Inductive inference

Based on Graham Priest's Logic: A very short introduction, 2000

Translation Petr Hromek (Logika, Dokořán 2007)

The Red-Headed League

Sherlock Holmes's quick eye took in my occupation, and he shook his head with a smile as he noticed my questioning glances. "Beyond the obvious facts that he has at some time done manual labor, that he takes snuff, that he is a Freemason, that he has been in China, and that he has done a considerable amount of writing lately, I can deduce nothing else."

Mr. Jabez Wilson started up in his chair, with his forefinger upon the paper, but his eyes upon my companion.

How, in the name of good fortune, did you know all that, Mr. Holmes?" he asked. "How did you know, for example, that I did manual labor? It's as true as gospel, for I began as a ship's carpenter."

"Your hands, my dear sir. Your right hand is quite a size larger than your left. You have worked with it and the muscles are more developed."

"Well, the snuff, then, and the Freemasonry?"

"I won't insult your intelligence by telling you how I read that, especially as, rather against the strict rules of your order, you use an arc and compass breastpin."

"Ah, of course, I forgot that. But the writing?"

"What else can be indicated by that right cuff so very shiny for five inches, and the left one with the smooth patch near the elbow where you rest it upon the desk."

"Well, but China?"

"The fish which you have tattooed immediately above your wrist could only have been done in China. I have made a small study of tattoo marks, and have even contributed to the literature of the subject. That trick of staining the fishes' scales of a delicate pink is quite peculiar to China. When, in addition, I see a Chinese coin hanging from your watch chain, the matter becomes even more simple."

Arthur Conan Doyle: Red-headed League

http://www.eastoftheweb.com/short-stories/UBooks/RedHead.shtml

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Example 1

Guess a number.

Example 1

Guess a number.

Given a number, guess the next

Example 1

Guess a number.

your guess

Example 1

Guess a number.

your guess

1 Next ?

Example 1

Guess a number.

your guess

1 Next ? 2

Example 1

Guess a number.

your guess

1 Next ? 2

Yes, 2. Next ?

Example 1

Guess a number.

your guess

1 Next ? 2

Yes, 2. Next ? 3

Example 1

Guess a number.

your guess

1 Next ? 2

Yes, 2. Next ? 3

Yes, 3. Next ?

Example 1

Guess a number.

your guess

1 Next ? 2

Yes, 2. Next ? 3

Yes, 3. Next ? 4

Example 1

Guess a number.

your guess

1 Next ? 2

Yes, 2. Next ? 3

Yes, 3. Next ? 4

No, 5. Next ?

Example 1

Guess a number.

	your guess
1 Next ?	2
Yes, 2. Next ?	3
Yes, 3. Next ?	4
No, 5. Next ?	7

Example 1

Guess a number.

your guess

1 Next ? 2

Yes, 2. Next ? 3

Yes, 3. Next ? 4

No, 5. Next ? 7

Yes, 7. Next ?

Example 1

Guess a number.

your guess

1	Next ?	2
Yes, 2.	Next ?	3
Yes, 3.	Next ?	4
No, 5.	Next ?	7
Yes, 7.	Next ?	11

Example 1

Guess a number.

your guess

1 Next ? 2

Yes, 2. Next ? 3

Yes, 3. Next ? 4

No, 5. Next ? 7

Yes, 7. Next ? 11

Yes, 11. Next?

Example 1

Guess a number.

	your guess
1 Next ?	2
Yes, 2. Next ?	3
Yes, 3. Next ?	4
No, 5. Next ?	7
Yes, 7. Next ?	11
Yes, 11. Next?	prime numbers ?

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Guess a number.

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Yes, 2. Next ?	3
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No, 5. Next ?	7
Yes, 7. Next ?	11
Yes, 11. Next?	prime numbers ?
Yes.	

Example 1: Guess a number

- Knowing $a_1, a_2, a_3, \ldots$, find a function $f(n) = a_n$

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Example:

0, 0, 0, Next?

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Example:

0, 0, 0, Next? 0

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Example:

0, 0, 0, Next? 0 No, 6

Example 1: Guess a number

- Knowing $a_1, a_2, a_3, \ldots$, find a function $f(n) = a_n$

- Can we always find a solution?
- f(i) must be fixed during the process of inference
- Can we always find a solution?
- f(i) must not be dependent on the answer
- Can we always find a solution?
- $a_1, a_2, a_3, \ldots$ must be "representative"

Example:

0, 0, 0, Next? 0 No, 6

f(n) = (n-1)*(n-2)*(n-3)

Example 2

	raining	warm
Mon		
Tue	yes	
Wed	yes	yes
Thu		yes
Fri		
Sat	yes	yes
Sun		yes

Example 2

	raining	warm	
Mon			P(raining) =
Tue	yes		
Wed	yes	yes	
Thu		yes	
Fri			
Sat	yes	yes	
Sun		yes	

Example 2

	raining	warm
Mon		
Tue	yes	
Wed	yes	yes
Thu		yes
Fri		
Sat	yes	yes
Sun		yes

P(raining) = 3/7

Example 2

	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) =
Wed	yes	yes	
Thu		yes	
Fri			
Sat	yes	yes	
Sun		yes	

Example 2

	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	
Thu		yes	
Fri			
Sat	yes	yes	
Sun		yes	

Example 2

	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) =
Thu		yes	
Fri			
Sat	yes	yes	
Sun		yes	

Example 2

	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) =
			= P(warm & raining)/p(raining)
Thu		yes	
Fri			
Sat	yes	yes	
Sun		yes	

Example 2

	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) =
			= P(warm & raining)/p(raining)
Thu		yes	= (2/7)/(3/7) = 2/3
Fri			
Sat	yes	yes	
Sun		yes	

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	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) =
			= P(warm & raining)/p(raining)
Thu		yes	= (2/7)/(3/7) = 2/3
			P(¬warm|raining) =
Fri			
Sat	yes	yes	
Sun		yes	

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	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) =
			= P(warm & raining)/p(raining)
Thu		yes	= (2/7)/(3/7) = 2/3
			P(¬warm|raining) =
Fri			= P(¬warm & raining)/p(raining)
Sat	yes	yes	
Sun		yes	

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	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) =
			= P(warm & raining)/p(raining)
Thu		yes	= (2/7)/(3/7) = 2/3
			P(¬warm|raining) =
Fri			= P(¬warm & raining)/p(raining)
			= (1/7)/(3/7) = 1/3
Sat	yes	yes	
Sun		yes	

Example 2

	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) =
			= P(warm & raining)/p(raining)
Thu		yes	= (2/7)/(3/7) = 2/3
			P(¬warm|raining) =
Fri			= P(¬warm & raining)/p(raining)
			= (1/7)/(3/7) = 1/3
Sat	yes	yes	P(warm|raining) > P(¬warm|raining)
Sun		yes	

Example 2

	raining	warm	
Mon			P(raining) = 3/7
Tue	yes		P(warm) = 4/7
Wed	yes	yes	P(warm|raining) = ?
			= P(warm & raining)/p(raining)
Thu		yes	= (2/7)/(3/7) = 2/3
			P(¬warm|raining) =
Fri			= P(¬warm & raining)/p(raining)
			= (1/7)/(3/7) = 1/3
Sat	yes	yes	P(warm|raining) > P(¬warm|raining)
			=>
Sun		yes	warm **inductively entails from the premise** raining

Inductive entailment

$P(H|E) > P(\neg H|E) \Rightarrow H$ inductively entails from E

E … description of a reference group (database,corpus)

$E = E^+ \cup E^-$

E^+ … positive examples, E^- … negative examples

more than 2 classes:

$E = \cup E_i$

$H_i = \text{argmax}(i)\ P(H_i|E)$

Back to the Read-headed league

H = Jabez Wilson did manual labor

¬H = Jabez Wilson did not manual labor

E = London inhabitants ?

= rich people ?

= Sherlock Holmes' clients ?

= human beings ?