MA010 Graph Theory—Homework Set #1

Each problem is worth 3 points. The solutions should be submitted using the university information system by **October 14**. Please submit your solutions as pdf files produced by a suitable text editor, e.g., LaTeX. Your solution should contain references to all sources, including those available on the web, that you have used.

1. Let G be an orientation of a connected graph with at least two vertices such that every vertex v of G satisfies that its in-degree and out-degree is the same. Show that G has a closed (directed) walk containing each edge exactly once.

2. Show that an n-vertex graph G is a tree if and only if G has no cycle and has $n-1$ edges.

3. Let G be an orientation of a complete graph with $n \geq 3$ vertices. Assume that the vertex set of G cannot be partitioned into two sets A and B such that all edges between the sets A and B are directed from A to B. Show that G contains a (directed) Hamilton cycle, i.e., a spanning cyclically oriented cycle.

4. Show that a planar graph with $n \geq 3$ vertices that does not contain C_3 as a subgraph has at most $2n-4$ edges.