Part I

Digital signatures

MURPHY LAW for CRYPTOGRAPHY

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It is not sufficient that a cryptographical system is very secure, or even perfectly secure - practically it is desirable that its implementations are secure enough what is very hard to achieve.

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In many countries it is already desirable, or even necessary, to use in important communications digital signatures and they have also legal significance.

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- Digital signatures employ public-key cryptography.

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Key observation: Digital signatures have to depend not only on the signer, but also on the document/message that is being signed.

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This chapter contains some of the main techniques for design and verification of digital signatures (as well as some possible attacks on them).

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Any public-key cryptosystem in which the plaintext and cryptotext spaces are the same can be used for digital signature.

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  $$\mathbf{ver}_k(w, s) = \begin{cases} true & \text{if } \ s = \mathbf{sig}_k(w); , \\ false & \text{otherwise.} \end{cases}$$

Algorithms sig_k and ver_k should be realizable in polynomial time.

Verification algorithms can be publicly known; signing algorithms (actually only their keys) should be kept secret

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such that the following two conditions are satisfied:

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Security:

For any w from M and k from K_v, it should be computationally unfeasible, without the knowledge of the private key corresponding to k, to find a signature s from S such that

$$ver_k(w, s) = \text{true}.$$

A COMMENT ON DIGITAL SIGNATURE SCHEMES

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Sometimes it is required that a digital signature scheme contains also a **keys generation phase**,

It is a phase that creates uniformly and randomly a secret (signing) key (from a set of potential secret keys) and outputs this secret key and the corresponding public (verification) key.

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- **In both cases, a more ambitious goal is to find the private key.**

ATTACKS MODELS on DIGITAL SIGNATURES

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ADAPTIVE CHOSEN SIGNATURES ATTACKS: The attacker is given valid signatures for several messages chosen by the attacker where messages chosen may depend on previous signatures given for chosen messages.

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Observe that to forge a signature scheme means to produce a new signature - it is not forgery to obtain from the signer a valid signature.

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$$s_b = f(k_b)??$$

SECURITY?

FROM RSA CRYPTOSYSTEM to RSA SIGNATURES

The idea of RSA cryptosystem is simple.
Public key: modulus $n = pq$ and encryption exponent e.
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  Indeed, is σ_1 and σ_2 are signatures for w_1 and w_2, then $\sigma_1\sigma_2$ and σ_1^{-1} are signatures for $w_1 w_2$ and w_1^{-1}.

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Verification: Given a message w and a signature (U, x) the versifier V computes x^2 and $h(wU)$ and verifies that they are equal.

IMPORTANT FACTS

Fact 1

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Fact 2

If a, b, n, x are integers and $gcd(x, n) = 1$, then

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PROOF

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by Fermat's little theorem.

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ElGamal SIGNATURES

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$$\textbf{Compute}:\ y = q^x \mod p$$

$$\textbf{key K} = (\text{p, q, x, y})$$

$$\textbf{public key}\ (\text{p, q, y})\ \text{-}\ \textbf{secret key}:\ \text{x}$$

Signature of a message w: Let $r \in Z_{p-1}^*$ be randomly chosen and kept secret.

$$\text{sig(w, r)} = \text{(a, b)},$$

$$\text{where } a = q^r \mod p$$

$$\text{and b} = (w - xa)r^{-1} \pmod{(p-1)}.$$

Verification: accept a signature (a,b) of w as valid if

$$y^a a^b = q^w \pmod p$$

(Indeed, for some integer k: $y^a a^b \equiv q^{ax} q^{rb} \equiv q^{ax+w-ax+k(p-1)} \equiv q^w \pmod p$)

SECURITY of ElGamal SIGNATURES

Let us analyze several ways an eavesdropper Eve can try to forge ElGamal signature (with x - secret; p, q and $y = q^x$ mod p - public):

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1. First suppose Eve tries to forge signature for a new message w, without knowing x.
   - If Eve first chooses a value a and tries to find the corresponding b, it has to compute the discrete logarithm

     $$lg_a q^w y^{-a},$$

     (because $a^b \equiv q^{r(w-xa)r^{-1}} \equiv q^{w-xa} \equiv q^w y^{-a}$) what is unfeasible.
   - If Eve first chooses **b** and then tries to find **a**, she has to solve the equation

     $$y^a a^b \equiv q^{xa} q^{rb} \equiv q^w \pmod{p}.$$

     It is not known whether this equation can be solved for any given b efficiently.

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     It is not known whether this equation can be solved for any given b efficiently.
2. If Eve chooses **a** and **b** and tries to determine **w** such that (a,b) is signature of w, then she has to compute discrete logarithm

   $$lg_q y^a a^b.$$

   Hence, Eve can not sign a “random” message this way.

From ElGamal to DSA (DIGITAL SIGNATURE STANDARD)

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However, with ElGamal this would lead to signatures with at least 1024 bits what is too much for such applications as smart cards.

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1. **The following global public key components** are chosen:
   - p - a random l-bit prime, $512 \leq l \leq 1024$, l = 64k.
   - q - a random 160-bit prime dividing p -1.
   - r $= h^{(p-1)/q} \mod p$, where h is a random primitive element of Z_p, such that $r > 1$, $r \neq 1$ (observe that r is a q-th root of 1 mod p).

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4. Key is K = (p, q, r, x, y)

Signing and Verification

Signing of a 160-bit plaintext w

- choose random $0 < k < q$
- compute a $= (r^k \mod p) \bmod$ q
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Verification of signature (a, b)

- compute $z = b^{-1} \mod q$
- compute $u_1 = wz \mod q$, $u_2 = az \mod q$

  verification:

$$ver_K(w, a, b) = true \Leftrightarrow (r^{u_1} y^{u_2} \mod p) \mod q = a$$

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Observe that y and a are also q-roots of 1. Hence any exponents of r,y and a can be reduced modulo q without affecting the verification condition.

This allowed to change ElGamal verification condition: $y^a a^b = q^w$.

Fiat-Shamir SIGNATURE SCHEME

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   Security of this signature scheme is 2^{-kt}.

   Advantage over the RSA-based signature scheme: only about 5% of modular multiplications are needed.

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Problem: Can Alice and Bob set up a subliminal channel, a covert communication channel between them, in full view of Walter, even though the messages themselves that they exchange contain no secret information?

Ong-Schnorr-Shamir SUBLUMINAL CHANNEL SCHEME

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Signature: (S_1, S_2). Alice then sends to Bob (w', S_1, S_2)

Ong-Schnorr-Shamir SUBLUMINAL CHANNEL SCHEME

Story Alice and Bob are in different jails. Walter, the warden, allows them to communicate by network, but he will not allow messages to be encrypted. Can they set up a subliminal channel, a covert communication channel between them, in full view of Walter, even though the messages themselves contain no secret information?

Yes. Alice and Bob create first the following communication scheme:

They choose a large n and an integer k such that **gcd(n, k) = 1**.

They calculate $h = k^{-2} \mod n = (k^{-1})^2 \mod n$.

They make h, n to be **public key**

They keep secret k as trapdoor information.

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Important note: Lamport signature scheme can be used safely to sign only one message. Why?

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The main reason why Merkle Signature Scheme is of interest, is that it is believed to be resistant to potential attacks using quantum computers.

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- Who knows.
- The possibility of having quite soon powerful quantum computers starts to be so realistic that in US decision has been made, on a very-high level of cares for national security, that the next generation of cryptographic primitives' standards (for encryptions, digital signatures, hash functions,...) should be secure even in case quantum computers would be available.

MERKLE SIGNATURES - II.

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As the next step a complete binary tree with 2^n leaves is designed and the value $h(PK_i)$ is stored in the i-the leave, counting from left to right. Moreover, to each internal node the hash of the concatenation of hashes of its two children is stored.

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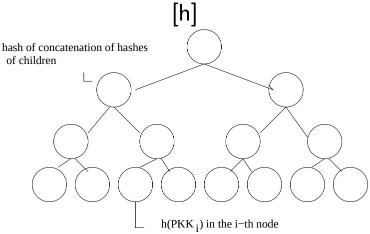
As the next step a complete binary tree with 2^n leaves is designed and the value $h(PK_i)$ is stored in the i-the leave, counting from left to right. Moreover, to each internal node the hash of the concatenation of hashes of its two children is stored. The hash assigned this way to the root is the public key of the Merkle signature scheme and the tree is called **Merkle tree**. See next figure for a Merkle tree.



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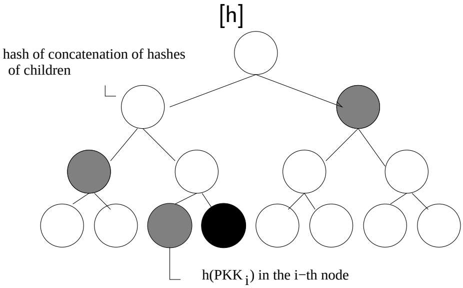
The verifier knows the public key - hash assigned to the root and signature created as above. This allows him to compute all hashes assigned to the root from the leave to the root and to verify that the value assigned this way agrees with he public key - hash assigned to the root.



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They also presented a new signature scheme, known nowadays as **GMR signature scheme**.

It was the first signature scheme that was proven as being robust against an adaptive chosen message attacks: an adversary who receives signatures of messages of his choice (where each message may be chosen in a way that depends on the signatures of previously chosen messages) cannot later forge the signature even of a single additional message.

TIMESTAMPING

There are various ways that a digital signature can be compromised.

For example: if Eve determines the secret key of Bob, then she can forge signatures of any Bob's message she likes. If this happens, authenticity of all messages signed by Bob before Eve got the secret key is to be questioned.

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Times tamping by Bob of a signature on a message w, using a hash function h.

- Bob computes $z = h(w)$;
- Bob computes $z' = h(z \parallel pub)$; – $\{ \parallel \}$ denotes concatenation
- Bob computes $y = sig(z')$;
- Bob publishes (z, pub, y) in the next day newspaper.

It is now clear that signature could not be done after the triple (z, pub, y) was published, but also not before the date pub was known.

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- Bob signs the message $m*$ to get a signature s_{m^*} (of $m*$) and sends s_{m*} to Alice. The signing is to be done in such a way that Alice can afterwards compute, using an unblinding procedure, from Bob's signature s_{m*} of $m*$ – Bob's signature s_m of m.

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Verification is similar to that of the RSA signature scheme.

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Assume now: $v_x = e_x$, $s_x = d_x$ for all users x.

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5. Mallot can then get w (observe that $v_X = e_X$ and $s_x = d_x$ for each user x).

   Indeed, Mallot can compute

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ANOTHER MAN-IN-THE-MIDDLE ATTACK

Consider the following protocol:

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- C is now able to learn w' and therefore also $e_A(w)$.
- C now sends to Alice the pair $(e_A(e_A(w)||C), A)$.
- Alice makes acknowledgment by sending the pair $(e_C(e_C(w)||A), C)$.
- C is now able to learn w.

PROBABILISTIC SIGNATURES SCHEMES - PSS

Let us have **integers** k, l, n such that $k + l < n$, a **trapdoor permutation**

$$f : D \to D, D \subset \{0,1\}^n,$$

a **pseudorandom bit generator**

$$G : \{0,1\}^l \to \{0,1\}^k \times \{0,1\}^{n-(l+k)}, \quad G(w) = (G_1(w), G_2(w))$$

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- Compute $r = t \oplus G_1(m)$.
- Accept signature σ if $h(w\|r) = m$ and $G_2(m) = u$; otherwise reject it.

Diffie-Hellman PUBLIC ESTABLISHMENT of SECRET KEYS - repetition

Main problem of the secret-key cryptography: a need to make a secure distribution (establishment) of secret keys ahead of transmissions.

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An eavesdropper seems to need, in order to determine x from **X, q, p** and y from **Y, q, p**, a capability to compute discrete logarithms, or to compute q^{xy} from q^x and q^y, what is believed to be unfeasible.

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An enhanced version of the above protocol is known as Station-to-Station protocol.

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The idea of a (t+1, n) threshold signature scheme is to distribute the power of the signing operation to (t+1) parties out of n.

A (t+1) threshold signature scheme should satisfy two conditions.

Unforgeability means that even if an adversary corrupts t parties, he still cannot generate a valid signature.

Robustness means that corrupted parties cannot prevent uncorrupted parties to generate signatures.

Shoup (2000) presented an efficient, non-interactive, robust and unforgeable threshold RSA signature schemes.

There is no proof yet whether Shoup's scheme is provably secure.

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- In 1988 Goldwasser, Micali and Rivest were first to rigorously define (perfect) security of digital signature schemes.

APPENDIX

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GENERAL OBSERVATIONS

- Digital signatures are often used to implement electronic signatures - this is a broader term that refers to any electronic data that carries the intend of a signature. Not all electronic signatures use digital signatures.
- The first broadly marketed software package to offer digital signature was Lotus Notes 1.0, released in 1989, which used RSA algorithm

SECURITY of ElGamal SIGNATURES

Let us analyze several ways an eavesdropper Eve can try to forge ElGamal signature (with x - secret; p, q and y $= q^x$ mod p - public):

$$\text{sig}(w, r) = (a, b);$$

where r is random and a $= q^r$ mod p; b $= (w - xa)r^{-1} \pmod{p-1}$.

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1. First suppose Eve tries to forge signature for a new message w, without knowing x.
   - If Eve first chooses a value a and tries to find the corresponding b, it has to compute the discrete logarithm

     $$lg_a q^w y^{-a},$$

     (because $a^b \equiv q^{r(w-xa)r^{-1}} \equiv q^{w-xa} \equiv q^w y^{-a}$) what is unfeasible.
   - If Eve first chooses b and then tries to find a, she has to solve the equation

     $$y^a a^b \equiv q^{xa} q^{rb} \equiv q^w \pmod{p}.$$

     It is not known whether this equation can be solved for any given b efficiently.

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2. If Eve chooses a and b and tries to determine such w that (a,b) is signature of w, then she has to compute discrete logarithm

   $$lg_q y^a a^b.$$

   Hence, Eve can not sign a "random" message this way.