FO Model Checking of Interval Graphs

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    ↑ $\mathbf{MSO_1}$ vs. $\mathbf{MSO_2}$ ↓
  - or, quantifies vertex and edge sets together $\exists X, Y, E, F$.

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  - nowhere dense classes in general. . . ???

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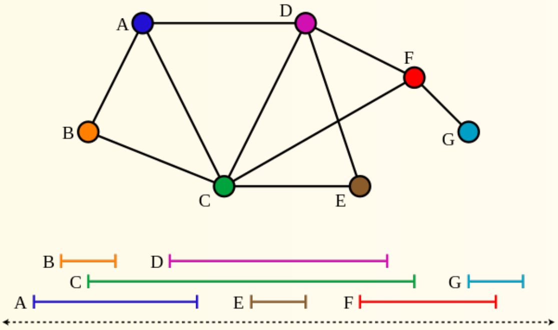
- INT: Intersection graphs of intervals on the real line.



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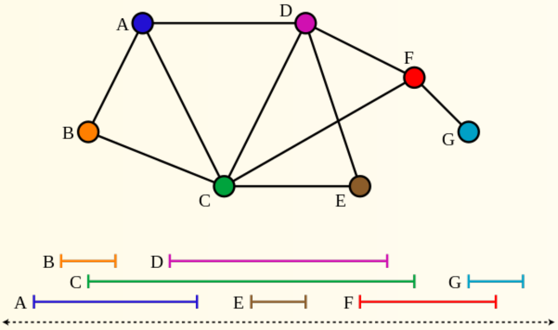

- ***L*-interval graphs** $=$ interval lengths only from a set L.

  (Unit-interval graphs: $L = \{1\}$.)

Technical remarks

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- Although the recognition problems for interval and for unit-interval graphs are in P, we do not know about L-interval graphs!

- Thus, we assume graphs are given by their interval representations, and these representations are handled by the real-precision RAM model (no tricks, though).

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  For any finite set $L \subseteq \mathbb{R}^+$, any FO property can be tested in time $\mathcal{O}(n \log n)$ on L-interval graphs.
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  - for example, independent and dominating set, subgraph isom., etc.
  - nearly tight result by the previous examples
  - rather easy to prove for rational L, but difficult otherwise

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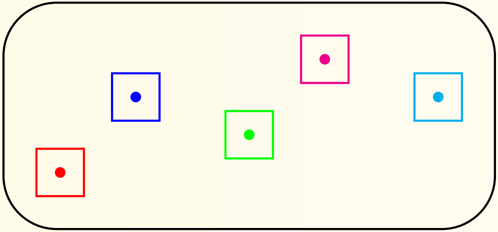
Gaifman's theorem. Every FO sentence is equivalent to a boolean combination of basic local sentences:

$$\exists \, {\color{red}\bullet}, {\color{green}\bullet}, \dots, {\color{blue}\bullet}. \ \text{“scattered”}({\color{red}\bullet}, \dots, {\color{blue}\bullet}) \wedge \boxed{{\color{red}\bullet}} \models \lambda({\color{red}\bullet}) \wedge \cdots \wedge \boxed{{\color{blue}\bullet}} \models \lambda({\color{blue}\bullet})$$



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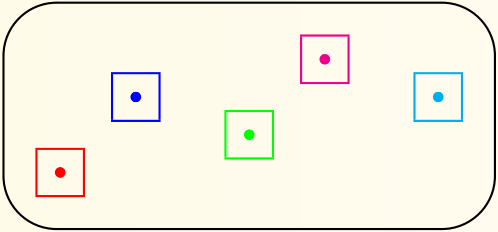

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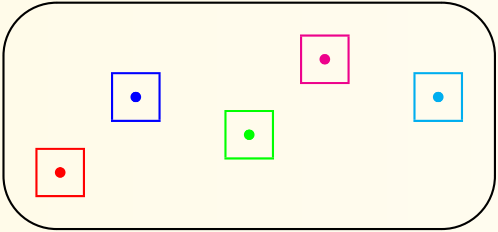

- Restriction to fixed-radius neighbourh. (above) definable inside FO.
- Hence, it is enough to solve any given FO property in
  every local neighbourhood!

Locality in interval graphs

For fin. $L \subseteq \mathbb{R}^+$, any FO prop. tested in $\mathcal{O}(n \log n)$ on L-interval graphs.

- Unit interval graphs have locally bounded clique-width.
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  - e.g., for unit interval $L = \{1\}$ these are $0, 1, 2, \dots$
- clique-width — simply order the intervals by their distance from the resp. accumulation points $\rightarrow$ linear k-expression
  - where $k \sim |L| \cdot$ #accum. points ! (finite in bounded radius)

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- Finished with any finite L.

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  - near to flagged – flag this one, and play the duplic. by $W \setminus w$

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