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DO YOU KNOW

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IV045, CODING THEORY, CRYPTOGRAPHY

and

CRYPTOGRAPHIC PROTOCOLS - 2020

Prof. Jozef Gruska
http://www.fi.muni.cz/usr/gruska/crypto20

Technické řešení této výukové pomůcky je spolufinancováno Evropským sociálním fondem a státním rozpočtem České republiky.

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

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  It ill be possible to ask questions related to tutorials (or lectures) via Google Hangout Meets (each Thursday in time 9.00-9.15 or at 10.00-10.15) or via discussion forums of tutorials or course.

Likely, the most efficient use of the lectures is to print materials of each lecture before the lecture and to make your comments into them during the lecture.

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- Whenever you find an error or a misprint in the lecture notes, let me know - extra points you get for that.

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RNDR. Matej Pivoluska PhD
RNDR Luděk Matyska, doctorant
RNDr Libor Caha, doctorant
Bc Henrieta Michelova, one of the best of the 2019 course IV054

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HISTORY OF CRYPTOGRAPHY

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Cryptography, when broadly understood, is an important tool to achieve such goals.

STORY I.

Mary - Queen of Scotts - picture

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Mary - Queen of Scotts - curriculum

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Born: 1542

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- After spending 19 years in jail a group of her admirers established a communication with Mary with the goal to free Mary (and to put Mary on the tron in England).
- Main cryptographer of Elisabeth I, Sir Francis Walsingham, expected that and was able to decrypt special encrypted communication between Mary and her admirers.

Mary - cryptosystem she used

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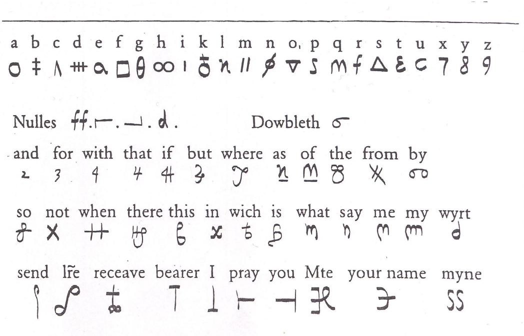

Figure 8 The nomenclator of Mary Queen of Scots, consisting of a cipher alphabet and codewords.

The above Cryptosystem was used for communication between Mary - the Queen of Scots and her admirers, headed by nobleman Anthony Babbington, trying to free her.

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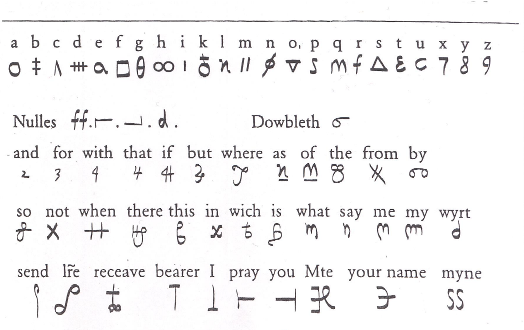

Figure 8 The nomenclator of Mary Queen of Scots, consisting of a cipher alphabet and codewords.

The above Cryptosystem was used for communication between Mary - the Queen of Scots and her admirers, headed by nobleman Anthony Babbington, trying to free her. She was then accused of a plot to kill the Queen Elizabeth I of England, her sister in law ,and sentence to death.

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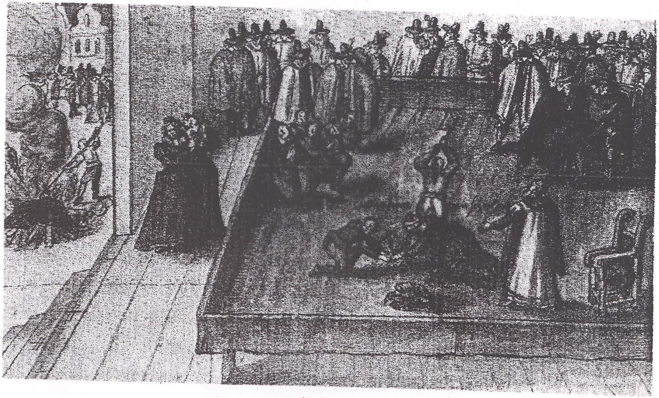

Figure 10 The execution of Mary Queen of Scots. 8.2.1587

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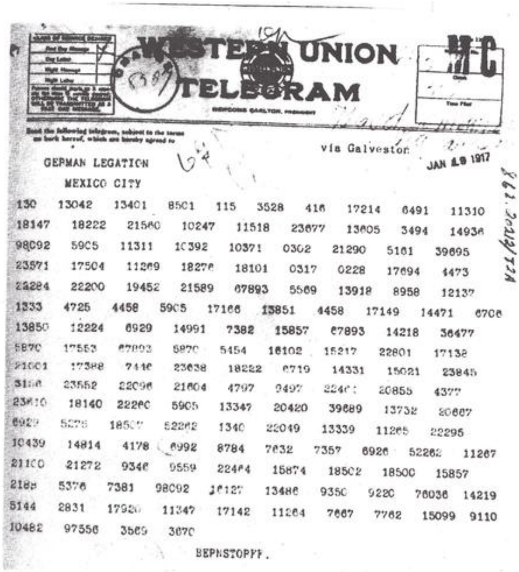
This telegram was captured and decoded by British. They used the telegram to convince US president to declare war to Germany what very much influenced the outcome of the WWI.

Zimmerman's telegram II.


WESTERN UNION TELEGRAM

via Galveston

JAN 19 1917

GERMAN LEGATION

MEXICO CITY

130 13042 13401 8501 115 3528 416 17214 6491 11310
18147 18222 21560 10247 11518 23677 13605 3494 14936
98092 5905 11311 10392 10371 0302 21290 5161 39695
23571 17504 11269 18276 18101 0317 0228 17694 4473
22284 22200 19452 21589 67893 5569 13918 8958 12137
1333 4725 4458 5905 17166 13851 4458 17149 14471 6706
13850 12224 6929 14991 7382 15857 67893 14218 36477
5870 17553 67893 5870 5454 16102 15217 22801 17138
21001 17388 7446 23638 18222 6719 14331 15021 23845
3156 23552 22096 21604 4797 9497 22464 20855 4377
23610 18140 22260 5905 13347 20420 39689 13732 20667
6929 5275 18507 52262 1340 22049 13339 11265 22295
10439 14814 4178 6992 8784 7632 7357 6926 52262 11267
21100 21272 9346 9559 22464 15874 18502 18500 15857
2188 5376 7381 98092 16127 13486 9350 9220 76036 14219
5144 2831 17920 11347 17142 11264 7667 7762 15099 9110
10482 97556 3569 3670

BERNSTORFF.

Figure 28 The Zimmermann telegram, as forwarded by von Bernstorff, the German Ambassador in Washington, to

Part I

Basics of the coding theory

PROLOGUE - I.

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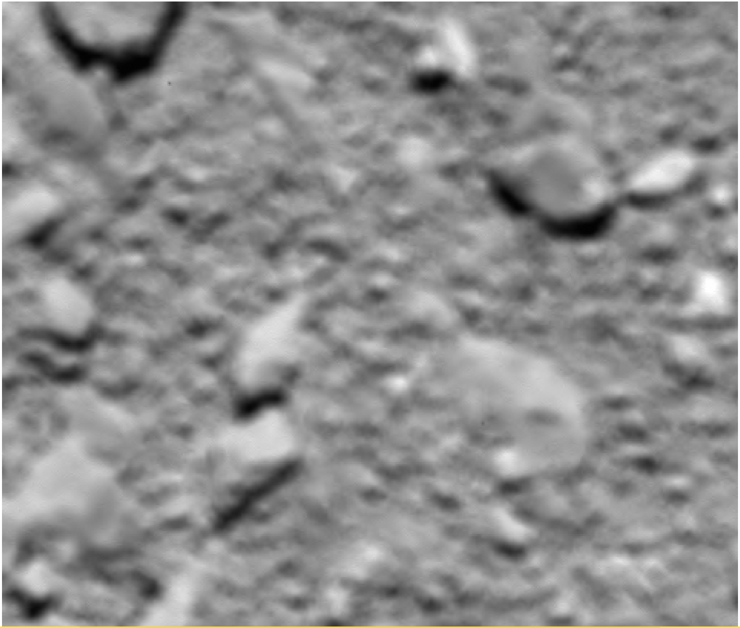
- **Since that time coding theory has made another enormous progress that has allowed, among other things, almost perfect mobile communication and transmission of music in time and space.**

ROSETTA spacecraft

ROSETTA LANDING - VIEW from 21 km -29.9.2016

ROSETTA LANDING - VIEW from 51 m -29.9.2016



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CHAPTER 1: BASICS of CODING THEORY

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This first chapter presents and illustrates the very basic problems, concepts, methods and results of coding theory.

PROLOGUE - II.

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using mainly classical, but also quantum tools.

CODING - BASIC CONCEPTS

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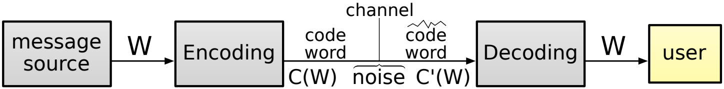
Error-correcting codes are used to correct messages when they are (erroneously) transmitted through noisy channels.



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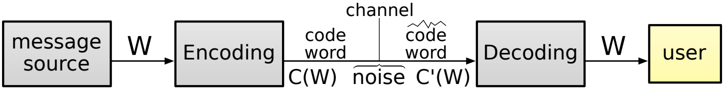

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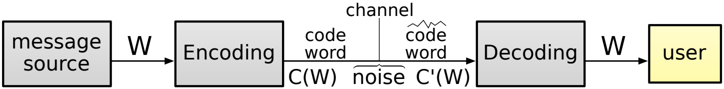

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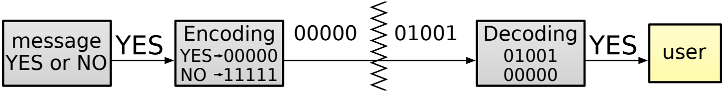

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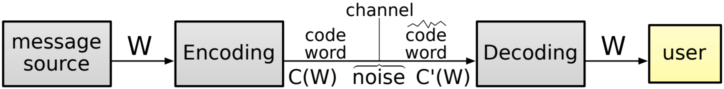

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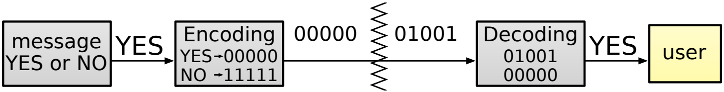

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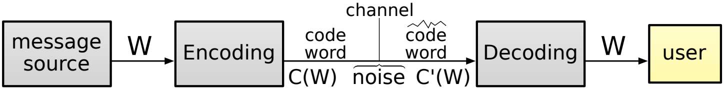

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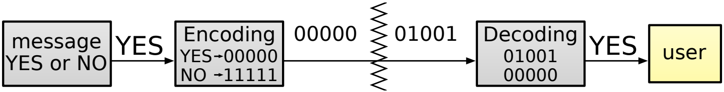

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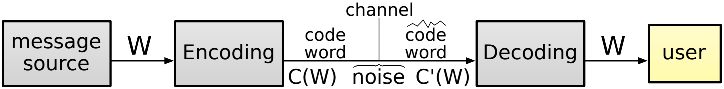

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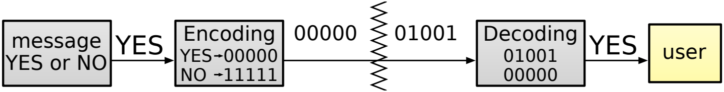

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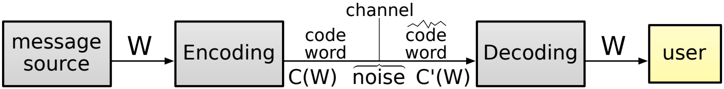

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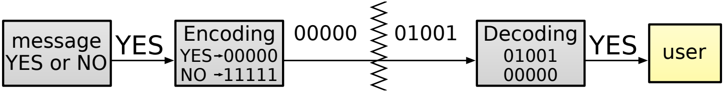

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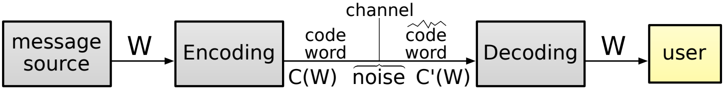

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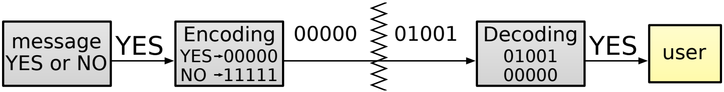

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Summary: The task of a communication channel coding is to encode the information to be sent over the channel in such a way that even in the presence of some channel noise, several (a specific number of) errors can be detected and/or corrected.

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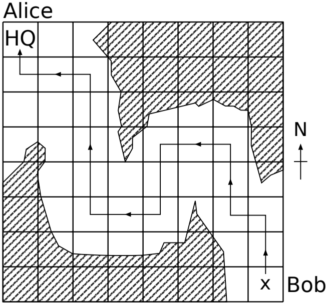

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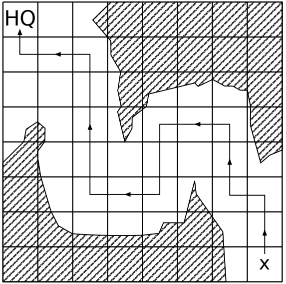

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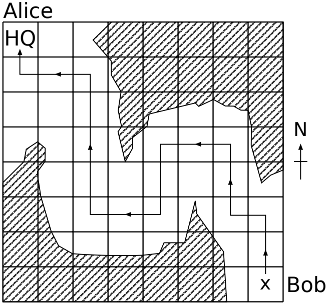

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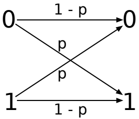
Consider a transition of binary symbols such that each symbol has probability of error $p < \frac{1}{2}$.



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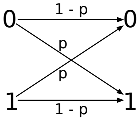

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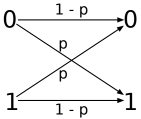

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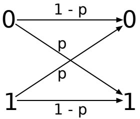

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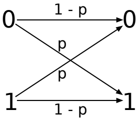

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Corollary One undetected error occurs only once every 2000 days! ($2000 \approx \frac{10^9}{5.5 \times 86400}$).

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is represented and encoded as follows

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Question How much better is two-dimensional encoding than one-dimensional encoding?

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Comment: A good (n, M, d)-code has small n, large M and also large d.

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Code rate (6/32 for Hadamard code), is an important parameter for real implementations, because it shows what fraction of the communication bandwidth is being used to transmit actual data.

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In such a case:

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For details about 13-digit ISBN see

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Lemma Any q-ary (n, M, d)-code over an alphabet $\{0, 1, \ldots, q-1\}$ is equivalent to an (n, M, d)-code which contains the all-zero codeword $00\ldots0$.

Proof Trivial.

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EXAMPLE

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Example Proof that $A_2(5,3) = 4$.

(a Code C_3, page (**??**), is a $(5,4,3)$-code, hence $A_2(5,3) \geq 4$.

(b Let C be a $(5,M,3)$-code with $M = 5$.

- By previous lemma we can assume that $00000 \in C$.
- C has to contain at most one codeword with at least four 1's. (otherwise $d(x,y) \leq 2$ for two such codewords x, y)
- Since $00000 \in C$, there can be no codeword in C with at most one or two 1.
- Since $d = 3$, C cannot contain three codewords with three 1's.
- Since $M \geq 4$, there have to be in C two codewords with three 1's. (say 11100, 00111), the only possible codeword with four or five 1's is then 11011.

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Find a position in which x, y differ and delete this position from all codewords of D.
Resulting code is an (n, M, d)-code.

A COROLLARY

Corollary:
If d is odd, then $A_2(n,d) = A_2(n+1, d+1)$.
If d is even, then $A_2(n,d) = A_2(n-1, d-1)$.

Example $A_2(5,3) = 4 \Rightarrow A_2(6,4) = 4$
$(5,4,3)$-code $\Rightarrow$ $(6,4,4)$-code

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{matrix}$$ by adding check.

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$$S(u, r) = \{v \in F_q^n \mid h(u, v) \leq r\}.$$

Theorem A sphere of radius r in F_q^n, $0 \leq r \leq n$ contains

$$\binom{n}{0} + \binom{n}{1}(q-1) + \binom{n}{2}(q-1)^2 + \ldots + \binom{n}{r}(q-1)^r$$

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A SPHERE and its VOLUME

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Proof Let u be a fixed word in F_q^n. The number of words that differ from u in m positions is

$$\binom{n}{m}(q-1)^m.$$

GENERAL UPPER BOUNDS on CODE PARAMETERS

Theorem **(The sphere-packing (or Hamming) bound)**
If C is a q-nary $(n, M, 2t+1)$-code, then

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Singleton bound: If C is an q-ary (n, M, d) code, then

$$M \leq q^{n-d+1}$$

A GENERAL UPPER BOUND on $A_q(n, d)$

Example An $(7, M, 3)$-code is perfect if

$$M\left(\binom{7}{0} + \binom{7}{1}\right) = 2^7$$

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An example of such a code:

$C4 = \{0000000, 1111111, 1000101, 1100010, 0110001, 1011000, 0101100,$
$0010110, 0001011, 0111010, 0011101, 1001110, 0100111, 1010011, 1101001, 1110100\}$

Table of $A_2(n,d)$ from 1981

n	$d = 3$	$d = 5$	$d = 7$
5	4	2	-
6	8	2	-
7	16	2	2
8	20	4	2
9	40	6	2
10	72-79	12	2
11	144-158	24	4
12	256	32	4
13	512	64	8
14	1024	128	16
15	2048	256	32
16	2560-3276	256-340	36-37

For current best results see http://www.codetables.de

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For example, two main requirements for many telegraphy codes used to be:

- Any two codewords had to have distance at least 2;
- No codeword could be obtained from another codeword by transposition of two adjacent letters.

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To transmit pictures Voyager used the so called Golay code G_{24}.

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Basic idea: Encode more (less) probable outputs of X by shorter (longer) binary words.
Example (Moorse code - 1838)

a .-	b -...	c -.-.	d -..	e .	f ..-.	g –.
h	i ..	j .—	k -.-	l .-..	m –	n -.
o —	p .–.	q –.-	r .-.	s ...	t -	u ..-
v ...-	w .–	x -..-	y -.–	z –..		

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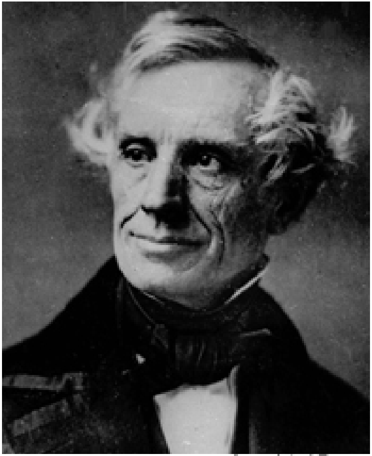
Samuel Moorse



Associated Press

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mess.	code	mess.	code	mess.	code	mess.	code
0000	10	0100	010	1000	011	1100	11101
0001	000	0101	11001	1001	11011	1101	111110
0010	001	0110	11010	1010	11100	1110	111101
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Observe that this is a **prefix code** - no codeword is a prefix of another codeword.

DESIGN of HUFFMAN CODE II

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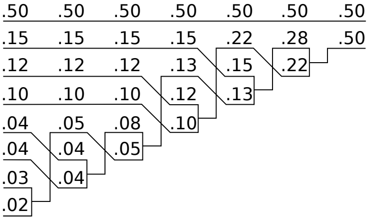

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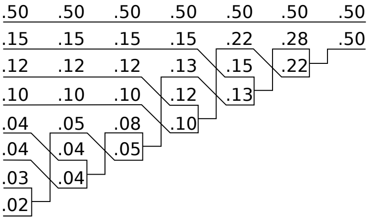

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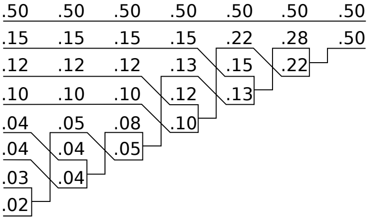

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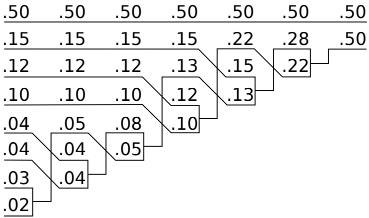

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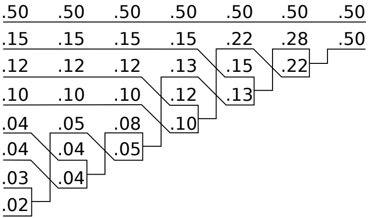

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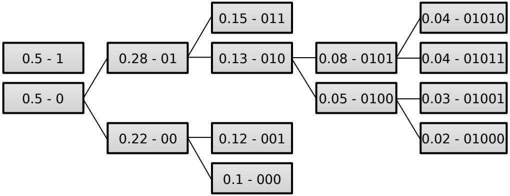

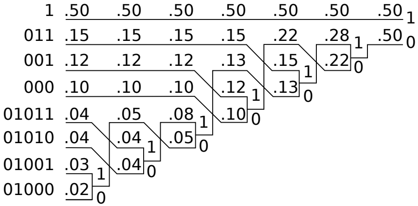

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Originally, information theory was a part of electrical engineering. Nowadays, it is an important part of mathematics and also of informatics.

The concept of **ENTROPY** is one of the most basic and important in modern science, especially in physics, mathematics and information theory.

So called **physical entropy** is a measure of the unavailable energy in a closed thermodynamics system (that is usually considered to be a measure of the system's disorder).

Entropy of an object is a measure of the amount of energy in the object which is unable to do some work.

Entropy is also a measure of the number of possible arrangements of the atoms a system can have.

Information entropy

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On the other side, if we have a process (random variable) Y producing value 0 with probability $\frac{1}{4}$ and value 1 with probability $\frac{3}{4}$, then we are more certain that the next value of the process will be 1 than 0.

History Rudolf Clausius coined the term **entropy** in 1865.

SHANNON's VIEW

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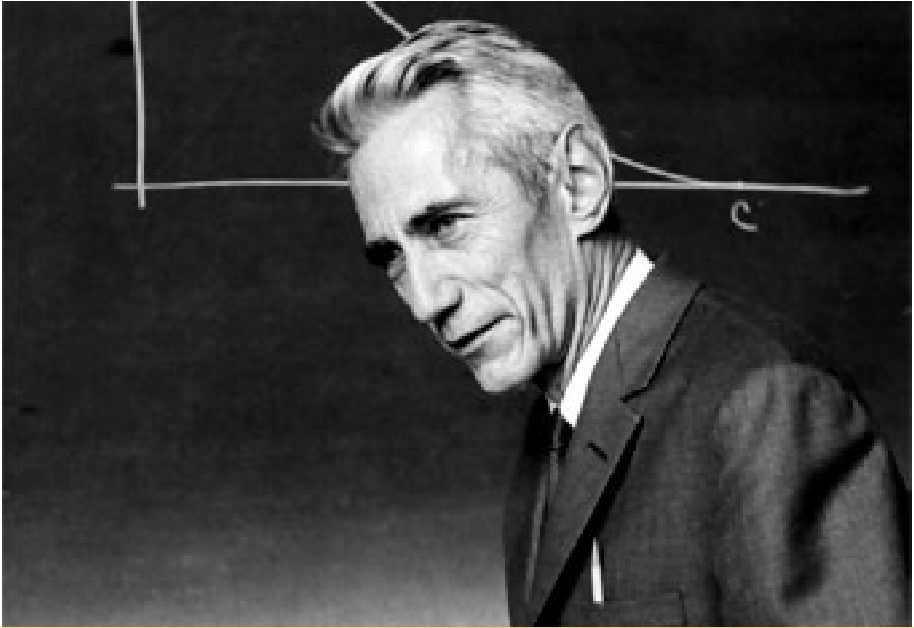
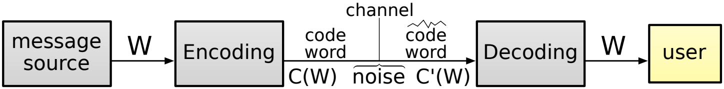


APPENDIX

HARD VERSUS SOFT DECODING I

At the beginning of this chapter the process **encoding-channel transmission-decoding** was illustrated as follows:



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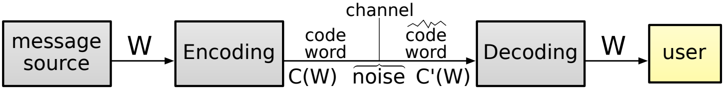

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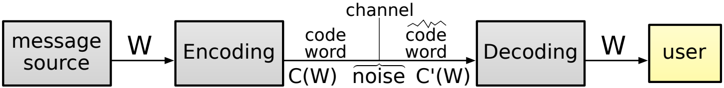

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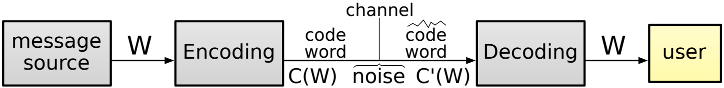

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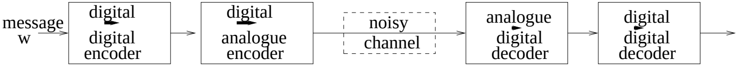

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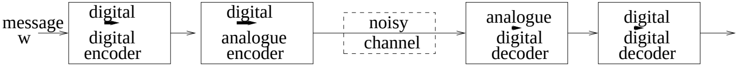

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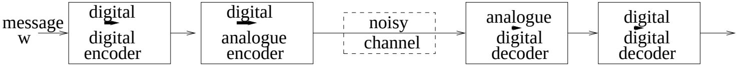

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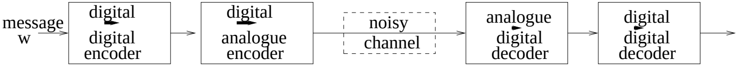

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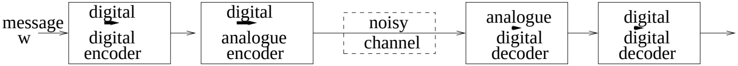

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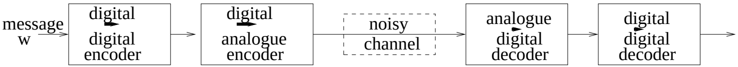

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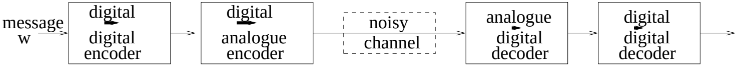

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In case the output of analogous-digital decoding is a pair (p_b, b) where p_b is the probability that the output is the bit b (or a weight of such a binary output (often given by a number from an interval $(-V_{max}, V_{max})$), we talk about a **soft decoding**.

HARD versus SOFT DECODING III

In order to deal with such a more general model of transmission and soft decoding, it is common to use, instead of the binary symbols 0 and 1 so-called **antipodal binary symbols** $+1$ and -1 that are represented electronically by voltage $+1$ and -1.

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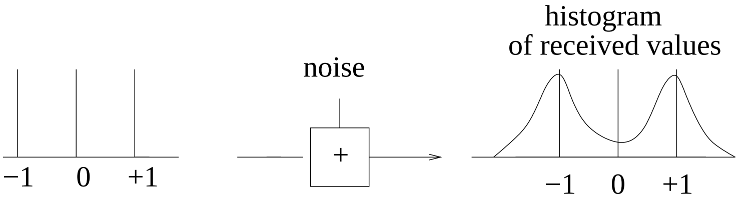

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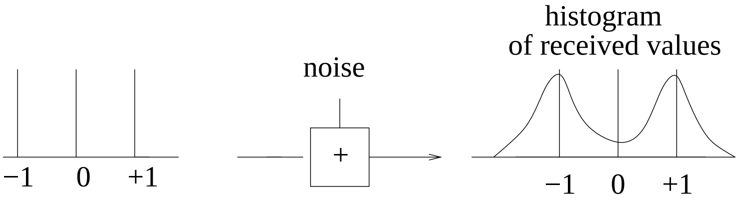

A very important case in practise, especially for space communication, is so-called additive white Gaussian noise (AWGN) and the channel with such a noise is called Gaussian channel.

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For example, in an important practical case of the Gaussian white noise one search at the minimal likelihood decoding for a codeword with minimal Euclidean distance.

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Hard decoding is used mainly for block codes and soft one for stream codes. However, distinctions between these two families of codes are tending to blur.

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For the same code there can be many encoding algorithms that map the same set of datawords into different codewords.

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- The era of Morse telegraph ended on 26.1.2006 when the main telegraph company in US, Western Union, announced cancelation of all telegraph services.

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The binary elements 0 and 1 were first called bits by J. W. Tuckley in 1943.