Propositional logic

- propositional letters, propositions
- connectives $\land$, $\lor$, $\rightarrow$, $\leftrightarrow$, $\neg$
- truth tables
- adequacy, adequate set of truth connectives
- truth assignment
  - assigns to each propositional letter a unique truth value
- truth valuation
  - assigns to each proposition a unique truth value
- valid proposition, tautology
- logically equivalent propositions
Propositional logic: axiomatic approach

- axioms \((A, B, C = \text{formulas})\):
  \(A_1\) \(A \Rightarrow (B \Rightarrow A)\)
  \(A_2\) \((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))\)
  \(A_3\) \((\neg B \Rightarrow \neg A) \Rightarrow (A \Rightarrow B)\)

- inference rule *modus ponens* (MP)

\[
\begin{array}{c|c|c}
A & A \Rightarrow B & B \\
\hline
\end{array}
\]
Resolution in propositional logic – example

\[ S \vdash_R \square ? \]

\[ S = (p \lor r) \land (q \leftrightarrow r) \land \neg q \land (t \leftrightarrow p) \land \neg s \land (s \leftrightarrow t) \]

\[ S = (p \lor r) \land (q \lor \neg r) \land \neg q \land (\neg p \lor t) \land \neg s \land (s \lor \neg t) \]

\[ S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\} \]

\[
\begin{array}{cccc}
\{p, r\} & \{q, \neg r\} & \{\neg p, t\} & \{s, \neg t\} \\
\{p, q\} & \{\neg q\} & \{\neg s\} & \{\neg p, s\} \\
\{p\} & \{\neg p\} & & \\
\end{array}
\]
Refining resolution I

- to narrow search space - SAT = \{ S \mid S \text{ is satisfiable} \} is NP-complete
  - to terminate the search along paths that are unpromising
  - to specify the order in which to go down alternative paths
Refining resolution II

- if there is a literal that is only positive(negative), remove all clauses that contain such a literal

- **T-resolution**: no parent clause is a tautology

- **Semantic resolution**. Let $\mathcal{I}$ be an interpretation. Semantic resolution with respect to $\mathcal{I}$ permits applications of the resolution rule only when at least one of their premises has a ground instance which is not satisfied by $\mathcal{I}$

- **Ordered resolution**. The propositional letters are indexed resolve on the literal with the higher index than any other in the parent clauses
• *Lock resolution.* Each occurrence of a literal has a distinct index the literal resolved on has in each parent the lowest index