

## Part I

### Secret-key cryptosystems basics

Decrypt cryptotexts:

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GBLVMUB JOGPSNBUJLZ

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RPNBMZ EBMFLP OFABKEFT

# PROLOGUE - II.

Decrypt:

Decrypt:

VHFUHW GH GHXA

VHFUHW GH GLHX,

VHFUHW GH WURLV,

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# CHAPTER 4: SECRET-KEY (SYMMETRIC) CRYPTOGRAPHY

- In this chapter we deal with some of the very old, or quite old, classical (secret-key or symmetric) cryptosystems and their cryptanalysis that were primarily used in the pre-computer era.

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- These cryptosystems are too weak nowadays, too easy to break, especially with computers.
- However, these simple cryptosystems give a good illustration of several of the important ideas of the cryptography and cryptanalysis.
- Moreover, most of them can be very useful in combination with more modern cryptosystem - to add a new level of security.

# BASICS

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## Importance of cryptography nowadays

- **Applications:** cryptography is the key tool to make modern information transmission secure, and to create secure information society.
- **Foundations:** cryptography gave rise to several new key concepts of the foundation of informatics: one-way functions, computationally perfect pseudorandom generators, zero-knowledge proofs, holographic proofs, program self-testing and self-correcting, ...

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Modern cryptography has

- (1) significantly enlarged its scope to the rigorous analysis of any system that can be potential subject to malicious threats and to designs of such versions of such systems that can guarantee that they withstand such treats.
- (2) started to develop cryptographic systems that also utilize elements and processes of the quantum world.

# CONTINUATION



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For example, digital signatures, authentication, privacy preservations, secret sharing, hashing, pseudorandom generators, zero-knowledge proofs, steganography, and so on.

As another consequence, cryptography has moved from an engineering art, built on heuristic techniques, to a scientific discipline based on mathematically rigorous design requirements, solution techniques and correctness proofs.

Such broadly developed modern cryptography is the subject of this lecture.

# APPROACHES and PARADOXES in CRYPTOGRAPHY

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## Paradoxes of modern cryptography:

- **Positive results** of modern cryptography are based on **negative results** of computational complexity theory.
- Computers, that were designed originally for **decryption**, seem to be now more useful for **encryption**.

# SECRET-KEY (SYMMETRIC) CRYPTOSYSTEMS - CIPHERS

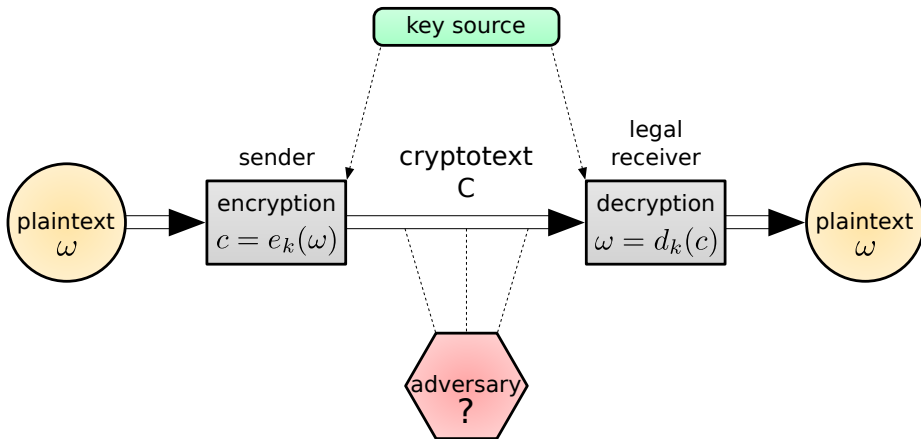
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**Secret-key (symmetric) cryptosystems scheme:**



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Security of such a cryptosystem depends solely on the secrecy of shared key.

# COMPONENTS of CRYPTOSYSTEMS:

**Plaintext-space:**  $P$  – a set of plaintexts (messages) over an alphabet  $\Sigma$

**Cryptotext-space:**  $C$  – a set of cryptotexts (ciphertexts) over alphabet  $\Delta$

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**Note:** As encryption algorithms we can use also **randomized algorithms**.

# SECRET-KEY CRYPTOGRAPHY BASICS - SUMMARY



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**Secret key cryptosystems provide secure transmission of messages along insecure channel** provided **the secret keys are transmitted over an extra secure channel.**

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**Practical security** is in the case no one was able to break the cryptosystem so far after many years and many attempts.

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- **Pre-computers era view:** Codebreakers or cryptanalysts are **linguistic alchemists** - a mystical tribe attempting to discover meaningful texts in the apparently meaningless sequences of symbols.
- **Current view** Codebreakers and cryptanalysts are **artists** that can superbly use modern mathematics, informatics and computing supertechnology for decrypting encrypted messages.





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- **Third World War** will be the war of informaticians (cryptographers and cryptanalysts).

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**Transposition ciphers** do not replace but only rearrange order of symbols in the plaintext - sometimes in a complicated way.

## PARTICULAR CRYPTOSYSTEMS

# CAESAR (100 - 42 B.C.) CRYPTOSYSTEM - SHIFT CIPHER I

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**The decryption algorithm  $d_k$**  for  $SC(k)$  substitutes any letter by the one occurring  $k$  positions backward (cyclically) in the alphabet.

# SHIFT CIPHER $SC(k)$ - $SC(3)$ is called CAESAR SHIFT

**Example**

$e_2(\text{EXAMPLE}) =$

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**Example** Find the plaintext to the following cryptotext obtained by the encryption with SHIFT CIPHER with  $k = ?$ .

Decrypt the  
cryptotext:

VHFUHW GH GHXA, VHFUHW GH GLHX,  
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**Numerical version of  $SC(k)$**  is defined, for English, on the set  $\{0, 1, 2, \dots, 25\}$  by the encryption algorithm:

$$e_k(i) = (i + k)(\text{mod } 26)$$

**Numerical version of the cipher Atbash used in the Bible.**

$$e(i) = 25 - i$$

# EXAMPLE

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Solution:



Decrypt:

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Solution:

Secret de deux  
secret de Dieu,  
secret de trois  
secret de tous.

# VATSYAYANA CIPHER - SC(2)

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This system is now believed, by some, to be the oldest cipher used.

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# FIRST INTERNET

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It is expected that Romans already used Polybious cryptosystem.

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- 6 The cryptosystem should **not** be **closed under composition**, i.e. not for every two keys  $k_1, k_2$  there is a key  $k$  such that
$$e_k(w) = e_{k_1}(e_{k_2}(w)).$$
- 7 The set of keys should be **very large**.

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# CRYPTANALYSIS ATTACKS I

The aim of cryptanalysis is to get as much information about the plaintext or the key as possible.

## Main types of cryptanalytic attacks

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where the cryptotexts  $c_i$  have been chosen by the cryptanalysts. The aim is to determine the key. (For example, if cryptanalysts get a temporary access to decryption machinery.)

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An eavesdropper can therefore be passive - Eve or active - Mallot.

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**Anonymity:** Alice does not want Bob to find out who sent the message

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**Encoding:** For a word  $w$  let  $c_w$  be the column vector of length  $n$  of the integer codes of symbols of  $w$ . ( $A \rightarrow 0, B \rightarrow 1, C \rightarrow 2, \dots$ )

**Encryption:**  $c_c = M c_w \bmod 26$

**Decryption:**  $c_w = M^{-1} c_c \bmod 26$

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**Encryption :**  $M_{w_{LO}} = \begin{bmatrix} 12 \\ 25 \end{bmatrix}$ ,  $M_{w_{ND}} = \begin{bmatrix} 21 \\ 16 \end{bmatrix}$ ,  $M_{w_{ON}} = \begin{bmatrix} 17 \\ 1 \end{bmatrix}$



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## Theorem

$$\text{If } M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ then } M^{-1} = \frac{1}{\det M} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

**Proof:** Exercise

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$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix} \pmod{11}.$$

The standard inverse of  $M$  in rational numbers is

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Since  $2^{-1} \equiv 6 \pmod{11}$ , the resulting matrix has the form

$$M^{-1} = \begin{pmatrix} 3 & 3 & 6 \\ 8 & 4 & 10 \\ 1 & 4 & 6 \end{pmatrix} \pmod{11}.$$





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in the journal **American Mathematical Monthly** in 1929.

Hill even tried to design a machine to use his cipher, but without a success.

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A monoalphabetic cryptosystem with letter-by-letter substitution is uniquely specified by a permutation of letters, (number of permutations (keys) is 26!)



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A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
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**Decryption:**  $d_{a,b}(y) = a^{-1}(y - b) \bmod 26$

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Frequency counts in English:

	%		%		%
E	12.31	L	4.03	B	1.62
T	9.59	D	3.65	G	1.61
A	8.05	C	3.20	V	0.93
O	7.94	U	3.10	K	0.52
N	7.19	P	2.29	Q	0.20
I	7.18	F	2.28	X	0.20
S	6.59	M	2.25	J	0.10
R	6.03	W	2.03	Z	0.09
H	5.14	Y	1.88		
	70.02		24.71		5.27

and for other languages:

English	%	German	%	Finnish	%	French	%	Italian	%	Spanish	%
E	12.31	E	18.46	A	12.06	E	15.87	E	11.79	E	13.15
T	9.59	N	11.42	I	10.59	A	9.42	A	11.74	A	12.69
A	8.05	I	8.02	T	9.76	I	8.41	I	11.28	O	9.49
O	7.94	R	7.14	N	8.64	S	7.90	O	9.83	S	7.60
N	7.19	S	7.04	E	8.11	T	7.29	N	6.88	N	6.95
I	7.18	A	5.38	S	7.83	N	7.15	L	6.51	R	6.25
S	6.59	T	5.22	L	5.86	R	6.46	R	6.37	I	6.25
R	6.03	U	5.01	O	5.54	U	6.24	T	5.62	L	5.94
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The 20 most common **digrams** are (in decreasing order) TH, HE, IN, ER, AN, RE, ED, ON, ES, ST, EN, AT, TO, NT, HA, ND, OU, EA, NG, AS.

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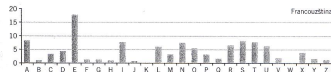
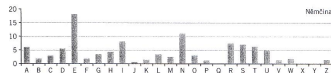
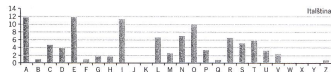
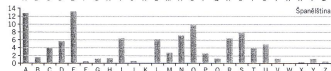
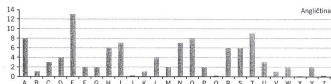


# FREQUENCY ANALYSIS for SEVERAL LANGUAGES

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## NEJČETNĚJŠÍ PÍSMENA V ZÁPADOEVROPSKÝCH JAZYCÍCH

Angličtina: E T A O I N S H R D L U  
Francouzština: E N A S R I U T O L D C  
Němčina: E N R I S T U D A H G L  
Italština: E I A O R L N T S C D P  
Španělština: E A O S R I N L D C T U



# OTHER CHARACTERISTICS of ENGLISH

## V ANGLIČTINĚ

Nejčastější písmena: **e t a o i n s h r d l u**

Nejčastější první písmena: **t a s o i c p b s h m**

Nejčastější poslední písmena: **e t s d n r y o f l a g**

Nejčastější dvojice písmen: **th er on an re he in ed nd ha at**

Nejčastější trojice písmen: **the and tha ent ion tio for nde**

Nejčastější zdvojení písmen: **ss ee tt ff ll mm oo**

Nejčastější písmena následující po E: **r d s n a c t m e p w o**

Nejčastější dvojpísmenná slova: **of to in it is be as at so we he**

Nejčastější trojpísmenná slova: **the and for are but not you all**

Nejčastější čtyřpísmenná slova: **that with have this will your from they**

# FREQUENCY COUNTS in CZECH and SLOVAK

First resource	<i>Czech</i>		<i>Slovak</i>	
	<i>o</i>	8.66	<i>a</i>	10.67
	<i>e</i>	7.69	<i>o</i>	9.12
	<i>n</i>	6.53	<i>e</i>	8.43
	<i>a</i>	6.21	<i>i</i>	5.74
	<i>t</i>	5.72	<i>n</i>	5.74
	<i>v</i>	4.66	<i>s</i>	5.02
	<i>s</i>	4.51	<i>t</i>	4.92
	<i>i</i>	4.35	<i>v</i>	4.60
Second resource:	<i>l</i>	3.84	<i>k</i>	3.96
	<i>Czech</i>		<i>Slovak</i>	
	<i>e</i>	10.13	<i>a</i>	9.49
	<i>a</i>	8.99	<i>o</i>	9.34
	<i>o</i>	8.39	<i>e</i>	9.16
	<i>i</i>	6.92	<i>i</i>	6.81
	<i>n</i>	6.64	<i>n</i>	6.34
	<i>s</i>	5.74	<i>s</i>	5.94
	<i>r</i>	5.33	<i>r</i>	5.12
	<i>t</i>	4.98	<i>t</i>	5.06
	<i>v</i>	4.50	<i>v</i>	4.85

# Discovery of FREQUENCY ANALYSIS - I.

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Frequency analysis was originally used to study Koran, to establish chronology of revelations by Muhammad in Koran.

## Discovery of FREQUENCY ANALYSIS - II.

[illegible]

مرآة الخ - والحمد لله رب العالمين صلوات الله على سيدنا محمد وآله

[illegible]

# CRYPTANALYSIS of AFFINE CRYPTOSYSTEM - EXAMPLE

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Cryptanalysis of a cryptotext encrypted using the AFFINE cryptosystem with an encryption algorithm

$$e_{a,b}(x) = (ax + b) \bmod 26$$

where  $0 \leq a, b \leq 25, \gcd(a, 26) = 1$ . (Number of keys:  $12 \times 26 = 312$ .)



# CRYPTANALYSIS of AFFINE CRYPTOSYSTEM - EXAMPLE

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**Example:** Assume that an English plaintext is divided into blocks of 5 letters and encrypted by an AFFINE cryptosystem (ignoring space and interpunctuations) as follows:

How to find the  
plaintext?

B H J U H	N B U L S	V U L R U	S L Y X H
O N U U N	B W N U A	X U S N L	U Y J S S
W X R L K	G N B O N	U U N B W	S W X K X
H K X D H	U Z D L K	X B H J U	H B N U O
N U M H U	G S W H U	X M B X R	W X K X L
U X B H J	U H C X K	X A X K Z	S W K X X
L K O L J	K C X L C	M X O N U	U B V U L
R R W H S	H B H J U	H N B X M	B X R W X
K X N O Z	L J B X X	H B N F U	B H J U H
L U S W X	G L L K Z	L J P H U	U L S Y X
B J K X S	W H S S W	X K X N B	H B H J U
H Y X W N	U G S W X	G L L K	

# CRYPTANALYSIS - CONTINUATION I

Frequency analysis of plaintext and  
frequency table for English:

X - 32	J - 11	D - 2
U - 30	O - 6	V - 2
H - 23	R - 6	F - 1
B - 19	G - 5	P - 1
L - 19	M - 4	E - 0
N - 16	Y - 4	I - 0
K - 15	Z - 4	Q - 0
S - 15	C - 3	T - 0
W - 14	A - 2	

	%		%		%
E	12.31	L	4.03	B	1.62
T	9.59	D	3.65	G	1.61
A	8.05	C	3.20	V	0.93
O	7.94	U	3.10	K	0.52
N	7.19	P	2.29	Q	0.20
I	7.18	F	2.28	X	0.20
S	6.59	M	2.25	J	0.10
R	6.03	W	2.03	Z	0.09
H	5.14	Y	1.88		
	70.02		24.71		5.27

**First guess:**  $E = X, T = U$

Encodings:  $4a + b = 23 \pmod{26}$

$xa + b = y$   $19a + b = 20 \pmod{26}$

**Solutions:**  $a = 5, b = 3 \rightarrow a^{-1} =$

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Translation table

crypto	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
plain	P	K	F	A	V	Q	L	G	B	W	R	M	H	C	X	S	N	I	D	Y	T	O	J	E	Z	U

B	H	J	U	H	N	B	U	L	S	V	U	L	R	U	S	L	Y	X	H
O	N	U	U	N	B	W	N	U	A	X	U	S	N	L	U	Y	J	S	S
W	X	R	L	K	G	N	B	O	N	U	U	N	B	W	S	W	X	K	X
H	K	X	D	H	U	Z	D	L	K	X	B	H	J	U	H	B	N	U	O
N	U	M	H	U	G	S	W	H	U	X	M	B	X	R	W	X	K	X	L
U	X	B	H	J	U	H	C	X	K	X	A	X	K	Z	S	W	K	X	X
L	K	O	L	J	K	C	X	L	C	M	X	O	N	U	U	B	V	U	L
R	R	W	H	S	H	B	H	J	U	H	N	B	X	M	B	X	R	W	X
K	X	N	O	Z	L	J	B	X	X	H	B	N	F	U	B	H	J	U	X
L	U	S	W	X	G	L	L	K	Z	L	J	P	H	U	U	L	S	Y	X
B	J	K	X	S	W	H	S	S	W	X	K	X	N	B	H	B	H	J	U
H	Y	X	W	N	U	G	S	W	X	G	L	L	K						

provides from the above cryptotext the plaintext that starts with KGWTG CKTMO OTMIT DMZEG, which does not make sense.

**Second guess:**  $E = X, A = H$

*Equations*  $4a + b = 23 \pmod{26}$

$$b = 7 \pmod{26}$$

**Solutions:**  $a = 4$  or  $a = 17$  and therefore  $a = 17$

# CRYPTANALYSIS - CONTINUATION II

**Second guess:**  $E = X, A = H$

**Equations**  $4a + b = 23 \pmod{26}$

$$b = 7 \pmod{26}$$

**Solutions:**  $a = 4$  or  $a = 17$  and therefore  $a = 17$

*This gives the translation table*

crypto	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
plain	V	S	P	M	J	G	D	A	X	U	R	O	L	I	F	C	Z	W	T	Q	N	K	H	E	B	Y

*and the following  
plaintext from the  
above cryptotext*

S A U N A I S N O T K N O W N T O B E A  
F I N N I S H I N V E N T I O N B U T T  
H E W O R D I S F I N N I S H T H E R E  
A R E M A N Y M O R E S A U N A S I N F  
I N L A N D T H A N E L S E W H E R E O  
N E S A U N A P E R E V E R Y T H R E E  
O R F O U R P E O P L E F I N N S K N O  
W W H A T A S A U N A I S E L S E W H E  
R E I F Y O U S E E A S I G N S A U N A  
O N T H E D O O R Y O U C A N N O T B E  
S U R E T H A T T H E R E I S A S A U N  
A B E H I N D T H E D O O R

## OTHER EXAMPLES of MONOALPHABETIC CRYPTOSYSTEMS

Symbols of the English alphabet will be replaced by squares with or without points and with or without surrounding lines using the following rule:

A:	B:	C:	J·	K·	L·	S	T	U
D:	E:	F:	M·	N·	O·	V	W	X
G:	H:	I:	P·	Q·	R·	Y	Z	

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D:	E:	F:	M.	N.	O.	V	W	X
G:	H:	I:	P.	Q.	R.	Y	Z	

For example the plaintext:

WE TALK ABOUT FINNISH SAUNA MANY TIMES LATER

results in the cryptotext:

□ □ □ : | . □ : | : □ | □ □ :  
□ □ : | □ : | : | □ : | □ : | □  
└ □ : | □ □ | . : | □ □ :



# OTHER EXAMPLES of MONOALPHABETIC CRYPTOSYSTEMS

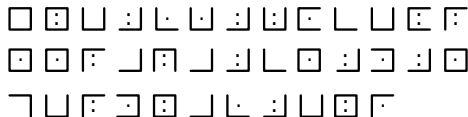
Symbols of the English alphabet will be replaced by squares with or without points and with or without surrounding lines using the following rule:

A:	B:	C:	J·	K·	L·	S	T	U
D:	E:	F:	M·	N·	O·	V	W	X
G:	H:	I:	P·	Q·	R·	Y	Z	

For example the plaintext:

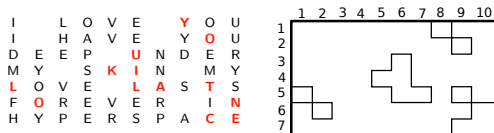
WE TALK ABOUT FINNISH SAUNA MANY TIMES LATER

results in the cryptotext:



**Garbage in between method:** the message (plaintext or cryptotext) is supplemented by “garbage letters”.

Richelieu cryptosystem used sheets of card board with holes.



# EXTREME CASES for FREQUENCY ANALYSIS

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British translation, due to Gilbert Adair, has appeared in 1994 under the title

### **A void**

## Appendix A

The Opening Paragraph of *A Void* by Georges Perec,  
translated by Gilbert Adair

Today, by radio, and also on giant hoardings, a rabbi, an admiral notorious for his links to masonry, a trio of cardinals, a trio, too, of insignificant politicians (bought and paid for by a rich and corrupt Anglo-Canadian banking corporation), inform us all of how our country now risks dying of starvation. A rumor, that's my initial thought as I switch off my radio, a rumor or possibly a hoax. Propaganda, I murmur anxiously—as though, just by saying so, I might allay my doubts—typical politicians' propaganda. But public opinion gradually absorbs it as a fact. Individuals start strutting around with stout clubs. "Food, glorious food!" is a common cry (occasionally sung to Bart's music), with ordinary hardworking folk harassing officials, both local and national, and cursing capitalists and captains of industry. Cops shrink from going out on night shift. In Mâcon a mob storms a municipal building. In Rocadamour ruffians rob a hangar full of foodstuffs, pillaging tons of tuna fish, milk and cocoa, as also a vast quantity of corn—all of it, alas, totally unfit for human consumption. Without fuss or ado, and naturally without any sort of trial, an indignant crowd hangs 26 solicitors on a hastily built scaffold in front of Nancy's law courts (this Nancy is a town, not a woman) and ransacks a local journal, a disgusting right-wing rag that is siding against it. Up and down this land of ours looting has brought docks, shops and farms to a virtual standstill.

First published in France as *La Disparition* by Editions Denöel in 1969, and in Great Britain by Harvill in 1994. Copyright © by Editions Denöel 1969; in the

# HOMOPHONIC CRYPTOSYSTEMS



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The number of substitutes of a letter is usually proportional to the frequency of the letter.

Though homophonic cryptosystems are not unbreakable, they are much more secure than ordinary monoalphabetic substitution cryptosystems.

The first known homophonic substitution cipher is from 1401.

# EXAMPLES of HOMOPHONIC CRYPTOSYSTEMS - I.

## Jindřich IV. Francouzský




Homofonní tabulku Jindřicha IV. (viz níže) určitě navrhoval François Viète, oficiální králův kryptograf, luštitel a matematik. Jde o praktickou a účinnou šifru, jakou lze čekat od autora, který zná všechny triky i jejich meze. Většina souhlásek má více variant podle jejich skutečné četnosti. Slovník obsahuje pouhá tři slova.

Tabulka zahrnuje i značkovací symbol: 

To stačí k označení všech začátků i konců bezvýznamných úseků, na rozdíl od označování textových částí z Montmorencyho tabulky.

A	B	C	D	E	F	G	H	I	J	L
ð	h	x	a	x	k	m	o	o	o	z
o		o	e	z	u			h	h	q
			u	=				h	h	
M	N	O	P	Q	R	S	T	U	X	Y
u	h	p	f	no	ff	h	h	h	x	z
z	o	h			h	o	z	q		
		h			h	h				

V kódovém seznamu najdeme jen tři slova:

odstavec =  že =  vy = 

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o		o	e	g	u			h	h	g
			u	=				u	u	
M	N	O	P	Q	R	S	T	U	X	Y
u	h	p	s	no	ff	et	t	h	x	z
g	do	g			u	o	u	g		
		h			g	h				

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## Vévoda z Montmorency

A	B	C	D	E	F	G	H	I	J	L
z	t	6	y	o	9	u	z	o	o	k
o	a	6	o	o	6	o	z	o		z
p		u		z		o		e		
		u						u		
M	N	O	P	Q	R	S	T	U	X	Y
o	z	>	h	4	f	x	z	u	x	z
o	u	g	r		l	f	z	g	x	z
□	f		x		r	f		z		
								u		
								z		

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**Example:** PLAYFAIR is encrypted as LCNMNFSC

Playfair was used in World War I by British army.

	S	D	Z	I	U
	H	A	F	N	G
Playfair square:	B	M	V	Y	W
	R	P	L	C	X
	T	O	E	K	Q

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**Second step:** For a plaintext  $w$  a key  $k$  has to be chosen that should be a word of the same length as  $w$ .

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**AUTOCLAVE-key cryptosystem:** a short keyword is chosen and appended by plaintext

# POLYALPHABETIC SUBSTITUTION CRYPTOSYSTEMS III

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## VIGENERE and AUTOCLAVE cryptosystems

Vigenère table:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

# POLYALPHABETIC SUBSTITUTION CRYPTOSYSTEMS III

## VIGENERE and AUTOCLAVE cryptosystems

Vigenère table:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Keyword:

H A M B U R G

Plaintext:

I N J E D E M M E N S C H E N G E S I C H T E S T E H T S E I N E G

Vigenere-key:

Autoclave-key:

Vigenere-encrypt..:

Autoclave-encrypt..:

# POLYALPHABETIC SUBSTITUTION CRYPTOSYSTEMS III

## VIGENERE and AUTOCLAVE cryptosystems

Vigenère table:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
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Keyword:

H A M B U R G

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I N J E D E M M E N S C H E N G E S I C H T E S T E H T S E I N E G

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B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Keyword:

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Plaintext:

I N J E D E M M E N S C H E N G E S I C H T E S T E H T S E I N E G

Vigenere-key:

H A M B U R G H A M B U R G H A M B U R G H A M B U R G H A M B U R

Autoclave-key:

H A M B U R G I N J E D E M M E N S C H E N G E S I C H T E S T E H

Vigenere-encrypt...:

P N V F X V S T E Z T W Y K U G Q T C T N A E E U Y Y Z Z E U O Y X

Autoclave-encrypt.:

P N V F X V S U R W W F L Q Z K R K K J L G K W L M J A L I A G I N

- Autoclave-key cipher is also called autokey cipher.

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- So called **running-key cipher** uses very long key that is a passage from a book (for example from Bible).



## BLAISE de VIGENERE (1523-1596)



The encryption method that is commonly called as Vigenere method was actually discovered in 1553 by Giovan Batista Belaso.

# VIGÉNERE CRYPTOSYSTEM

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- It seems that the reason for ignorance of the VIGENERE cryptosystem was its apparent complexity.

# CRYPTANALYSIS of cryptotexts produced by VIGENERE-key cryptosystems

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CHRGQPWOEIRULYANDOSHCHRIZKEBUSNOFKYWROPDCHRGKAXBNRHROAKERBKSCHRIWK

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Substring “CHR” occurs in positions 1, 21, 41, 66: expected keyword length is therefore 5.

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**Method.** Determine the greatest common divisor of the distances between identical subwords (of length 3 or more) of the cryptotext.

## Charles Babbage (1791-1871)



# FRIEDMAN METHOD to DETERMINE KEY LENGTH

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Then it holds, as shown on next slide:

$$L = \frac{0.027n}{(n-1)I - 0.038n + 0.065}, \quad I = \sum_{i=1}^{26} \frac{n_i(n_i - 1)}{n(n-1)}$$

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Once the length of the keyword is found it is easy to determine the key using the frequency analysis method for monoalphabetic cryptosystems.

# DERIVATION of the FRIEDMAN METHOD I

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- 1 Let  $n$  be the length of a cryptotext  $w$  and  $o_i$  be the number of occurrences of the  $i$ -th symbol of the alphabet in  $w$ .

# DERIVATION of the FRIEDMAN METHOD I

- 1 Let  $n$  be the length of a cryptotext  $w$  and  $o_i$  be the number of occurrences of the  $i$ -th symbol of the alphabet in  $w$ . The probability that if one selects a pair of symbols from  $w$ , then they are the same is

$$I = \frac{\sum_{i=1}^{26} o_i(o_i-1)}{n(n-1)} = \sum_{i=1}^{26} \frac{\binom{o_i}{2}}{\binom{n}{2}}$$

and it is called the **index of coincidence**.



# DERIVATION of the FRIEDMAN METHOD I

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Since  $I = \frac{A}{\frac{n(n-1)}{2}} = \frac{1}{L(n-1)} [0.027n + L(0.038n - 0.065)]$

one gets the formula for  $L$  from one of the previous slides.

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to decode columns use decoding method for Caesar

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The reuse of keys by Soviet Union spies (due to the manufacturer's accidental duplication of one-time-pad pages) enabled US cryptanalysts to unmask the atomic spy Klaus Fuchs in 1949.

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**Example** ONE-TIME PAD cryptosystem is perfectly secure because for any pair  $c, p$  there exists a key  $k$  such that

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**IDEA:** Find a simple way to generate almost perfectly random key shared by both communicating parties and make them to use this key for one-time pad encoding and decoding!!!!

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- Moreover, modern block ciphers often include smaller substitution tables, called S-boxes.



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**Example**

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The **basic idea** is very simple: **permute the plaintext to get the cryptotext**. Less clear it is how to specify and perform efficiently permutations.

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$$a^2 c d e f^3 g^2 i^2 j k m n^3 o^5 p r s^2 t^2 u^3 z$$

**Solution: ??**

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**Example:** keyword: HOW MANY ELKS,  $k = 8$

0 8  
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z  
P Q R T U V X Z H O W M A N Y E L K S B C D F G I J

## KEYWORD CAESAR - Example I

**Example** Decrypt the following cryptotext encrypted using the KEYWORD CAESAR and determine the keyword and  $k$

T I V D Z C R T I C F Q N I Q T U T F  
Q X A V F C Z F E Q X C P C Q U C Z W K  
Q F U V B C F N R R T X T C I U A K W T Y  
D T U P M C F E C X U U V U P C B V A N H C  
V R U P C F E Q X C U P C F U V B C  
X V I U Q T I F F U V I C F N F N Q A A K  
V I U P C U V E U V U Q G C Q F Q N I Q  
W Q U P T U T F Q A F V I C X C F F Q M K  
U P Q U U P C F U V B C T F E M V E C M A K  
P C Q U C Z Q I Z U P Q U K V N P Q B C  
U P C R Q X T A T U K V R U P M V D T I Y  
D Q U C M V I U P C F U V I C F

# KEYWORD CAESAR - Example II

Step 1. Make the frequency counts:

	Number		Number		Number
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UPC is likely to be THE.

Let us now decrypt the remaining letters in the high frequency group: F,V,I

From the words TU, TF  $\Rightarrow$  F=S

From UV  $\Rightarrow$  V=O

From VI  $\Rightarrow$  I=N

## CONTINUATION

So we have: T=I, Q=A, U=T, P=H, C=E, F=S, V=O, I=N and now in

```
T   I V D   Z C R T I C   F Q N I Q   T U   T F
Q   X A V F C Z   F E Q X C   P C Q U C Z   W K
Q   F U V B C   F N R R T X T C I U A K   W T Y
D T U P   M C F E C X U   U V   U P C   B V A N H C
V R   U P C   F E Q X C   U P C   F U V B C
X V I U Q T I F   F U V I C F   N F N Q A A K
V I   U P C   U V E   U V   U Q G C   Q   F Q N I Q
W Q U P   T U   T F   Q A F V   I C X C F F Q M K
U P Q U   U P C   F U V B C   T F   E M V E C M A K
P C Q U C Z   Q I Z   U P Q U   K V N   P Q B C
U P C   R Q X T A T U K   V R   U P M V D T I Y
D Q U C M   V I   U P C   F U V I C F
```

we have several words with only one unknown letter what leads to another guesses and the table:

```
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
L V E W P S K M N ? Y ? R U ? H A F ? I T O B C G D
```

This leads to the keyword **CRYPTOGRAPHY GIVES ME FUN** and  $k = 4$  -

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- Possible plaintexts are **thatis, ofyour, season, oxford, thatof,....** but there is no way to determine the plaintext uniquely.

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Empirical evidence indicates that if a simple substitution cryptosystem is applied to a meaningful English message, then about 25 cryptotext characters are enough for an experienced cryptanalyst to recover the plaintext.

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- If  $K$  is chosen random, then  $H(K) = |K|$ , so for randomly chosen letters in English,  
 $H(K) = |K| = \lg(26) = 4.7$  bits.
- $H(M)$  has been empirically found to be 2.9 bits for English.
- Therefore the unicity distance for English is 1 when  
 $|M| = (4.7/1.8)|K|$

# ANAGRAMS – EXAMPLES

German:

IRI BRÄTER, GENF

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IRI BRÄTER, GENF	Briefträgerin
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## English:

algorithms	logarithms
antagonist	stagnation
compressed	decompress
coordinate	decoration
creativity	reactivity
deductions	discounted
descriptor	predictors
impression	permission
introduces	reductions
procedures	reproduces

# SOME SOLUTIONS



FRANK PEKL, REGEN

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PEER ASTIL, MELK

Krankenpfleger

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## APPENDIX I

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- Till recently it was assumed that secret codebooks are necessary for secret communication.



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- Famous was the nomenclator designed by very famous French cryptologist Rosignol, for Ludvig XIV, that was not broken for several hundred of years.
- It was the design of the telegraph and the need for *field ciphers* to be used in combat that ended the massive use of nomenclators and started a new history of cryptography dominated by polyalphabetic substitution cryptosystems.