Erratum to "On the structure of graphs in the Caucal hierarchy"

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Recently Pawel Parys [3] pointed out an error in the article "On the structure of graphs in the Caucal hierarchy" [1], which contains two main results: Theorems 15 and 61. Theorem 61 is a pumping lemma for higher-order pushdown automata. The proof consists in two parts: (1) a series of technical lemmas that, given a run of the automaton containing a so-called *pumping pair,* constructs a longer run and (2) a proof that every sufficiently long run contains a pumping pair. The error found by Parys is in the proof of (2). He presented a counterexample [2] invalidating the following results of [1]: all the material from Lemma 50 to Corollary 55 in Section 8 and Lemma 60 in Section 9. This counterexample uses an automaton $\mathcal{A}$ of level 3 with an unary stack alphabet $\{a\}$ that after performing the operations

$$\mathrm{push}_a,\ \mathrm{push}_a$$

indefinitely repeats the following sequence of operations:

$$\mathrm{clone}_2,\ \mathrm{clone}_3,\ \mathrm{pop}_1,\ \mathrm{clone}_3,\ \mathrm{pop}_2,\ \mathrm{clone}_3\,.$$

Consider the following words of level 2

$$\alpha := \varepsilon : a : a\,, \quad \beta := aa : \varepsilon : a\,, \quad \gamma := aa : a : a\,.$$

The sequence of stack contents of the run of $\mathcal{A}$ is shown in Figure 1. In general, after an initial segment the run consists of pieces of the form

$$(\gamma\beta\alpha)^n : \alpha$$
$$(\gamma\beta\alpha)^n : \gamma$$
$$(\gamma\beta\alpha)^n\gamma : \gamma$$
$$(\gamma\beta\alpha)^n\gamma : \beta$$
$$(\gamma\beta\alpha)^n\gamma\beta : \beta$$
$$(\gamma\beta\alpha)^n\gamma\beta : \alpha$$

for every $n \in \mathbb{N}$. Such a run does not contain a pumping pair.

$$\begin{array}{l}
\varepsilon \\
\varepsilon : \varepsilon : \varepsilon : a \\
\varepsilon : \alpha \\
\varepsilon : \gamma \\
\gamma : \gamma \\
\gamma : \beta \\
\gamma\beta : \beta \\
\gamma\beta : \alpha \\
\gamma\beta\alpha : \alpha \\
\gamma\beta\alpha : \gamma \\
\gamma\beta\alpha\gamma : \gamma \\
\quad \vdots
\end{array}$$

Figure 1. The run of the automaton $\mathcal{A}$

Since the proof of Theorem 61 relies on Lemma 60, the statement of the pumping lemma has to be considered as open. Note that the paper of Parys [3] also contains a pumping lemma for higher-order pushdown automata. On the one hand, this version is even stronger than Theorem 61 since it gives better bounds on the length of the run. On the other hand, it is weaker than Theorem 61 in the sense that it only applies to configuration graphs that are locally finite.

Finally, I like to stress that the counterexample of Parys does not invalidate Theorem 15 of [1], the second main result of that article.

References

[1] A. Blumensath, *On the structure of graphs in the Caucal hierarchy*, Theoretical Computer Science, 400 (2008), pp. 19–45.

[2] P. Parys, *The pumping lemma is incorrect?* unpublished, 2011.

[3] ——, *A Pumping Lemma for Pushdown Graphs of Any Level*, in 29th Int. Symp. on Theoretical Aspects of Computer Science (STACS 2012), 2012, pp. 54–65.