The *crossing number* $cr(G)$ of a graph G is a measure of the nonplanarity of G. It is well understood exactly under which circumstances a graph can be drawn in the plane without any crossings of edges. When this is not possible, one is interested in finding an *optimal* drawing, that is, a drawing with the minimum possible number of crossings of edges, or at least a drawing close to optimal. Crossing number questions were first raised by Turán in the mid 1940's. Besides their intrinsical theoretical interest, they have important applications, most importantly in the area of VLSI layout.

Turán's original problem was the calculation of the crossing number $cr(K_{m,n})$ of the complete bipartite graph $K_{m,n}$. There are natural drawings of $K_{m,n}$ with exactly $Z_{m,n}$ crossings, where $Z_{m,n} := [(m-1)/2][m/2][(n-1)/2][n/2]$. No drawing of $K_{m,n}$ with fewer crossings has ever been found, and so it has been long conjectured that $cr(K_{m,n}) = Z_{m,n}$. This has been verified only for $m < 7$ (and arbitrary n). The exact knowledge of $cr(K_{6,n})$ implies the best general bound known previous to our work, namely $cr(K_{m,n}) >= 0.8Z_{m,n}$.

In this talk we will report on some recent research on this problem. We attacked the problem of estimating $cr(K_{7,n})$ for large values of n. We analyzed combinatorial and topological properties of drawings of $K_{7,n}$, and came up with a quadratic programming problem whose solution gives a lower bound for $cr(K_{7,n})$.

The quadratic programming problem we originally obtained is quite large (the associated matrix is of size 720×720). To solve this problem, we used state-of-the-art quadratic programming techniques, combined with a bit of invariant theory of permutation groups. As a result, we were finally able to give the improved bound $cr(K_{7,n}) >= 2.1796n^2 - 4.5n$, quite close to $Z_{7,n} = 2.25n^2 + O(n)$. Our bound for $cr(K_{7,n})$ implies that, for each fixed $m > 8$, $lim_{n->infty}cr(K_{m,n})/Z(m,n) >= 0.83m/(m-1)$. We also obtained as a byproduct an improved bound for the crossing number of the complete graph K_n.