Part I

Protocols to do seemingly impossible

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The point is that if $d = d_k d_{k-1} \dots d_1$, then at the computation of c^d, in the i-th iteration, a multiplication is performed only if $d_i = 1$ (and that requires time and energy).

EXAMS

CHAPTER 10: PROTOCOLS DOING SEEMINGLY IMPOSSIBLE

CHAPTER 10: PROTOCOLS DOING SEEMINGLY IMPOSSIBLE
and

CHAPTER 10: PROTOCOLS DOING SEEMINGLY IMPOSSIBLE and ZERO-KNOWLEDGE PROTOCOLS

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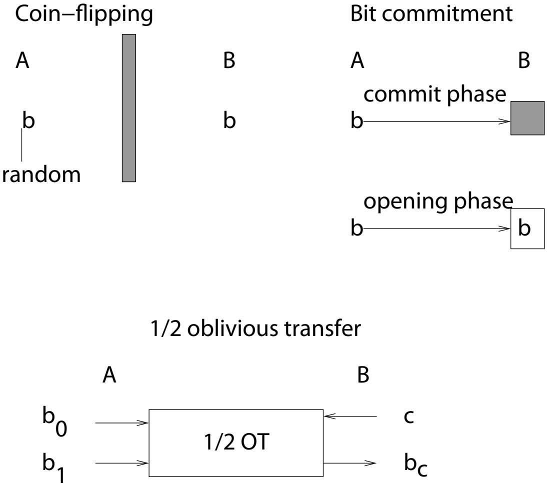
PICTORIAL SCHEMES for PRIMITIVES of CRYPTOGRAPHICAL PROTOCOLS



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In 1-**out**-2 **oblivious transfer protocols** Alice puts into a channel to Bob two messages (elements), **first** and **second**e. Bob will inform the channel which one he wants to receive (first or second), but not both, and gets it, in such a way that Alice will have no idea which of them Bob asked and received.

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Alice computes four square roots $(x_1, n - x_1)$ and $(x_2, n - x_2)$ of x and

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Bob tells Alice whether her guess was correct.

(Later, if necessary, Alice reveals p and q, and Bob reveals x.)

KEY FACT

Key fact used in the previous protocol is that Alice is able to compute (in polynomial time) squares of $x \bmod n$ because she knows decomposition of n into the product of two primes.

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Problem: In some coin tossing protocols one party can find out the outcome sooner than the second party. In such a case if she is not happy with the outcome she can disrupt the protocol – to produce reject or to say "I do not continue in performing the protocol". A way out is to require that in case of correct behavior no outcome should have probability $> \frac{1}{2}$.

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The protocol is computationally secure. Indeed, to cheat, Alice should be able to find, for randomly chosen r_1, r_2, such one-way function f that $f(r_1) = f(r_2)$.

BIT COMMITMENT - BASIC IDEA

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BIT COMMITMENT PROTOCOLS (BCP)

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Each bit commitment scheme should have three properties:

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Correctness: If both, the sender and the receiver, follow the protocol, then the receiver will always learn (recover) the committed value b.

BIT COMMITMENT with ONE-WAY FUNCTIONS

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Bit commitment scheme II. Let p be a large Blum prime, $X = Z_p^* = Y$, α be a primitive element of Z_p^*.

$$f(b, x) = \alpha^x \mod p, \text{ if SLB(x) = b;}$$
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Binding property of this bit commitment scheme follows from the fact that in the case of discrete logarithms modulo Blum primes there is no effective way to determine second least significant bit (SLB) of the discrete logarithm.

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Note: Observe that after step 2 Alice will know what the outcome is, but Bob does not. So Alice can disrupt the protocol if the outcome is to be not good for her. This is a weak point of this protocol.

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s can now be derived from v^s by computing $v^1, v^2, v^3, \ldots$ and comparing with v^s if the number of voters is not too large.

OBLIVIOUS TRANSFER (OT) PROBLEM

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Oblivious transfer problem: Design a protocol for sending a message from Alice to Bob in such a way that Bob receives the message with probability $\frac{1}{2}$ and "garbage" with the probability $\frac{1}{2}$. Moreover, Bob knows whether he got the message or garbage, but Alice has no idea which one he got.

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Oblivious transfer problem: **Design a protocol for sending a message from Alice to Bob in such a way that Bob receives the message with probability $\frac{1}{2}$ and "garbage" with the probability $\frac{1}{2}$.** Moreover, Bob knows whether he got the message or garbage, but Alice has no idea which one he got.

An Oblivious transfer protocol:

1. Alice chooses two large primes p and q and sends $n = pq$ to Bob.
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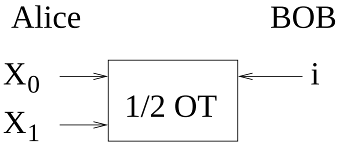

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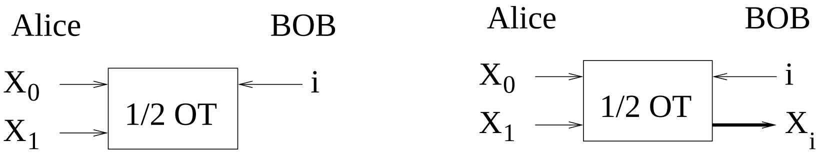

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4. Alice, who cannot read encrypted messages from Bob and Carol, decrypt them with her key and sends back to the senders,

$$\text{five } d_A(e_B(e_A(w_i))) = e_B(w_i) \text{ to Bob,}$$
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7. Alice decrypt messages to learn her hand.

Additional cards can be dealt with in a similar manner. If either Bob or Carol wants a card, they take an encrypted message $e_A(w_i)$ and go through the protocol with Alice. If Alice wants a card, whoever currently has the deck sends her a card.

ZERO-KNOWLEDGE PROOFS/PROTOCOLS

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- Zero-knowledge proofs and probabilistic proofs represent a new type of proofs – proofs that provide convincing evidence – so much convincing as needed.

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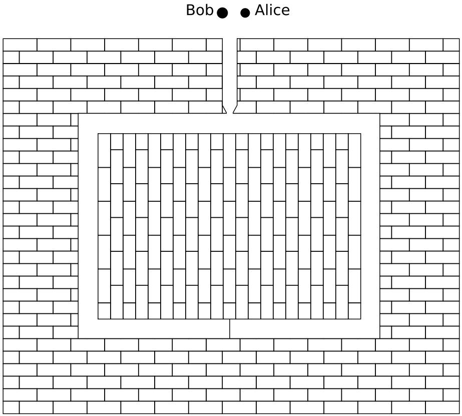
By a theorem we understand in the following a claim that a specific object has a specific property. For example, that a specific graph is 3-colorable.

(A cave with a magic door opening on a secret word)

Alice knows a secret word opening the door in cave. How can she convince Bob about it without revealing this secret word?



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- It was not until the sixteenth century that zero began to play a useful role in commerce.

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- Empirically the topic of vacuum was studied only since 17th century.

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- String theory is believed to have huge number of vacua - the so-called string theory landscape of it.

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This is a particular case known as **zero-knowledge proof of knowledge**.

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At the end, Verifier either accepts or rejects Prover's attempts to convince Verifier.

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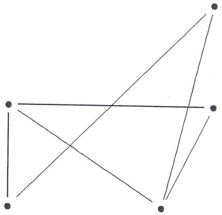
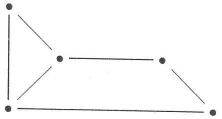
Intuitively, one may think about interactions between verifier and prover as consisting of "tricky" questions asked by the verifier to which the prover has to reply "convincingly".

GRAPH ISOMORPHISM - EXAMPLE

Are the following two graphs isomorphic?





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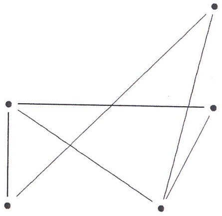
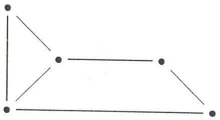
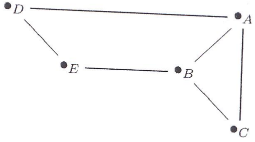
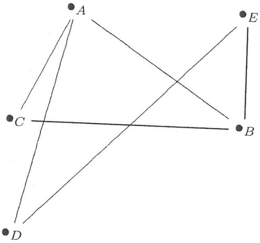




GRAPH ISOMORPHISM - EXAMPLE

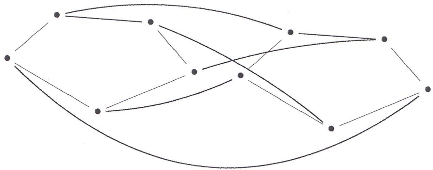
Here is isomorphism of the graphs from the previous slide.

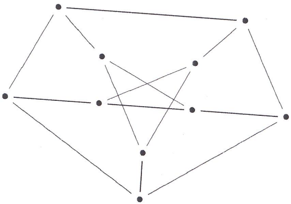




HOMEWORK -1

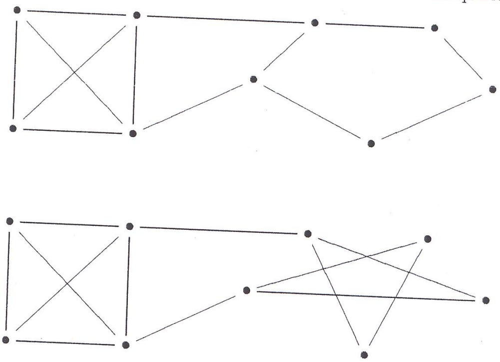
Problem 3. Show that the following two graphs are isomorphic:





HOMEWORK -2

Problem 2. Show the following two graphs are not isomorphic:



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Soundness: If G_1 is isomorphic to G_2, then Peggy can deceive Vic if and only if she correctly guesses n times those i's Vic chooses randomly.

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1. Vic chooses randomly an integer $i \in \{1, 2\}$ and a permutation π of $\{1, \ldots, n\}$.Vic then computes the image H of G_i under the permutation π and sends H to Peggy.
2. Peggy determines the value j such that G_J is isomorphic to H, and sends j to Vic.
3. Vic checks to see if $i = j$.

Vic accepts Peggy's proof if $i = j$ in each of n rounds.

Completeness: If G_1 is not isomorphic to G_2, then probability that Vic accepts is 1 because Peggy will have no problem to answer correctly.

Soundness: If G_1 is isomorphic to G_2, then Peggy can deceive Vic if and only if she correctly guesses n times those i's Vic chooses randomly. The probability that this can happen is 2^{-n}.

Observe that Vic's computations can be performed in polynomial time (with respect to the size of graphs).

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Computational zero-knowledge refer to the case there is no polynomial time distinguishability.

ZERO-KNOWLEDGE PROOF PROTOCOLS - VERY INFORMALLY

Very informally An interactive "proof protocol" at which a Prover tries to convince a Verifier about the truth of a statement, or about possession of a knowledge, is called "zero-knowledge" protocol if the Verifier does not learn from communication anything more except that the statement is true or that Prover has knowledge (secret) she claims to have.

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Example The proof $n = 670592745 = 12345 \times 54321$ is not a zero-knowledge proof that n is not a prime.

ZERO-KNOWLEDGE PROOF PROTOCOLS - MORE FORMALLY

Informally, a zero-knowledge proof is an interactive proof protocol that provides **highly convincing evidence** that a statement is true or that Prover has certain knowledge (of a secret) and that Prover knows a (standard) proof of it while providing **not a single bit of information** about the proof (knowledge or secret).

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In addition, during interactions, Prover does not reveal to Verifier any other information, except whether T is true or not. Consequently, whatever Verifier can do after he gets convinced, he can do just believing that T is true.

Similar arguments hold for the case Prover possesses a secret.

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In the following definition both prover (P) and verifier (V) as well as a simulator (S) will be Turing machines.

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is undistinguishable from what can be obtained from the transcript of the communication between P and V for the input x.

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2. Alice first computes the numbers $y_u = d_A(s + u); 1 \leq u \leq 100$, then chooses a large random prime p and computes numbers

$$z_u = y_u \mod p, \qquad 1 \leq u \leq 100 \qquad (*)$$

and verifies that for all $u \neq v$

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$$z_1, \ldots, z_i, z_{i+1} + 1, \ldots, z_{100} + 1, p$$
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3. Bob checks whether j-th number in the above sequence is congruent to x modulo p. If yes, Bob knows that $i \geq j$, otherwise $i < j$.

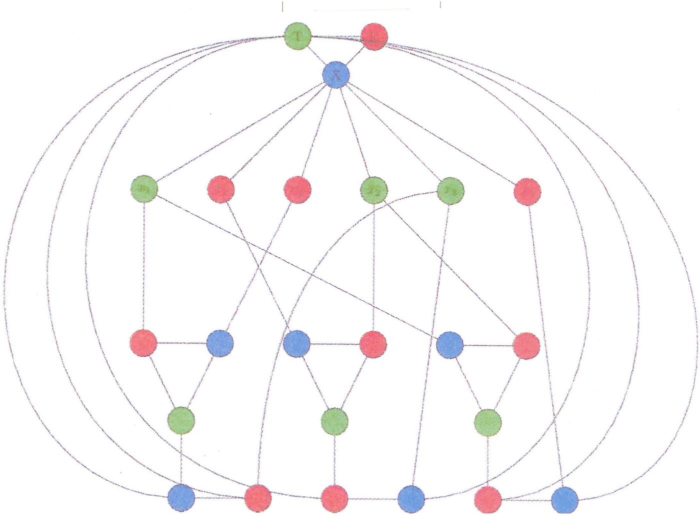
$$i \geq j \Rightarrow z'_j = z_j \equiv y_j = d_A(k) \equiv x \pmod p$$
$$i < j \Rightarrow z'_j = z_j + 1 \not\equiv y_j = d_A(k) \equiv x \pmod p$$

MILLIONAIRE

MILLIONAIRE

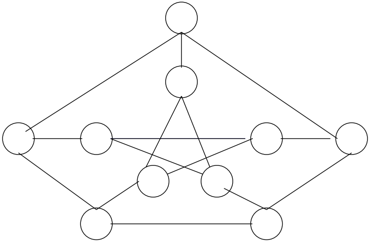
- The previous problem is ofter referred to as **Millionaire problem** that want to know who of them is richer without disclosing any additional information about their wealth.
- The problem is also often seen as an example of **two-party (multi-party) secure computation** at which both parties want to know some outcomes that depends on their inputs, but they do not want to disclose any information about their inputs.

3-COLORABILITY of GRAPHS



3-COLORABILITY of GRAPHS - EXAMPLE

Are the nodes of the following graph colorable by three colors in such a way that no edge connects nodes of the same color?



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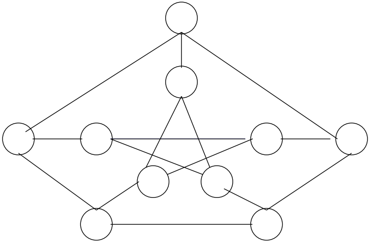
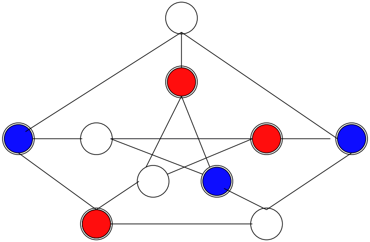


Yes, they are:



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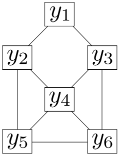

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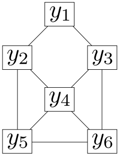

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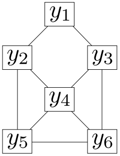

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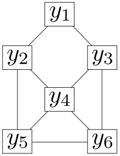

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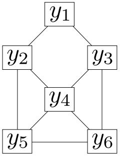

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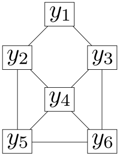

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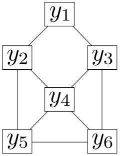

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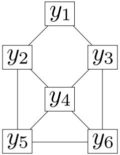

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(b)

Protocol: Peggy colors the graph $G = (V, E)$ with colors (red, blue, green) and she performs with Vic $|E|^2$- times the following interactions, where $v_1, \ldots, v_n$ are nodes of V.

1. Peggy chooses a random permutation of colors, recolors G, and encrypts, for i = 1,2,. . . ,n, the color c_i of node v_i by an encryption procedure e_i – for each i different. Peggy then removes colors from nodes, labels the i-th node of G with cryptotext $y_i = e_i(c_i)$, and designs Table (b).
   Peggy finally shows Vic the graph with nodes labeled by cryptotexts.
2. Vic chooses an edge and asks Peggy to show him coloring of the corresponding nodes.
3. Peggy shows Vic entries of the table corresponding to the nodes of the chosen edge.
4. Vic performs desired encryptions to verify that nodes really have colors as shown.

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- Vic checks whether colors are different and match the commitment received in the first step.

Zero-knowledge proofs for other **NP**-complete problems can be obtained using the standard reduction.

HISTORY of ZERO-KNOWLEDGE PROOFS

Research in zero-knowledge proofs have been motivated by identification problems and an approach where one party wants to prove his identity by demonstrating some secret knowledge (say a password) but does not want that other parties learn anything about this knowledge.

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The wide applicability of zero-knowledge proofs was first demonstrated in 1986 by Goldreich, Micali, Wigderson, who showed how to construct zero-knowledge proofs for any **NP**-set.

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Soundness: If graphs G_1 and G_2 are not isomorphic, then Peggy can deceive Vic only if she is able to guess in each round the j Vic chooses and then sends as H the graph G_j. However, the probability that this happens is 2^{-n}.

Observe that Vic can perform all computations in polynomial time. However, why is this proof a zero-knowledge proof?

WHY is the last "PROOF" a "ZERO-KNOWLEDGE PROOF"?

Because Vic gets convinced, by the overwhelming statistical evidence, that graphs G_1 and G_2 are isomorphic, but he does not get any information ("knowledge") that would help him to create isomorphism between G_1 and G_2.

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Information that Vic can receive during the protocol, called transcript, contains:

- The graphs G_1 and G_2.
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Transcript has therefore the form

$$T = ((G_1, G_2); (H_1, i_1, r_1), \ldots, (H_n, i_n, r_n)).$$

The essential point, which is the basis for the formal definition of zero-knowledge proof, is that Vic can forge transcript, without participating in the interactive proof, that look like "real transcripts", if graphs are isomorphic, by means of the following forging algorithm called simulator.

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- $T = (G_1, G_2)$,
- **for** j $=1$ **to** n **do**
  - Chose randomly $i_j \in \{1, 2\}$.
  - Chose ρ_j to be a random permutation of $\{1, \ldots, n\}$.
  - Compute H_j to be the image of G_{i_j} under ρ_j;
  - Concatenate (H_j, i_j, ρ_j) at the end of T.

CONSEQUENCES and FORMAL DEFINITION

The fact that a simulator can forge transcripts has several important consequences.

- Anything Vic can compute using the information obtained from the transcript can be computed using only a forged transcript and therefore participation in such a communication does not increase Vic capability to perform any computation.
- Participation in such a proof does not allow Vic to prove isomorphism of G_1 and G_2.
- Vic cannot convince someone else that G_1 and G_2 are isomorphic by showing the transcript because it is indistinguishable from a forged one.

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Formal definition of what this means that a forged transcript "looks like" a real one:

Definition Suppose that we have an interactive proof system for a decision problem Π and a polynomial time simulator S.

Denote by $\Gamma(x)$ the set of all possible transcripts that could be produced during the interactive proof communication for a yes-instance x.

Denote F(x) the set of all possible forged transcripts produced by the simulator S.

For any transcript $T \in \Gamma(x)$, let $p_\Gamma(T)$ denote the probability that T is the transcript produced during the interactive proof. Similarly, for $T \in F(x)$, let $p_F(T)$ denote the probability that T is the transcript produced by S.

If $\Gamma(x) = F(x)$ and, for any $T \in \Gamma(x)$, $p_\Gamma(T) = p_F(T)$, then we say that the interactive proof system is a zero-knowledge proof system.

Is the above interactive protocol for graph non-isomorphism also a zero-knowledge protool?

NO

Because....

Why?

APPENDIX

WHAT IS A PROOF?

- A proof is whatever convinces me (M. Even).
- A nice proof makes us wiser (Yu. Manin).
- A proof is a sequence of statements each of them is either an axiom or follows from previous statements by am easy deduction rule - whether a to-be-proof is indeed a proof it should be checkeable by a computer. (A proof is therefore a computation process.)

HISTORY of PROOFS

- The concept of the proof (of a theorem from axioms) was introduced during the first golden era of mathematics, in Greece, 600-300 BC.
- Most of their proofs were actually proofs of correctness of geometric algorithms.
- After 300 BC, Greek's ideas concerning proofs were actually ignored for 2000 years.
- During the second golden era of mathematics, in 17th century, the concept of the proof did not play very important role. Famous was encouragement of those times "Go on, God will be with you" whenever rigour of some methods or correctness of some theorem was questioned.
- An understanding that proofs are important has developed again at the end of 19th century and especially at the beginning of 20th century because
  - a lot of counter-intuitive phenomena have appeared in mathematics (for example a function that is everywhere continuous but has nowhere derivative);
  - paradoxes have appeared in the set theory. - For example, Does there exist a set of all sets?

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However, it is possible to resolve this problem by considering zero-knowledge proofs of knowledge about knowledge.

In such a setting the goal is not to prove that input is (or is not) in the given language, but that Prover knows whether the input is (or is not) in the language.