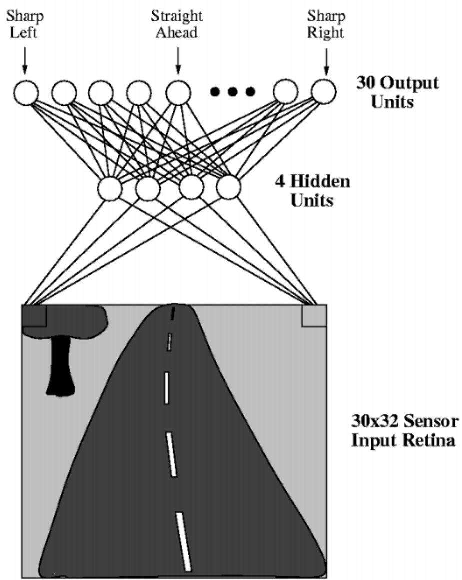
  Simplified view: Preserve lower parts of the network trained to solve the related problem (feature extractors). Add your own top part and then train only the new top part (or train the whole network but carefully).

Applications

- Image recognition, segmentation, etc.
- Machine translation and other text processing
- Text to Speech and vice versa
- Finance, business predictions, fraud detection
- Game playing (backgammon is a classical example, AlphaGo is the famous one, computer games, **bridge**)
- (artificial brain and intelligence)
- ...

Text and image processing are possibly the most advanced deep learning applications.

ALVINN


Sharp
Left
Straight
Ahead
Sharp
Right
30 Output
Units
4 Hidden
Units
30x32 Sensor
Input Retina

ALVINN

- Two layer MLP, $960 - 4 - 30$ (sometimes $960 - 5 - 30$)

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- Inputs correspond to pixels.
- Sigmoidal activation function (logistic sigmoid).
- Direction corresponds to the center of gravity.

  I.e., output neurons are considered as points of mass evenly distributed along a line. Weight of each neuron corresponds to its value.

ALVINN – Training

Trained while driving.

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- the values $\vec{d}_k$ computed using Gaussian distribution:

$$d_{ki} = e^{-D_i^2/10}$$

where D_i is the distance between the i-th output from the one that corresponds to the real direction of the steering wheel.

(This is better than the binary output because similar road directions induce similar reaction of the driver.)

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Naive approach: just take images from the camera.

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- Images are very similar (the network basically sees the road from the right lane), overfitting.

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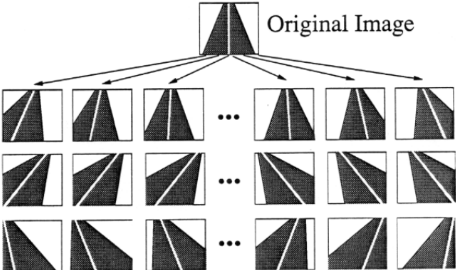
- every image of the road has been has been transformed to 15 slightly different copies



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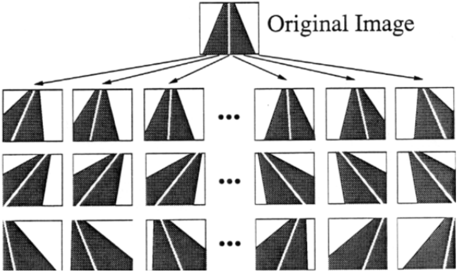

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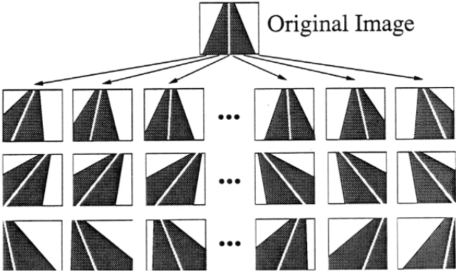

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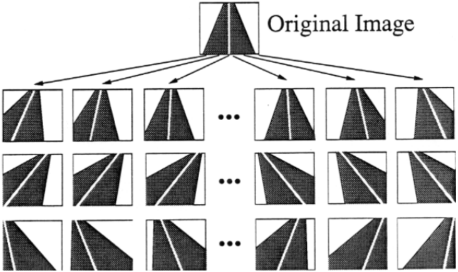

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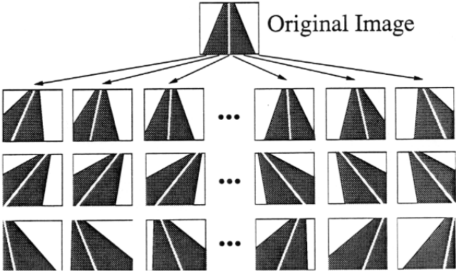

Repetitiveness of images was solved as follows:

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- afterwards, a new image is captured, 15 copies made, and these new 15 substitute 15 selected from the buffer (10 with the smallest training error, 5 randomly)

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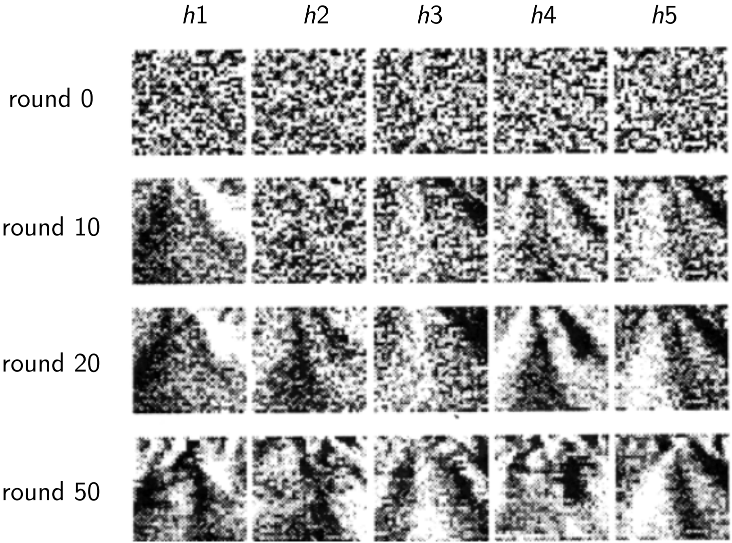
  (The speed was limited by the hydraulic controller of the steering wheel not the learning algorithm.)
- ALVINN was able to go through roads it never "seen" and in different weather

ALVINN – Weight Learning



Here $h1, \ldots, h5$ are values of hidden neurons.

Deep Learning

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- In 2006 a solution was found by Hinton et al:
  - Use *unsupervised* methods to initialize the weights layer by layer so that they capture important features in data.
    More precisely: The lowest hidden layer learns patterns in data, second lowest learns patterns in data transformed through the first layer, and so on.

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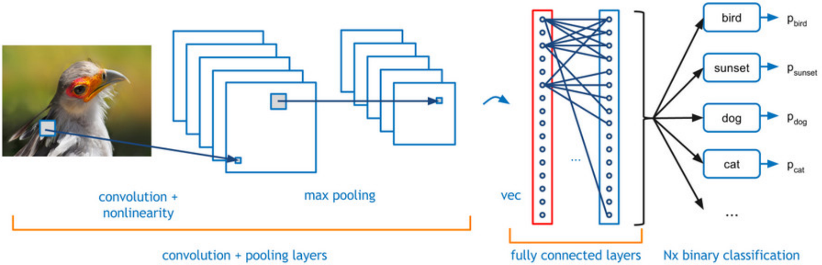
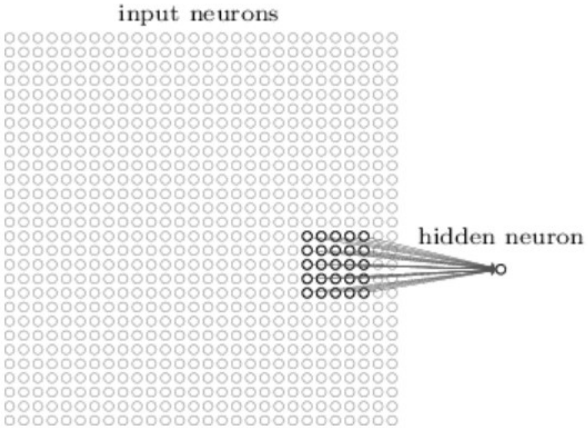
    More precisely: The lowest hidden layer learns patterns in data, second lowest learns patterns in data transformed through the first layer, and so on.
  - Then use a supervised learning algorithm to only *fine tune* the weights to the desired input-output behavior.
- ... but the true revolution started with convolutional networks trained on several GPUs.

Convolutional network

A specific architecture of neural networks from 80s.



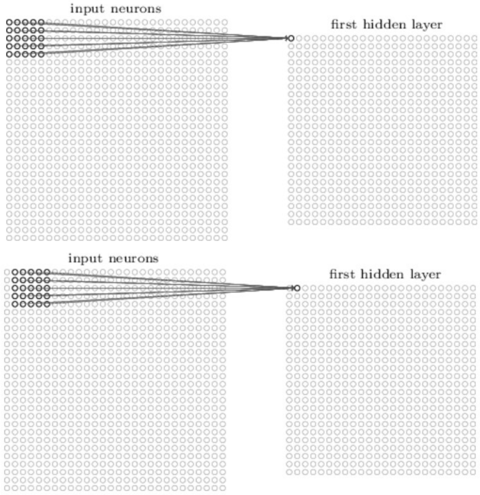
Convolutional layers



Every neuron is connected with a (typically small) *receptive field* of neurons in the lower layer.

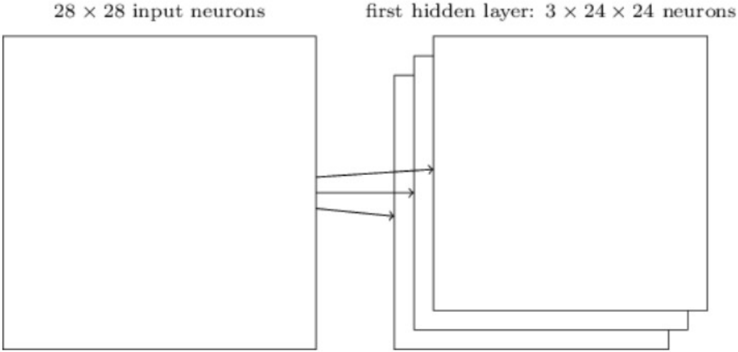
Neuron is "standard": Computes a weighted sum of its inputs, applies an activation function.

Convolutional layers



Neurons grouped into *feature maps* sharing weights.

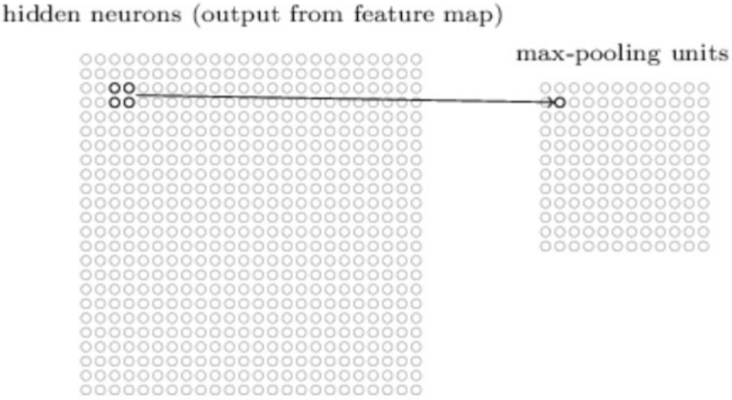
Convolutional layers



Each feature map represents a property of the input that is supposed to be spatially invariant.

Typically, we consider several feature maps in a single layer.

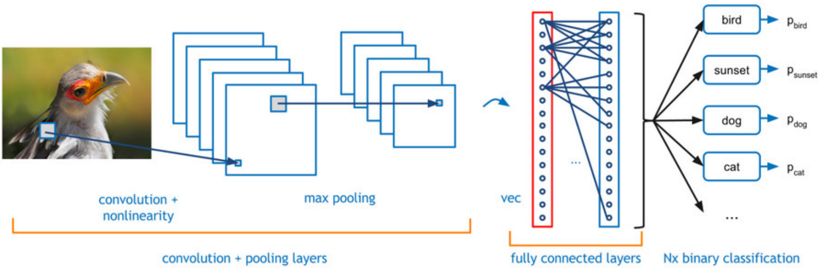
Pooling layers



Neurons in the pooling layer compute simple functions of their receptive fields (the fields are typically disjoint):

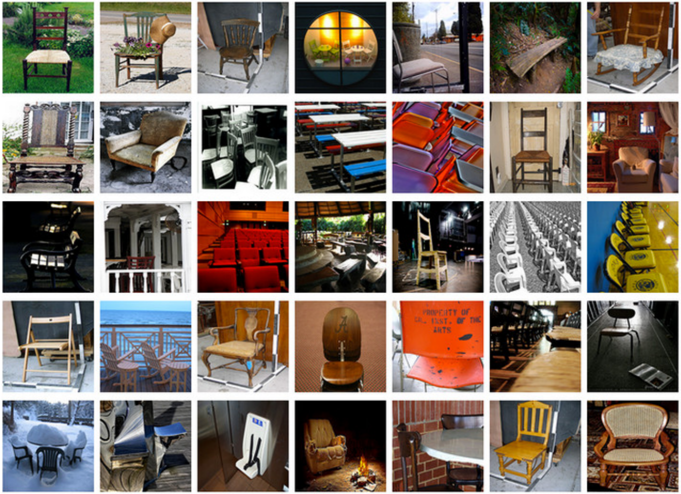
- **Max-pooling** : maximum of inputs
- **L2-pooling** : square root of the sum of squres
- **Average-pooling** : mean
- ...

Convolutional network



ImageNet Large-Scale Visual Recognition Challenge (ILSVRC)

ImageNet database (16,000,000 color images, 20,000 categories)



ImageNet Large-Scale Visual Recognition Challenge (ILSVRC)

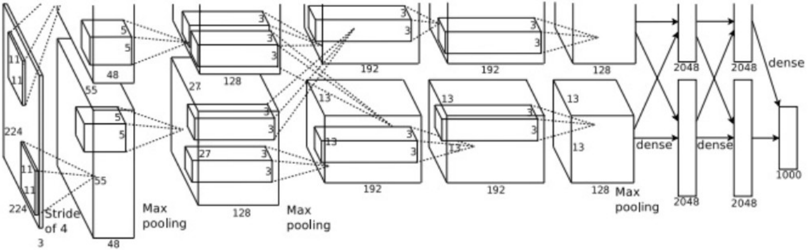
Competition in classification over a subset of images from ImageNet.

In 2012: Training se 1,200,000 images, 1000 categories. Validation set 50,000, Test set 150,000.

Many images contain several objects $\rightarrow$ typical rule is top-5 highest probability assigned by the net.

KSH síť

ImageNet classification with deep convolutional neural networks, by Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton (2012).



Trained on two GPUs (NVIDIA GeForce GTX 580)

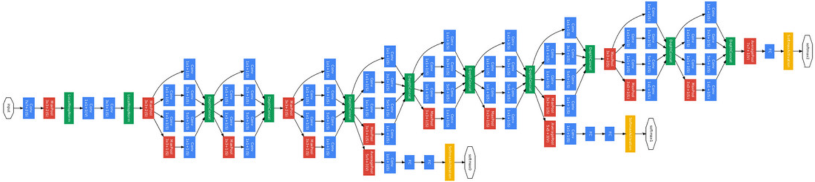
Results:

- Accuracy 84.7% in top-5 (second best alg. at the time: 73.8%)
- 63.3% in "perfect" classification (top-1)

ILSVRC 2014

The same set of images as in 2012, top-5 criterium.

GoogLeNet: deep convolutional net, 22 layers



Results:

- 93.33% in top-5

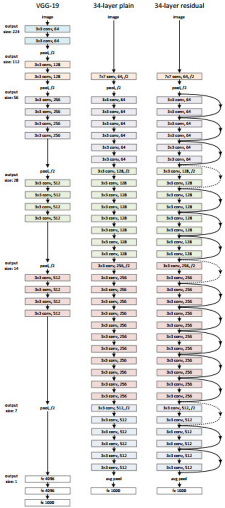
Superhuman power?

Superhuman GoogLeNet?!

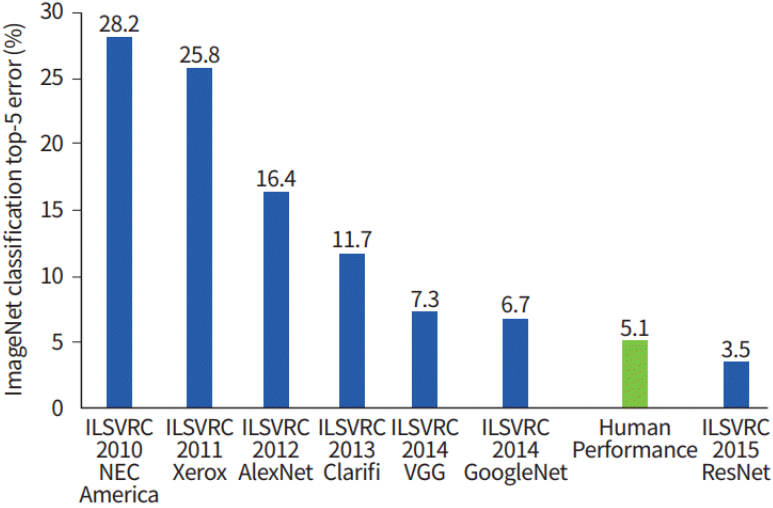
Andrej Karpathy: ...the task of labeling images with 5 out of 1000 categories quickly turned out to be extremely challenging, even for some friends in the lab who have been working on ILSVRC and its classes for a while. First we thought we would put it up on [Amazon Mechanical Turk]. Then we thought we could recruit paid undergrads. Then I organized a labeling party of intense labeling effort only among the (expert labelers) in our lab. Then I developed a modified interface that used GoogLeNet predictions to prune the number of categories from 1000 to only about 100. It was still too hard - people kept missing categories and getting up to ranges of 13-15% error rates. In the end I realized that to get anywhere competitively close to GoogLeNet, it was most efficient if I sat down and went through the painfully long training process and the subsequent careful annotation process myself... The labeling happened at a rate of about 1 per minute, but this decreased over time... Some images are easily recognized, while some images (such as those of fine-grained breeds of dogs, birds, or monkeys) can require multiple minutes of concentrated effort. I became very good at identifying breeds of dogs... Based on the sample of images I worked on, the GoogLeNet classification error turned out to be 6.8%... My own error in the end turned out to be 5.1%, approximately 1.7% better.

ILSVRC 2015

- Microsoft network ResNet: 152 layers, complex architecture
- Trained on 8 GPUs
- 96.43% **accuracy** in top-5



ILSVRC



ImageNet classification top-5 error (%)
30
25
20
15
10
5
0
28.2
25.8
16.4
11.7
7.3
6.7
5.1
3.5
ILSVRC 2010 NEC America
ILSVRC 2011 Xerox
ILSVRC 2012 AlexNet
ILSVRC 2013 Clarifi
ILSVRC 2014 VGG
ILSVRC 2014 GoogleNet
Human Performance
ILSVRC 2015 ResNet

ILSVRC 2016

Trimps-Soushen (The Third Research Institute of Ministry of Public Security)

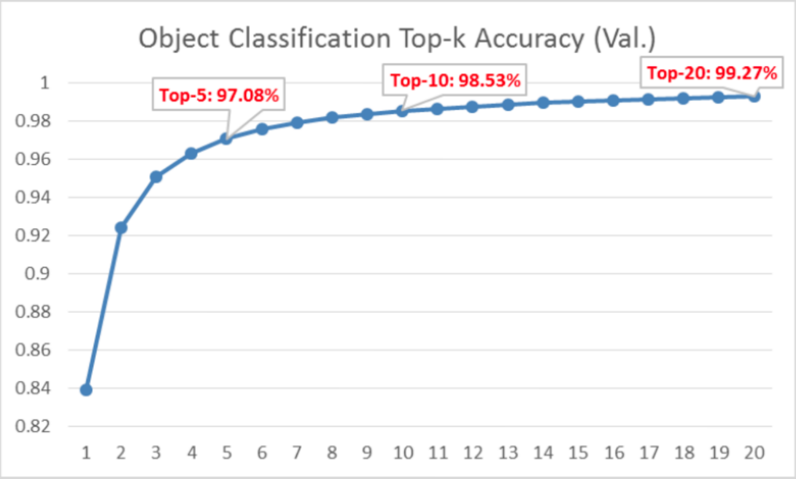
There is no new innovative technology or novelty by Trimps-Soushen.

Ensemble of the pre-trained models from previous years.

Each of the models are strong at classifying some categories, but also weak at classifying some categories.

Test error: 2.99%

Top-k accuracy analyzed



https://towardsdatascience.com/review-trimps-soushen-winner-in-ilsvrc-2016-image-classification-dfbc423111dd

Top-20 typical errors

Out of 1458 misclassified images in Top-20:

Error Categories	Numbers	Percentages(%)
Label May Wrong	221	15.16
Multiple Objects (>5)	118	8.09
Non-Obvious Main Object	355	24.35
Confusing Label	206	14.13
Fine-grained Label	258	17.70
Obvious Wrong	234	16.05
Partial Object	66	4.53

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Top-k accuracy analyzed

Predict:
1 *pencil box*
2 *diaper*
3 *bib*
4 *purse*
5 *running shoe*

Ground Truth:
sleeping bag

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Top-k accuracy analyzed

Predict:
1 *dock*
2 *submarine*
3 *boathouse*
4 *breakwater*
5 *lifeboat*

Ground Truth:
paper towel

https://towardsdatascience.com/review-trimps-soushen-winner-in-ilsvrc-2016-image-classification-

Top-k accuracy analyzed

Predict:

1 *bolete*

2 *earthstar*

3 *gyromitra*

4 *hen of the woods*

5 *mushroom*

Ground Truth:

stinkhorn

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Top-k accuracy analyzed

Predict:
1 *apron*
2 *plastic bag*
3 *sleeping bag*
4 *umbrella*
5 *bulletproof vest*

Ground Truth:
poncho

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Note that $\frac{\partial E}{\partial w_{ji}}(\vec{w}^{(t)})$ is a component of ∇E, i.e. the weight change in the step $t+1$ can be written as follows: $\vec{w}^{(t+1)} = \vec{w}^{(t)} - \varepsilon(t) \cdot \nabla E(\vec{w}^{(t)})$.

MLP – Gradient Computation

For every weight w_{ji} we have (obviously)

$$\frac{\partial E}{\partial w_{ji}} = \sum_{k=1}^{p} \frac{\partial E_k}{\partial w_{ji}}$$

So now it suffices to compute $\frac{\partial E_k}{\partial w_{ji}}$, that is the error for a fixed training example $(\vec{x}_k, d_k)$.

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Applying the chain rule we obtain

$$\frac{\partial E_k}{\partial w_{ji}} = \frac{\partial E_k}{\partial y_j} \cdot \sigma_j'(\xi_j) \cdot y_i$$

where (more applications of the chain rule)

$\frac{\partial E_k}{\partial y_j}$ is computed directly for the output neurons $j \in Y$

$$\frac{\partial E_k}{\partial y_j} = \sum_{r \in j^{\rightarrow}} \frac{\partial E_k}{\partial y_r} \cdot \sigma_r'(\xi_r) \cdot w_{rj} \qquad \text{for } j \in Z \smallsetminus (Y \cup X)$$

(Here $y_r = y[\vec{w}](\vec{x}_k)$ where $\vec{w}$ are the current weights and $\vec{x}_k$ is the input of the k-th training example.)

Multilayer Perceptron – Backpropagation

Input: A training set $D = \left\{ \left(\vec{x}_k, \vec{d}_k\right) \quad \middle| \quad k = 1, \ldots, p \right\}$ and the current vector of weights $\vec{w}$.

Note that the backprop. is repeated in every iteration of the gradient descent!

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- Evaluate all values y_i of neurons using the standard bottom-up procedure with the input $\vec{x}_k$.
- For every training example $(\vec{x}_k, \vec{d}_k)$ compute $\frac{\partial E_k}{\partial y_j}$ using *backpropagation* through layers top-down :
  - For all $j \in Y$ compute $\frac{\partial E_k}{\partial y_j}$ by taking the derivative of the error.

    e.g., in the case of the squared error we have $\frac{\partial E_k}{\partial y_j} = y_j - d_{kj}$.

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  - In the layer ℓ, assuming that $\frac{\partial E_k}{\partial y_r}$ has been computed for all neurons r in the layer $\ell + 1$, compute
    $$\frac{\partial E_k}{\partial y_j} = \sum_{r \in j^{\rightarrow}} \frac{\partial E_k}{\partial y_j} \cdot \sigma_r'(\xi_r) \cdot w_{rj}$$
    for all j from the ℓ-th layer. Here σ_r' is the derivative of σ_r.
  - Put $\frac{\partial E_k}{\partial w_{ji}} = \frac{\partial E_k}{\partial y_j} \cdot \sigma_j'(\xi_j) \cdot y_i$

Output: $\frac{\partial E}{\partial w_{ji}} = \sum_{k=1}^{p} \frac{\partial E_k}{\partial w_{ji}}$.