Foundations of Computing

Jozef Gruska

Additional Exercises

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Chapter 1

1. Fermat's little theorem implies that if $n > 2$ is a prime, then $2^{n-1} \equiv 1 \pmod{n}$. (a) Show, by computing $2^{58482} \bmod 58483$, that 58483 is not prime. (The resulting number has 5 digits, 3 of them are different and the total sum of all digits is 23). (b) Find factors of 58483

Chapter 5

1. Has it a sense to consider **P**-completeness with respect to $\leq_p^m$ reduction?
2. * Show that propositional logic programming is **P**-complete.
3. ** Show that propositional logic programming remains to be **P**-complete even if all rules are of the form: $p_i \wedge p_j \rightarrow p_r$, $p_j \rightarrow p_r$, $\rightarrow p_r$.
4. ** Show that simple logic programming (with all rules of the form $p \rightarrow q$, $\rightarrow q$) is **NL**-complete.
5. Show that to any deterministic TM M one can construct a propositional logic program $LP(M, w, t)$ that simulates t steps of M on input w.

Chapter 10

1. Let $n = m^d = s^{d-1}$ for some integers $n, m, s, d \in \mathbf{N}^+$. (1) Show that if $d = 3$, then $[\sqrt{3}, n]$-array can be embedded into the $[\sqrt{n}, 2]$-array with dilation 2 and congestion 2; (b) Show that $[\sqrt{d}n, d]$-array can be embedded into the $[\sqrt{d-1}n, d-1]$-array with dilation and congesion $n^{1/r(r-1)}$.