Exercises – set 10

Random Walks and Markov Chains I

Dr. Shenggen (Timothy) Zheng , May 16th, 2013, 8:30–9:45 B411

1. Consider a Markov chain with state space {0,1,2,3} and a transition matrix

$$P = \begin{bmatrix} 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix}.$$

   (a) What is the corresponding directed, weighted graph representation of the Markov chain?

   (b) Does the Markov chain have a stationary distribution? Why? If it does, find the stationary distribution of the Markov chain.

   Consider a Markov chain with state space {0,1,2,3} and a transition matrix

$$P' = \begin{bmatrix} 0 & 1/2 & 1/4 & 1/4 \\ 1/4 & 0 & 1/2 & 1/4 \\ 1/4 & 1/4 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 & 0 \end{bmatrix}.$$

   (c) What is the corresponding directed, weighted graph representation of the Markov chain?

   (d) Does the Markov chain have a stationary distribution? Why? If it does, find the stationary distribution of the Markov chain.

2. (Poor man wants to buy a pen) There is a poor men Bob. He just has one dollar, but he wants to buy an n dollars pen. Because he has no enough money, he does a fair gambling with Alice. In each round Bob wins a dollar with probability $\frac{1}{2}$ or loses a dollar with probability $\frac{1}{2}$. Once Bob has enough money to buy the pen or has no money, he will stop the gambling.

   (a) If $n = 2, 3, 4$, what is the probability that Bob succeeds in buying the pen?

   (b) Guess the probability for the case $n = k$ and prove it.

Figure 1: Drunkard's walk.

3. (Drunkard's walk) Let a drunken man walk on a linear graph (as figure 1)between his home and the bar, each time choosing a step right or left with the same probability. Suppose he begins the walk at position p, his home is l steps on the left, and the bar is r steps on the right. Once he gets home, his wife will catch him and make him stay at home. Once he gets to the bar, he will stay at the bar and have more alcohol. (Hints: What is the difference between exercises 2 and 3?)

   (a) What is the probability that the drunken man gets home?

   (b) What is the probability that the drunken man gets to the bar?