MA010 Graph Theory—Homework Set #1

Each problem is worth 4 points. The solutions should be submitted using the university information system by **November 14**. Please submit your solutions as pdf files produced by a suitable text editor, e.g., LaTeX; solutions that are not submitted as pdf files may be assigned zero points. Your solutions should contain references to all sources, including those available on the web, that you have used.

1. Let G be an orientation of a connected graph with at least two vertices such that every vertex v of G satisfies that its in-degree and out-degree are the same. Show that G has a closed (directed) walk containing each edge exactly once.

2. Show that an n-vertex graph G is a tree if and only if G has no cycle and has $n-1$ edges.

3. Show that every quadrangulation of the plane is 2-colorable (recall that a quadrangulation is a plane graph such that each face is bounded by a cycle of length four).

4. Show that for every positive integer n, there exists a positive integer N such that every coloring of the edges of $K_{N,N}$ with two colors contains a monochromatic copy of $K_{n,n}$.