Propositional logic

- propositional letters, propositions
- connectives $\wedge$, $\vee$, $\longrightarrow$, $\leftrightarrow$, $\neg$
- truth tables
- adequacy, adequate set of truth connectives
- truth assignement

  assigns to each propositional letter a unique truth value

  truth valuation

  assigns to each proposition a unique truth value
- valid proposition, tautology
- logically equivalent propositions

Propositional logic: axiomatic approach

- axioms (A, B, C = formulas):

  $\mathbf{A_1}$ $A \Rightarrow (B \Rightarrow A)$

  $\mathbf{A_2}$ $(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$

  $\mathbf{A_3}$ $(\neg B \Rightarrow \neg A) \Rightarrow (A \Rightarrow B)$

- inference rule *modus ponens (MP)*

$$\frac{A \qquad A \Rightarrow B}{B}$$

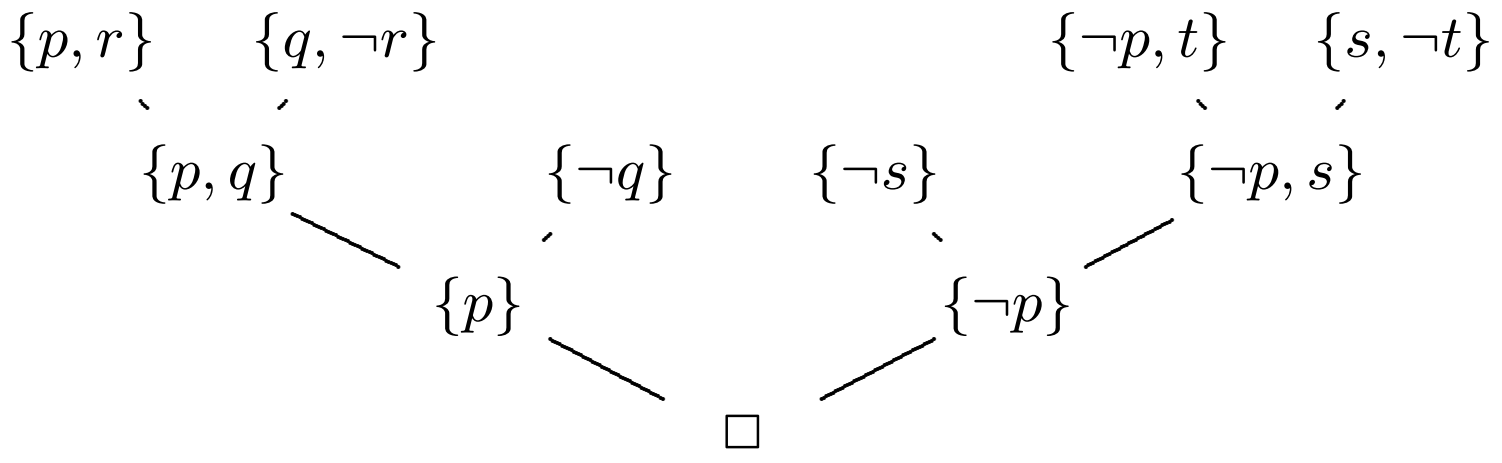
Resolution in propositional logic – example

- $S \vdash_R \square$?

  $S = (p \vee r) \wedge (q \Leftarrow r) \wedge \neg q \wedge (t \Leftarrow p) \wedge \neg s \wedge (s \Leftarrow t)$

  $S = (p \vee r) \wedge (q \vee \neg r) \wedge \neg q \wedge (\neg p \vee t) \wedge \neg s \wedge (s \vee \neg t)$

  $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$



Refining resolution I

- to narrow search space - SAT=$\{S \mid S$ is satisfiable $\}$ is NP-complete
  - to terminate the search along paths that are unpromising
  - to specify the oder in which to go down alternative paths

Refining resolution II

- if there is a literal that is only positive(negative), remove all clauses that contain such a literal
- *T-resolution* : no parent clause is a tautology
- *Semantic resolution*. Let $\mathcal{I}$ be an interpretation. Semantic resolution with respect to $\mathcal{I}$ permits applications of the resolution rule only when at least one of their premises has a ground instance which is not satisfied by $\mathcal{I}$
- *Ordered resolution.* The propositional letters are indexed resolve on the literal with the higher indexthan any othe in the parent clauses

- *Lock resolution.* Each occurence of a literal has a distinct index the literal resolved on has in each parent the lowest index