Modal Logic

to make precise the properties of

possibility, necessity, belief, knowledge, temporal reasoning

$\Box\phi$ - "it is necessary that ϕ" "ϕ will always be true"

$\Diamond\phi$ - "it is possible that ϕ" "ϕ will eventually be true"

Modal Logic II

If $\mathcal{L}$ is a language for (classical) predicate logic, we extend it to a *modal language* $\mathcal{L}_{\Box,\Diamond}$ by adding two new primitive symbols $\Box$ and $\Diamond$, and a new clause to the definition of formulas

If ϕ is a formula, then so are $(\Box\phi)$ and $(\Diamond\phi)$.

Semantics(informal) Relation between $\Box$ and $\Diamond$ similar to that between $\forall$ and $\exists$

Kripke frames - a collection W of *possible worlds*
$w \Vdash \phi$ "ϕ is true at w"

Semantics for modal logic

$\mathcal{L}$ has at least one constant symbol but no function symbols other than constants

$C = \{W, S, \{C(p)\}_{p \in W}\}$

W ... a set of words
S ... an accessibility (or successor) relation between worlds
$C(p)$ ... an assignement of a classical structure $C(p)$ for $\mathcal{L}$ to each $p \in W$

C is a *frame* for the language $\mathcal{L}$ ($\mathcal{L}$*-frame*) if, for every p and q in W,
pSq implies that $C(p) \subseteq C(q)$ and
the interpretation of constants in $\mathcal{L}\ (p) \subseteq \mathcal{L}\ (q)$ are the same in $C(p)$ as in $C(q)$.

Forcing for frames

Let $C = \{W, S, \{C(p)\}_{p \in W}\}$ be a frame for a language $\mathcal{L}$, p be in W and ϕ be a sentence of the language $\mathcal{L}\ (p)$.

p forces ϕ, written $p \Vdash \phi$

1. For atomic ϕ, $p \Vdash \phi \Leftrightarrow \phi$ is true in $C(p)$.
2. $p \Vdash (\phi \to \psi) \Leftrightarrow p \Vdash \phi$ implies $p \Vdash \psi$.
3. $\wedge$, vee ; $\forall$, $\exists$ ... c in $\mathcal{L}\ (p)$
4. $p \Vdash \neg\phi \Leftrightarrow$. p does not force ϕ, $p \nVdash \phi$.
5. $p \Vdash \Box\phi \Leftrightarrow$ for all $q \in W$ such that pSq, $q \Vdash \phi$.
6. $p \Vdash \Diamond\phi \Leftrightarrow$ there is a $q \in W$ such that pSq and $q \Vdash \phi$.

Modal tableaux

signed forcing assertions $Tp \Vdash \psi, Fp \Vdash \psi$