Chapter 5

COMPLEXITY

INTRODUCTION

The study of complexity questions and of complexity classes, computational and communicational, has proved to be very enlightening and important for classical computation. It has developed a firm theoretical basis for our understanding of the potentials and limitations of computational resources, models, and modes. There is reason to expect the same for the complexity investigations in quantum computation and communication.

It is of utmost importance to determine whether quantum classification of inherent computational complexity is indeed different from the classical one. Would this prove to be the case the very basic foundations of computing would be shaken.

Quantum computational complexity theory is characterized, as its classical counterpart, by a number of fundamental open problems concerning the proper inclusions of complexity classes. In order to get a better insight into these problems, and to test potential methods to solve them, the relativized quantum complexity theory is of interest and importance.

It is also of importance to find out how much quantum features can speed up computations, shorten communications and achieve efficiency in size or space.

Investigations of the potential impacts on the power of computing of the existence of slightly non-linear evolutions in quantum physics are also of interest.

LEARNING OBJECTIVES

The aim of the chapter is to learn:

1. the way universal quantum Turing machines can be constructed;
2. the basic quantum complexity classes and their properties;
3. the basic relations between classical and quantum complexity classes;
4. the basic results concerning relativized quantum complexity;
5. the basic concepts of quantum communication complexity;
6. a reduction of quantum communication protocols to quantum computation problems;
7. the potential impacts of non-linearity on the power of quantum computing.