# How to Parallelize Sequential Processes

Antonín Kučera*
e-mail: tony@fi.muni.cz

Faculty of Informatics, Masaryk University
Botanická 68a, 60200 Brno
Czech Republic

**Abstract.** A process is prime if it cannot be decomposed into a parallel product of nontrivial processes. We characterize all non-prime normed BPA processes together with their decompositions by means of normal forms which are designed in this paper. Using this result we demonstrate decidability of the problem whether a given normed BPA process is prime; moreover, we show that non-prime normed BPA processes can be decomposed into primes effectively. This brings other positive decidability results. Finally, we prove that bisimilarity is decidable in a large subclass of normed PA processes.

## 1 Introduction

A general problem considered by many researchers is how to improve performance of sequential programs by parallelization. In this paper we study this problem within a framework of process algebras. They provide us with a pleasant formalism which allows to specify sequential as well as parallel programs.

Here we adopt normed BPA processes as a simple model of sequential behaviours (they are equipped with a binary sequential operator). We examine the problem of effective decomposability of normed BPA processes into a parallel product of primes (a process is prime if it cannot be decomposed into nontrivial components). We design special normal forms for normed BPA processes which allow us to characterize all non-prime normed BPA processes together with their decompositions up to bisimilarity. As a consequence we also obtain a refinement of the result achieved in [4].

Next we show that any normed BPA process can be decomposed into a parallel product of primes effectively. We also prove several related decidability results. Finally, we prove that bisimilarity is decidable in a large subclass of normed PA processes (see [2]), which consists of processes of the form $\Delta_1 \| \cdots \| \Delta_n$, where each $\Delta_i$ is a normed BPA or BPP process.

In many parts of our paper we rely on results established by other researchers. The question of possible decomposition of processes into a parallel product of primes was first addressed by Milner and Moller in [15]. A more general result was later proved by Christensen, Hirshfeld and Moller (see [8])—it says that

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*each* normed process has a unique decomposition into primes up to bisimilarity. However, the proof is non-constructive.

Bisimilarity was proved to be decidable for normed BPA processes (see [1, 11, 10]) and normed BPP processes (see [7, 9]). Blanco proved in [3] that bisimilarity is decidable even in the union of normed BPA and normed BPP processes. The same problem was independently examined by Černá, Křetínský and Kučera in [5]. They demonstrated decidability of the problem whether for a given normed BPA (or BPP) process $\Delta$ there is some unspecified normed BPP (or BPA) process $\Delta'$ such that $\Delta \sim \Delta'$. If the answer is positive, then it is also possible to *construct* an example of such $\Delta'$. Decidability of bisimilarity in the union of normed BPA and normed BPP processes is an immediate consequence.

Another property of normed BPA and BPP processes which is important for us is *regularity*. A process is regular if it is bisimilar to a process with finitely many states. Kučera proved in [13] that regularity is decidable for normed BPA and normed BPP processes in polynomial time.

## 2 Preliminaries

Let $Act = \{a, b, c, \ldots\}$ be a countably infinite set of *atomic actions*. Let $Var = \{X, Y, Z, \ldots\}$ be a countably infinite set of *variables* such that $Var \cap Act = \emptyset$. The class of BPA (or BPP) expressions is composed of all terms over the signature $\{\epsilon, a, ., +\}$ (or $\{\epsilon, a, \|, +\}$) where '$\epsilon$' is a constant denoting the empty expression, '$a$' is a unary operator of action prefixing ('$a$' ranges over $Act$), and '.', '$\|$' and '$+$' are binary operators of sequential composition, parallel composition and nondeterministic choice, respectively. In the rest of this paper we do not distinguish between expressions related by *structural congruence* which is the smallest congruence relation over BPA and BPP expressions such that the following laws hold: associativity and '$\epsilon$' as a unit for '.', '$\|$', '$+$' operators, and commutativity for '$\|$' and '$+$' operators. Moreover, we also abbreviate $a\epsilon$ as $a$.

As usual, we restrict our attention to *guarded* expressions. A BPA or BPP expression $E$ is guarded if every variable occurrence in $E$ is within the scope of an atomic action.

A *guarded BPA (or BPP) process* is defined by a finite family $\Delta$ of recursive process equations $\Delta = \{X_i = E_i \mid 1 \leq i \leq n\}$ where $X_i$ are distinct elements of $Var$ and $E_i$ are guarded BPA (or BPP) expressions, containing variables from $\{X_1, \ldots, X_n\}$. The set of variables which appear in $\Delta$ is denoted by $Var(\Delta)$.

The variable $X_1$ plays a special role ($X_1$ is sometimes called *the leading variable*)—it is a root of a labelled transition system, defined by the process $\Delta$ and the rules of Figure 1.

Nodes of the transition system generated by $\Delta$ are BPA (or BPP) expressions, which are often called *states of* $\Delta$, or just "states" when $\Delta$ is understood from the context. We also define the relation $\stackrel{w}{\rightarrow}^*$, where $w \in Act^*$, as the reflexive and transitive closure of $\stackrel{a}{\rightarrow}$ (we often write $E \rightarrow^* F$ instead of $E \stackrel{w}{\rightarrow}^* F$ if $w$ is irrelevant). Given two states $E, F$, we say that $F$ is *reachable from* $E$, if $E \rightarrow^* F$. States of $\Delta$ which are reachable from $X_1$ are said to be *reachable*.

$$\frac{}{aE \xrightarrow{a} E} \qquad \frac{E \xrightarrow{a} E'}{E.F \xrightarrow{a} E'.F} \qquad \frac{E \xrightarrow{a} E'}{E+F \xrightarrow{a} E'} \qquad \frac{F \xrightarrow{a} F'}{E+F \xrightarrow{a} F'}$$

$$\frac{E \xrightarrow{a} E'}{E\|F \xrightarrow{a} E'\|F} \qquad \frac{F \xrightarrow{a} F'}{E\|F \xrightarrow{a} E\|F'} \qquad \frac{E \xrightarrow{a} E'}{X \xrightarrow{a} E'} \ (X = E \in \Delta)$$

**Fig. 1.** SOS rules

**Remark 1.** Processes are often identified with their leading variables. Furthermore, if we assume fixed processes $\Delta_1, \Delta_2$ such that $Var(\Delta_1) \cap Var(\Delta_2) = \emptyset$, then we can view any process expression $E$ (not necessarily guarded) whose variables are defined in $\Delta_1, \Delta_2$ as a process too—if we denote this process by $\Delta$, then the leading equation of $\Delta$ is $X = E'$, where $X \notin Var(\Delta_1) \cup Var(\Delta_2)$ and $E'$ is a process expression which is obtained from $E$ by substituting each variable in $E$ with the right-hand side of its corresponding defining equation in $\Delta_1$ or $\Delta_2$ ($E'$ must be guarded now). Moreover, def. equations from $\Delta_1, \Delta_2$ are added to $\Delta$. All notions originally defined for processes can be used for process expressions in this sense too.

**Bisimulation** The equivalence between process expressions (states) we are interested in here is *bisimilarity* [16], defined as follows: A binary relation R over process expressions is a *bisimulation* if whenever $(E, F) \in R$ then for each $a \in Act$

- if $E \xrightarrow{a} E'$, then $F \xrightarrow{a} F'$ for some $F'$ such that $(E', F') \in R$
- if $F \xrightarrow{a} F'$, then $E \xrightarrow{a} E'$ for some $E'$ such that $(E', F') \in R$

Processes $\Delta$ and $\Delta'$ are *bisimilar*, written $\Delta \sim \Delta'$, if their leading variables are related by some bisimulation.

**Normed processes** An important subclass of BPA and BPP processes can be obtained by an extra restriction of *normedness.* A variable $X \in Var(\Delta)$ is *normed* if there is $w \in Act^*$ such that $X \xrightarrow{w}^* \epsilon$. In that case we define the *norm* of $X$, written $|X|$, to be the length of the shortest such $w$. A process $\Delta$ is *normed* if all variables of $Var(\Delta)$ are normed. The norm of $\Delta$ is then defined to be the norm of $X_1$. Note that bisimilar normed processes must have the same norm which is easily computed by the following rules: $|a| = 1$, $|E+F| = \min\{|E|, |F|\}$, $|E.F| = |E|+|F|$, $|E\|F| = |E|+|F|$ and if $X_i = E_i$ and $|E_i| = n$, then $|X_i| = n$.

**Greibach normal form** Any BPA or BPP process $\Delta$ can be effectively presented in a special normal form which is called 3-Greibach normal form by analogy with CF grammars (see [1] and [6]). Before the definition we need to introduce the set $Var(\Delta)^*$ of all finite sequences of variables from $Var(\Delta)$, and the set $Var(\Delta)^{\otimes}$ of all finite multisets over $Var(\Delta)$. Each multiset of $Var(\Delta)^{\otimes}$ denotes a BPP expression by combining its elements in parallel using the '$\|$' operator.

A BPA (or BPP) process $\Delta$ is said to be in *Greibach normal form (GNF)* if all its equations are of the form $X = \sum_{j=1}^{n} a_j \alpha_j$ where $n \in N$, $a_j \in Act$ and $\alpha_j \in Var(\Delta)^*$ (or $\alpha_j \in Var(\Delta)^{\otimes}$). We also require that each $Y \in Var(\Delta)$ appears in some reachable state of $\Delta$. If $length(\alpha_j) \leq 2$ (or $card(\alpha_j) \leq 2$) for each $j$, $1 \leq j \leq n$, then $\Delta$ is said to be in 3-GNF.

From now on we assume that all BPA and BPP processes we are working with are presented in GNF. This justifies also the assumption that all reachable states of a BPA process $\Delta$ are elements of $Var(\Delta)^*$ and all reachable states of a BPP process $\Delta'$ are elements of $Var(\Delta')^{\otimes}$.

**Regular processes** A process $\Delta$ is *regular* if there is a process $\Delta'$ with finitely many states such that $\Delta \sim \Delta'$. A regular process $\Delta$ is said to be in normal form if all its equations are of the form $X = \sum_{j=1}^{n} a_j X_j$ where $n \in N$, $a_j \in Act$ and $X_j \in Var(\Delta)$.

It is easy to see that a process is regular iff it can reach only finitely many states up to bisimilarity. In [14] it is shown, that regular processes can be represented in the normal form just defined. Thus a process $\Delta$ is regular iff there is a regular process $\Delta'$ in normal form such that $\Delta \sim \Delta'$. A proof of the following proposition can be found in [13].

**Proposition 2.** *Let $\Delta$ be a normed BPA or BPP process. The problem whether $\Delta$ is regular is decidable in polynomial time. Moreover, if $\Delta$ is regular then a regular process $\Delta'$ in normal form such that $\Delta \sim \Delta'$ can be effectively constructed.*

**Special notation** Here we summarize special notation used in this paper.

- **nBPA** and **nBPP** are abbreviations for normed BPA and normed BPP, respectively.
- if $\alpha$ is a state of a nBPA or nBPP process such that $\alpha$ is regular (see Remark 1), then $\Delta^{\mathcal{R}}(\alpha)$ denotes a bisimilar regular process in normal form, which can be effectively constructed due to Proposition 2. Furthermore, we always assume that $\Delta^{\mathcal{R}}(\alpha)$ contains completely fresh variables which are not contained in any other process we deal with.
- the class of all processes for which there is a bisimilar nBPA (or nBPP) process is denoted $\mathcal{S}$*(nBPA)* (or $\mathcal{S}$*(nBPP)*).
- if $\Delta_1, \ldots, \Delta_n$ are processes from nBPA$\cup$nBPP and $X_i$ is the leading variable of $\Delta_i$ for $1 \leq i \leq n$, then $\Delta_1 \| \cdots \| \Delta_n$ denotes the process $X_1 \| \cdots \| X_n$ in the sense of Remark 1.
- square brackets '[' and ']' indicate optional occurrence—if we say that some expression is of the form $a[A][B]$, we mean that this expression is either $a$, $aA$, $aB$ or $aAB$.
- upper indexes are used heavily; they appear in two forms:

$$\alpha^i = \underbrace{\alpha \| \cdots \| \alpha}_{i} \qquad\qquad \alpha^{\cdot i} = \underbrace{\alpha . \cdots . \alpha}_{i}$$

**Decidability of bisimilarity in nBPA $\cup$ nBPP** Bisimilarity is known to be decidable for nBPA (see [1, 11, 10]) and nBPP (see [7, 9]) processes. The following result due to Černá, Křetínský and Kučera (see [5]) says that bisimilarity is decidable even in the union of nBPA and nBPP processes.

**Proposition 3.** *Let $\Delta$ be a nBPA (or nBPP) process. It is decidable, whether $\Delta \in \mathcal{S}(nBPP)$ (or whether $\Delta \in \mathcal{S}(nBPA)$) and if the answer is positive, then a bisimilar nBPP (or nBPA) process can be effectively constructed.*

**Decomposability, prime processes** Let *nil* be a special name for the process which cannot emit any action (i.e., $nil \sim \epsilon$). A nBPA or nBPP process $\Delta$ is *prime* if $\Delta \not\sim nil$ and whenever $\Delta \sim \Delta_1 \| \Delta_2$ we have that either $\Delta_1 \sim nil$ or $\Delta_2 \sim nil$.

Natural questions are, what processes have a decomposition into a finite parallel product of primes and whether this decomposition is unique. This problem was first examined by Milner and Moller in [15]. They proved that each normed finite-state process has a unique decomposition up to bisimilarity. A more general result is due to Christensen, Hirshfeld and Moller—they proved the following proposition (see [8]):

**Proposition 4.** *Each nBPP process has a unique decomposition into a parallel product of primes (up to bisimilarity).*

**Remark 5.** Proposition 4 holds for *any* normed process in fact (namely for nBPA). The proof is independent of a concrete syntax—it could be easily formulated in terms of normed transition systems. This proposition thus says that each normed process can be parallelized in the "best" way and that this way is in some sense unique. However, this nice theoretical result is non-constructive.

## 3 Decomposability of nBPP processes

Each nBPP processes $\Delta$ can be easily decomposed into a parallel product of primes—all what has to be done is a construction of a bisimilar *canonical* process (see [6]).

**Theorem 6.** *Let $\Delta$ be a nBPP process. It is decidable whether $\Delta$ is prime and if not, its decomposition into primes can be effectively constructed.*

*Proof.* By induction on $n = |\Delta|$:

- **n=1:** each nBPP process whose norm is 1 is prime.
- **Induction step:** Suppose $\Delta \sim \Delta_1 \| \Delta_2$. As $\Delta_1, \Delta_2$ are reachable states of $\Delta_1 \| \Delta_2$, there are $\alpha_1, \alpha_2 \in Var(\Delta)^{\otimes}$ such that $\Delta_1 \sim \alpha_1$ and $\Delta_2 \sim \alpha_2$, thus $\Delta \sim \alpha_1 \| \alpha_2$. Furthermore, $|\Delta| = |\alpha_1| + |\alpha_2|$. We show that there are only finitely many candidates for $\alpha_1, \alpha_2$. First, there are only finitely many pairs $[k_1, k_2] \in N \times N$ such that $k_1 + k_2 = |\Delta|$. For each such pair $[k_1, k_2]$ there are

only finitely many pairs $[\beta_1, \beta_2]$ such that $\beta_1, \beta_2 \in Var(\Delta)^{\otimes}$, $|\beta_1| = k_1$ and $|\beta_2| = k_2$. It is obvious that the set $\mathcal{M}$ of all such pairs can be effectively constructed. For each element $[\beta_1, \beta_2]$ of $\mathcal{M}$ we check whether $\Delta \sim \beta_1 \| \beta_2$ (it can be done because bisimilarity is decidable for nBPP processes). If there is no such pair then $\Delta$ is prime. Otherwise, we check whether $\beta_1, \beta_2$ are primes (it is possible by ind. hypothesis) and construct their decompositions. If we combine these decompositions in parallel, we get a decomposition of $\Delta$. $\square$

As each normed regular process in normal form can be seen as a nBPP process in GNF, Theorem 6 (and especially its constructive proof) can be also used for regular nBPA processes (see Proposition 2). In the next section we can thus concentrate on non-regular nBPA processes.

## 4 Decomposability of nBPA processes

It this section we give an exact characterization of non-prime nBPA processes. We design special normal forms which allow us to characterize all non-prime nBPA processes together with their decompositions (up to bisimilarity). Our results bring also interesting consequences—we obtain a refinement of the result achieved in [4] (see Remark 18) and we also show that any nBPA process can be decomposed into prime processes effectively. Further positive decidability results are discussed in the end of the second subsection. Finally, we demonstrate decidability of bisimilarity in a natural subclass of normed PA processes.

### 4.1 Normal forms for non-prime nBPA processes

In this subsection we design the promised normal forms for non-prime nBPA processes and for prime processes which appear in corresponding decompositions. As we already know from the previous section, the problem of possible decomposition of a nBPA process into a parallel product of primes is actually interesting only for non-regular nBPA processes, hence the main characterization theorem does not concern regular nBPA processes.

The layout of this subsection is as follows: first we present two technical lemmas (Lemma 7 and 8). Then we consider the following problem: if $\Delta$ is a non-regular nBPA process such that $\Delta \sim \Delta_1 \| \Delta_2$, where $\Delta_1, \Delta_2$ are some (unspecified) processes, how do the processes $\Delta, \Delta_1, \Delta_2$ look like? It is clear that $\Delta_1, \Delta_2 \in \mathcal{S}(nBPA)$, hence the assumption that $\Delta_1, \Delta_2$ are nBPA processes can be used w.l.o.g. This problem is solved by Proposition 11 and 16 with a help of several definitions. Having this, the proof of Theorem 21 is easy to complete.

**Lemma 7.** *Let $\Delta$ be a nBPA process. Let $\alpha, \gamma \in Var(\Delta)^+$, $Q, C \in Var(\Delta)$ such that $|Q| = |C| = 1$ and $\alpha \| Q \sim C.\gamma$. Then $\alpha \sim Q^{|\alpha|}$.*

*Proof.* We prove that for each $1 \leq i \leq |\alpha|$ there is $\beta \in Var(\Delta)^*$ such that $\beta \| Q^i \sim C.\gamma$. This is clearly sufficient, because then $\alpha \| Q \sim C.\gamma \sim Q^{|\alpha|+1}$ and thus $\alpha \sim Q^{|\alpha|}$. We proceed by induction on $i$.

– **i=1:** choose $\beta = \alpha$.
– **Induction step:** Let $\beta \| Q^i \sim C.\gamma$. As $|C| = 1$, all states which are reachable from $\beta \| Q^i$ in one norm-decreasing step are bisimilar. As $\Delta$ is normed, there is $\beta' \in Var(\Delta)^*$ such that $\beta \xrightarrow{a} \beta'$ where $|\beta| = |\beta'| + 1$. Hence $\beta \| Q^{i-1} \sim \beta' \| Q^i$ and by substitution we obtain $\beta \| Q^i \sim \beta' \| Q^{i+1}$. □

**Lemma 8.** *Let $\Delta$ be a nBPA process, $\alpha, \beta, \gamma \in Var(\Delta)^*$ such that $\alpha$ is non-regular and $\alpha \| \beta \sim \gamma$. Let $\beta \rightarrow^* Q$ where $|Q| = 1$. Then $\beta \sim Q^{|\beta|}$.*

The proof of Lemma 8 is omitted due to the lack of space (it is rather technical). It can be found in [12].

**Definition 9 (simple processes).** A nBPA process $\Delta$ is *simple* if $Var(\Delta)$ contains just one variable, i.e., $card(Var(\Delta)) = 1$.

We will often identify simple processes with their leading (and only) variables in the rest of this paper. Moreover, it is easy to see that a simple process $Q$ is non-regular iff the def. equation for $Q$ contains a summand of the form $aQ^{\cdot k}$ where $a \in Act$ and $k \geq 2$. The norm of $Q$ is one, because $Q$ could not be normed otherwise. Another important property of simple processes is presented in the remark below:

**Remark 10.** Each simple nBPA process $Q$ belongs to $\mathcal{S}$*(nBPP)*—a bisimilar nBPP process can be obtained just by replacing the '.' operator with the '$\|$' operator in the def. equation for $Q$. Consequently, any process expressions built over $k$ copies of $Q$ using '.' and '$\|$' operators are bisimilar (e.g., $(Q.(Q\|Q))\|Q \sim (Q\|Q).(Q\|Q)$).

**Proposition 11.** *Let $\Delta_1, \Delta_2$ be non-regular nBPA processes. Then $\Delta_1 \| \Delta_2 \in \mathcal{S}$(nBPA) iff $\Delta_1 \sim Q^{|\Delta_1|}$ and $\Delta_2 \sim Q^{|\Delta_2|}$ for some non-regular simple process $Q$.*

*Proof.*
"$\Leftarrow$" Easy—see Remark 10.
"$\Rightarrow$" Assume there is some nBPA process $\Delta$ such that $\Delta_1 \| \Delta_2 \sim \Delta$. Then there are $\alpha_1, \alpha_2 \in Var(\Delta)^*$ such that $\Delta_1 \sim \alpha_1$ and $\Delta_2 \sim \alpha_2$. Thus $\alpha_1 \| \alpha_2 \sim \Delta$ and as $\alpha_1, \alpha_2$ are non-regular, we can use Lemma 8 and conclude that there are $Q_1, Q_2 \in Var(\Delta)$ such that $|Q_1| = |Q_2| = 1$, $\alpha_1 \rightarrow^* Q_1$, $\alpha_2 \rightarrow^* Q_2$ and $\alpha_1 \sim Q_1^{|\alpha_1|}$, $\alpha_2 \sim Q_2^{|\alpha_2|}$. First we prove that $Q_1 \sim Q$ for some simple process $Q$. To do this, it suffices to prove that if $a\gamma$ is a summand in the def. equation for $Q_1$, then $\gamma \sim Q_1^{|\gamma|}$ (if this is the case, then also $\gamma \sim Q_1^{\cdot|\gamma|}$ — see Remark 10). As $\alpha_1 \| \alpha_2 \rightarrow^* Q_1 \| \alpha_2 \xrightarrow{a} \gamma \| \alpha_2$, the process $\gamma \| \alpha_2$ belongs to $\mathcal{S}$*(nBPA)*. Let $\gamma \rightarrow^* R$ where $|R| = 1$. Then $\gamma \sim R^{|\gamma|}$ (due to Lemma 8) and as $\alpha_1 \rightarrow^* \gamma \rightarrow^* R$, we also have $\alpha_1 \sim R^{|\alpha_1|}$. Hence $R \sim Q_1$ and $\gamma \sim Q_1^{|\gamma|}$.

To finish the proof we need to show that $Q_1 \sim Q_2$. Let $m = \max\{|X|,\ X \in Var(\Delta)\}$. As $\alpha_1$ is non-regular, it can reach a state of an arbitrary norm—let $\alpha_1 \rightarrow^* \alpha_1'$ where $|\alpha_1'| = m$. Then $\alpha_1' \| Q_2 \sim \delta$ for some $\delta \in Var(\Delta)^*$ whose length is at least two—$\delta = A.B.\delta'$. Clearly $\alpha'_1 \sim Q_1^{|\alpha'_1|}$ (we can use the same argument as in the first part of this proof—$Q_2$ is non-regular and $\alpha'$ plays the role of $\gamma$), hence $Q_1^{|\alpha'_1|} \| Q_2 \sim A.B.\delta'$. As $Q_1^{|\alpha'_1|-|A|} \| Q_2 \sim B.\delta'$ and $Q_1^{|\alpha'_1|-|A|+1} \sim B.\delta'$, we have $Q_1^{|\alpha'_1|-|A|} \| Q_2 \sim Q_1^{|\alpha'_1|-|A|+1}$ by transitivity and thus $Q_1 \sim Q_2$. $\square$

Proposition 11 in fact says that if $\Delta$ is a non-regular nBPA process such that $\Delta \sim \Delta_1 \| \Delta_2$, where $\Delta_1, \Delta_2$ are non-regular processes, then each of those three processes can be equivalently represented as a power of some non-regular simple process. This representation is very special and can be seen as normal form.

If $\Delta$ is a non-regular nBPA process such that $\Delta \sim \Delta_1 \| \Delta_2$, it is also possible that $\Delta_1$ is non-regular and $\Delta_2$ regular. Before we start to examine this sub-case, we introduce a special normal form for nBPA processes (as we shall see, $\Delta$ and $\Delta_1$ can be represented in this normal form):

**Definition 12 (DNF(Q)).** Let $\Delta$ be a non-regular nBPA process in GNF, $Q \in Var(\Delta)$. We say that $\Delta$ is in $DNF(Q)$ if all summands in all defining equations from $\Delta$ are of the form $a([Y].[Q^{\cdot i}])$, where $Y \in Var(\Delta)$, $i \in N$ and $a \in Act$. Furthermore, all summands in the def. equation for $Q$ must be of the form $a[Q]$, where $a \in Act$.

**Example 13.** The following process is in $DNF(Q)$:

$$
\begin{aligned}
X &= a(Y.Q.Q) + bX + a(Q.Q.Q) + c \\
Y &= bQ + cX + c(Y.Q) + b \\
Q &= aQ + bQ + a + c
\end{aligned}
$$

**Remark 14.** Reachable states of a process $\Delta$ in $DNF(Q)$ are of the form $[Y].[Q^{\cdot i}]$ where $Y \in Var(\Delta)$ and $i \in N \cup \{0\}$. As $\Delta$ is non-regular, the state $Q^{\cdot k}$ is reachable for each $k \in N$.

Note that the variable $Q$ itself is a regular simple process. The next lemma says that if $\Delta$ is a process in $DNF(Q)$, then the variable $Q$ is in some sense unique:

**Lemma 15.** *Let $\Delta$ and $\Delta'$ be processes in $DNF(Q)$ and $DNF(R)$, respectively. If $\Delta \sim \Delta'$, then $Q \sim R$.*

**Proposition 16.** *Let $\Delta_1, \Delta_2$ be nBPA processes such that $\Delta_1$ is non-regular and $\Delta_2$ is regular. Then $\Delta_1 \| \Delta_2 \in \mathcal{S}(nBPA)$ iff there is a process $\Delta'_1$ in $DNF(Q)$ such that $\Delta_1 \sim \Delta'_1$ and $\Delta_2 \sim Q^{|\Delta_2|}$.*

*Proof.*
"$\Rightarrow$" Let $\Delta_2 \to^* Q'$ where $Q' \in Var(\Delta_2)$, $|Q'| = 1$. Using the same kind of argument as in the proof of Proposition 11 we obtain that $Q' \sim Q$ for some regular simple process $Q$ such that $\Delta_2 \sim Q^{|\Delta_2|}$. It remains to prove that there is a process $\Delta'_1$ in $DNF(Q)$ such that $\Delta_1 \sim \Delta'_1$. We show that each summand of each defining equation from $\Delta_1$ can be transformed to a form which is admitted by $DNF(Q)$. First, let us realize two facts about summands—if $a\alpha$ is a summand in a def. equation from $\Delta_1$, then

1. If $\alpha = \beta.Y.\gamma$ where $Y$ is a non-regular variable, then each variable $P$ of $\gamma$ is bisimilar to $Q^{|P|}$.
2. $\alpha$ contains at most one non-regular variable.

The first fact is a consequence of Lemma 7—let $\Delta$ be a nBPA process such that $\Delta_1 \| \Delta_2 \sim \Delta$. As $\Delta_1$ is normed, $\Delta_1 \to^* Y.\gamma.\delta$ for some $\delta \in \mathit{Var}(\Delta_1)^*$. As $Y$ is non-regular, it can reach a state of an arbitrary length—let $m = \max\{|X|,\ X \in \mathit{Var}(\Delta_1)\}$ and let $Y \to^* \omega$ where $\mathit{length}(\omega) = m$. As $\Delta_1 \| \Delta_2 \to^* \omega.\gamma.\delta \| Q'$, there is $\varphi \in \mathit{Var}(\Delta)^*$ such that $\omega.\gamma.\delta \| Q' \sim \varphi$. Let $\varphi = C.\varphi'$ and let $s$ be a norm-decreasing sequence of actions such that $\mathit{length}(s) = |C| - 1$ and $\omega \xrightarrow{s}{}^* \omega'$. Then $\omega'.\gamma.\delta \| Q' \sim C'.\varphi'$ where $|C'| = 1$ and due to Lemma 7 (and the fact that $Q' \sim Q$) we have $\omega'.\gamma.\delta \sim Q^{|\omega'.\gamma.\delta|}$, hence $\gamma \sim Q^{|\gamma|}$ and $P \sim Q^{|P|}$ for each variable $P$ which appears in $\gamma$.

The second fact is a consequence of the first one—assume that $\alpha = \beta.Y.\gamma.Z.\delta$ where $Y, Z$ are non-regular. Then $Z \sim Q^{|Z|}$ and as $Q$ is regular, $Q^{|Z|}$ is regular too. Hence $Z$ is regular and we have a contradiction.

Now we can describe the promised transformation of $\Delta_1$ to $\Delta_1'$: if $X = \sum_{i=1}^{n} a_i\alpha_i$ is a def. equation in $\Delta_1$, then $X = \sum_{i=1}^{n} a_i\mathcal{T}(\alpha_i)$ is a def. equation in $\Delta_1'$, where $\mathcal{T}$ is defined as follows:

– If $\alpha_i$ does not contain any non-regular variable, then $\mathcal{T}(\alpha_i) = A$, where $A$ is the leading variable of $\Delta^{\mathcal{R}}(\alpha_i)$. Moreover, defining equations of $\Delta^{\mathcal{R}}(\alpha_i)$ are added to $\Delta_1'$.
– If $\alpha_i = \beta.Y.\gamma$ where $Y$ is a non-regular variable, then $\mathcal{T}(\alpha_i) = A$, where $A$ is the leading variable of the process $\Delta'$ which is obtained by the following modification of the process $\Delta^{\mathcal{R}}(\beta)$: each summand in each def. equation of $\Delta^{\mathcal{R}}(\beta)$ which is of the form $b$, where $b \in \mathit{Act}$, is replaced with $b(Y.Q^{\cdot|\gamma|})$ — remember $\gamma \sim Q^{|\gamma|} \sim Q^{\cdot|\gamma|}$. Moreover, def. equations of $\Delta'$ are added to $\Delta_1'$.

The defining equation for $Q$ is also added to $\Delta_1'$. The resulting process is in $\mathit{DNF}(Q)$ and as $\mathcal{T}$ preserves bisimilarity, $\Delta_1 \sim \Delta_1'$.

"$\Leftarrow$" We show how to construct a nBPA process $\Delta$ which is bisimilar to $\Delta_1' \| Q^{|\Delta_2|}$. Let $k = |\Delta_2|$. The set of variables of $\Delta$ looks as follows:

$$\mathit{Var}(\Delta) = \{Q\} \cup \{Y_i,\ Y \in \mathit{Var}(\Delta_1'), Y \neq Q \text{ and } i \in \{0, \ldots, k\}\}$$

Defining equations of $\Delta$ are constructed using the following rules:

– the def. equation for $Q$ is the same as in $\Delta_1'$
– if $a(Y.Q^{\cdot j})$, where $j \in N \cup \{0\}$, $Y \neq Q$, is a summand in the def. equation for $Z \in \mathit{Var}(\Delta_1')$, then $a(Y_i.Q^{\cdot j})$ is a summand in the def. equation for $Z_i$ for each $i \in \{0, \ldots, k\}$
– if $a(Q^{\cdot j})$ where $j \in N \cup \{0\}$ is a summand in the def. equation for $Z \in \mathit{Var}(\Delta_1')$, then $a(Q^{\cdot j+i})$ is a summand in the def. equation for $Z_i$ for each $i \in \{0, \ldots, k\}$
– if $aQ$ is a summand in the def. equation for $Q$ and $Z \in \mathit{Var}(\Delta_1')$, $Z \neq Q$, then $aZ_i$ is a summand in the def. equation for $Z_i$ for each $i \in \{1, \ldots, k\}$

– if $a$ is a summand in the def. equation for $Q$ and $Z \in Var(\Delta_1')$, $Z \neq Q$, then $aZ_{i-1}$ is a summand in the def. equation for $Z_i$ for each $i \in \{1, \dots, k\}$

The intuition which stands behind this construction is that lower indexes of variables indicate how many copies of $Q$ in $Q^{|\Delta_2|}$ have not disappeared yet. The fact $\Delta_1' \| Q^{|\Delta_2|} \sim \Delta$ is easy to check. □

**Example 17.** If we apply the algorithm presented in the "⇐" part of the proof of Proposition 16 to the process $X\|Q^2$, where $X, Q$ are variables of the process presented in Example 13, we obtain the following output:

$$
\begin{aligned}
X_2 &= a(Y_2.Q.Q) + bX_2 + a(Q.Q.Q.Q.Q) + c(Q.Q) + aX_2 + bX_2 + aX_1 + cX_1\\
X_1 &= a(Y_1.Q.Q) + bX_1 + a(Q.Q.Q.Q) + cQ + aX_1 + bX_1 + aX_0 + cX_0\\
X_0 &= a(Y_0.Q.Q) + bX_0 + a(Q.Q.Q) + c\\
Y_2 &= b(Q.Q.Q) + cX_2 + c(Y_2.Q) + b(Q.Q) + aY_2 + bY_2 + aY_1 + cY_1\\
Y_1 &= b(Q.Q) + cX_1 + c(Y_1.Q) + bQ + aY_1 + bY_1 + aY_0 + cY_0\\
Y_0 &= bQ + cX_0 + c(Y_0.Q) + b\\
Q &= aQ + bQ + a + c
\end{aligned}
$$

**Remark 18.** Proposition 16 can also be seen as a refinement of the result presented in [4]—Burkart and Steffen proved that PDA processes are closed under parallel composition with finite-state processes, while BPA processes lack this property. Proposition 16 says precisely, which nBPA processes can remain nBPA if they are combined in parallel with a regular process. Moreover, it also characterizes all such regular processes.

It is easy to see that the algorithm from the proof of Proposition 16 always outputs a process in $DNF(Q)$ (see Example 17). Moreover, the structure of this process is very specific; we can observe that each variable belongs to a special "level". This intuition is formally expressed by the following definition (it is a little complicated—but it pays because we will be able to characterize all non-prime nBPA processes):

**Definition 19.** Let $\Delta$ be a nBPA process in $DNF(Q)$. The *level* of $\Delta$, denoted $Level(\Delta)$, is the maximal $l \in N$ such that the set $Var(\Delta) - \{Q\}$ can be divided into $l$ disjoint linearly ordered subsets $L_1, \dots, L_l$ of the same cardinality $k$. Moreover, the following conditions must be true (the $j^{th}$ element of $L_i$ is denoted $A_{i,j}$):

– $A_{l,1}$ is the leading variable of $\Delta$.
– Defining equations for variables of $L_1$ contain only variables from $L_1 \cup \{Q\}$
– The defining equation for $A_{i,j}$, where $i \geq 2$, $1 \leq j \leq k$, contains exactly those summands which can be derived by one of the following rules:
  1. If $aQ$ is a summand in the defining equation for $Q$, then $aA_{i,j}$ is a summand in the defining equation for $A_{i,j}$ for each $2 \leq i \leq l$, $1 \leq j \leq k$.
  2. If $a$ is a summand in the defining equation for $Q$, then $aA_{i-1,j}$ is a summand in the defining equation for $A_{i,j}$ for each $2 \leq i \leq l$, $1 \leq j \leq k$.

3. If $a(A_{1,m}.Q^{\cdot n})$ is a summand in the defining equation for $A_{1,j}$ such that $A_{1,m} \neq Q$, then $a(A_{i,m}.Q^{\cdot n})$ is a summand in the defining equation for $A_{i,j}$ for each $2 \leq i \leq l$.
4. If $aQ^{\cdot n}$ is a summand in the defining equation for $A_{1,j}$, then $aQ^{\cdot(n+i-1)}$ is a summand in the defining equation for $A_{i,j}$, where $2 \leq i \leq l$.

**Example 20.** The process of Example 17 has the level 3; $L_1 = \{X_0, Y_0\}$, $L_2 = \{X_1, Y_1\}$ and $L_3 = \{X_2, Y_2\}$.

Now we can present the first main theorem of this paper:

**Theorem 21.** *Let $\Delta$ be a non-regular nBPA process and let $\Delta \sim \Delta_1 \| \cdots \| \Delta_n$, where $n \geq 2$, $\Delta_i$ is a prime process for each $1 \leq i \leq n$ and $\Delta_1$ is non-regular. Then one of the following possibilities holds:*

- *There is a non-regular simple process $Q$ such that $\Delta \sim Q^{\cdot|\Delta|}$ and $\Delta_i \sim Q$ for each $1 \leq i \leq n$.*
- *There are nBPA processes $\Delta', \Delta'_1$ in $DNF(Q)$ such that $\Delta \sim \Delta'$, $\Delta_1 \sim \Delta'_1$, $Level(\Delta') = n$, $Level(\Delta'_1) = 1$ and $\Delta_i \sim Q$ for each $2 \leq i \leq n$.*

*Proof.* By a straightforward induction on $n$—see [12]. □

### 4.2 Decidability results

In this subsection we present several positive decidability results. We show that it is decidable whether a given nBPA process is prime and if the answer is negative, then its decomposition into primes can be effectively constructed. There are also other decidable properties which are summarized in Theorem 26. Finally, we demonstrate decidability of bisimilarity in a natural subclass of normed PA processes.

**Lemma 22.** *Let $\Delta$ be a nBPA process. It is decidable whether there is a nBPA process $\Delta'$ in $DNF(Q)$ such that $\Delta \sim \Delta'$. Moreover, if the answer to the previous question is positive, then the process $\Delta'$ can be effectively constructed.*

*Proof.* We can assume (w.l.o.g.) that $\Delta$ is in 3-GNF. If there is a process $\Delta'$ in $DNF(Q)$ such that $\Delta \sim \Delta'$, then there is $R \in Var(\Delta)$ such that $R \sim Q$, because $Q$ is a reachable state of $\Delta'$. As $Q$ is a regular simple process, each summand in the def. equation for $R$ must be of the form $a[P]$, where $R \sim P$. As bisimilarity is decidable for nBPA processes, we can construct the set $\mathcal{M}$ of all variables of $Var(\Delta)$ with this property. Each variable from this set is a potential candidate for the variable which is bisimilar to $Q$ (if the set $\mathcal{M}$ is empty, then $\Delta$ cannot be bisimilar to any process in $DNF(Q)$).

For each variable $V \in \mathcal{M}$ we now modify the process $\Delta$ slightly—we replace each summand of the form $aP$ in the def. equation for $V$ with $aV$. The resulting process is denoted $\Delta_V$ (clearly $\Delta \sim \Delta_V$). For each $\Delta_V$ we check whether $\Delta_V$ can be transformed to a process in $DNF(V)$. To do this, we first need to realize the following fact: if there is $\Delta'_V$ in $DNF(V)$ such that $\Delta_V \sim \Delta'_V$ and $a(A.B)$ is a summand in a def. equation from $\Delta_V$ such that $A$ is non-regular, then $B \sim V^{\cdot|B|}$. It is easy to prove by the technique we already used many times in this paper—as $A$ is non-regular, it can reach a state of an arbitrary norm. Furthermore, there is a reachable state of $\Delta_V$ which is of the form $A.B.\gamma$ where $\gamma \in Var(\Delta_V)^*$. We choose sufficiently large $\alpha$ such that $A \to^* \alpha$ and $\alpha.B.\gamma$ must be bisimilar to a state of $\Delta'_V$ which is of the form $[Y].V^{\cdot i}$ where $i \geq |B.\gamma|$. From this we get $B \sim V^{\cdot|B|}$.

Now we can describe the promised transformation $\mathcal{T}$ of $\Delta_V$ to a process $\Delta'_V$ in $DNF(V)$. If this transformation fails, then there is *no* process in $DNF(V)$ bisimilar to $\Delta_V$. $\mathcal{T}$ is invoked on each summand of each def. equation from $\Delta_V$ and works as follows:

- $\mathcal{T}(a) = a$
- $\mathcal{T}(aA) = aA$
- $\mathcal{T}(a(A.B)) = aN$ if $A$ is regular. The variable $N$ is the leading variable of $\Delta^{\mathcal{R}}(A)$, whose def. equations are also added to $\Delta'_V$ after the following modification: each summand in each def. equation of $\Delta^{\mathcal{R}}(A)$ which is of the form $b$ where $b \in Act$ is replaced with $bB$.
- $\mathcal{T}(a(A.B)) = a(A.V^{\cdot|B|})$ if $A$ is non-regular and $B \sim V^{\cdot|B|}$. If $A$ is non-regular and $B \not\sim V^{\cdot|B|}$, then $\mathcal{T}$ *fails*.

If there is $V \in \mathcal{M}$ such that $\mathcal{T}$ succeeds for $\Delta_V$, then the process $\Delta'_V \sim \Delta$ is the process we are looking for. Otherwise, there is no process in $DNF(Q)$ bisimilar to $\Delta$. □

**Proposition 23.** *Let $\Delta_1, \ldots, \Delta_n$, $n \geq 2$ be nBPA processes. It is decidable whether $\Delta_1 \| \cdots \| \Delta_n \in \mathcal{S}(nBPA)$. Moreover, if the answer to the previous question is positive, then a nBPA process $\Delta$ such that $\Delta_1 \| \cdots \| \Delta_n \sim \Delta$ can be effectively constructed.*

*Proof.* By induction on $n$:

- **n=2:** we distinguish three possibilities (it is decidable which one actually holds—see Proposition 2):
  1. $\Delta_1$ and $\Delta_2$ are regular. Then $\Delta_1 \| \Delta_2 \in \mathcal{S}(nBPA)$ and a bisimilar regular process $\Delta$ in normal form can be easily constructed.
  2. $\Delta_1$ and $\Delta_2$ are non-regular. Proposition 11 says that there is a non-regular simple process $Q$ such that $\Delta_1 \sim Q^{|\Delta_1|} \sim Q^{\cdot|\Delta_1|}$ and $\Delta_2 \sim Q^{|\Delta_2|} \sim Q^{\cdot|\Delta_2|}$. As $Q$ is a reachable state of $Q^{\cdot|\Delta_2|}$, there is $R \in Var(\Delta_1)$ such that $Q \sim R$. As reachable states of $Q$ are of the form $Q^{\cdot i}$ where $i \in N \cup \{0\}$, each summand $a\alpha$ in the def. equation for $R$ has the property $\alpha \sim R^{\cdot|\alpha|}$. As bisimilarity is decidable for nBPA processes, we can find all variables of $Var(\Delta)$ having this property—we obtain a set of possible candidates for $R$ (if this set is empty, then $\Delta_1 \| \Delta_2 \notin \mathcal{S}(nBPA)$). Now we check whether the constructed set of candidates contains a variable $R$ such that $\Delta_1 \sim R^{\cdot|\Delta_1|}$. If not, then $\Delta_1 \| \Delta_2 \notin \mathcal{S}(nBPA)$. Otherwise we have $R$ which is bisimilar to $Q$.

The same procedure is now applied to $\Delta_2$. If it succeeds, it outputs some $S \in Var(\Delta)$. Now we check whether $R \sim S$. If not, then $\Delta_1 \| \Delta_2 \notin \mathcal{S}(nBPA)$. Otherwise $\Delta_1 \| \Delta_2 \in \mathcal{S}(nBPA)$ and $\Delta_1 \| \Delta_2 \sim R^{\cdot |\Delta_1| + |\Delta_2|}$.

3. $\Delta_1$ is non-regular and $\Delta_2$ is regular (or $\Delta_1$ is regular and $\Delta_2$ is non-regular—this is symmetric). Due to Proposition 16 we know that there is a regular simple process $Q$ and a nBPA process $\Delta_1'$ in $DNF(Q)$ such that $\Delta_1 \sim \Delta_1'$ and $\Delta_2 \sim Q^{|\Delta_2|} \sim Q^{\cdot |\Delta_2|}$. An existence of $\Delta_1'$ can be checked effectively (see Lemma 22). If it does not exist, then $\Delta_1 \| \Delta_2 \notin \mathcal{S}(nBPA)$. If it exists, it can be also constructed and thus the only thing which remains is to test whether $\Delta_2 \sim Q^{\cdot |\Delta_2|}$. If this test succeeds, then $\Delta_1 \| \Delta_2 \in \mathcal{S}(nBPA)$ and we invoke the algorithm from the proof of Proposition 16 with $\Delta_1' \| Q^{|\Delta_2|}$ on input—it outputs a nBPA process which is bisimilar to $\Delta_1 \| \Delta_2$.

– **Induction step:** if $\Delta_1 \| \cdots \| \Delta_n \in \mathcal{S}(nBPA)$, then also $\Delta_1 \| \cdots \| \Delta_{n-1} \in \mathcal{S}(nBPA)$ and this is decidable by induction hypothesis—if the answer is negative, then $\Delta_1 \| \cdots \| \Delta_n \notin \mathcal{S}(nBPA)$ and if it is positive, then we can construct a nBPA process $\Delta'$ such that $\Delta_1 \| \cdots \| \Delta_{n-1} \sim \Delta'$. Now we check whether $\Delta' \| \Delta_n \in \mathcal{S}(nBPA)$ and construct a bisimilar nBPA process $\Delta$. □

As an immediate consequence of Proposition 23 we get:

**Proposition 24.** *Let $\Delta, \Delta_1, \ldots, \Delta_n$ be nBPA processes. It is decidable whether $\Delta \sim \Delta_1 \| \cdots \| \Delta_n$.*

**Theorem 25.** *Let $\Delta$ be a nBPA process. It is decidable whether $\Delta$ is prime and if not, its decomposition into primes can be effectively constructed.*

*Proof.* The technique is the same as in the proof of Theorem 6. We can almost copy the whole proof—the crucial result which allows us to do so is Proposition 24. □

Decidability results which were proved in this subsection are summarized by the following theorem:

**Theorem 26.** *Let $\Delta, \Delta_1, \ldots, \Delta_n$ be nBPA processes. The following problems are decidable:*

– *Is $\Delta$ prime? (If not, its decomposition can be effectively constructed)*
– *Is $\Delta$ bisimilar to $\Delta_1 \| \cdots \| \Delta_n$?*
– *Does the process $\Delta_1 \| \cdots \| \Delta_n$ belong to $\mathcal{S}(nBPA)$?*
– *Is there any process $\Delta'$ such that $\Delta \| \Delta' \in \mathcal{S}(nBPA)$? (if so, an example of such a process can be effectively constructed).*
– *Is there any process $\Delta'$ such that $\Delta \sim \Delta_1 \| \cdots \| \Delta_n \| \Delta'$? (if so, $\Delta'$ can be effectively constructed).*

A "structural" way how to construct new processes from older ones is to combine them in parallel. If we do this with nBPA and nBPP processes, we obtain a natural subclass of normed PA processes denoted sPA (**s**imple PA processes).

**Definition 27 (sPA processes).** The class of sPA processes is defined as follows: sPA $= \{\Delta_1 \| \cdots \| \Delta_n \mid n \in N,\ \Delta_i \in \text{nBPA} \cup \text{nBPP} \text{ for each } 1 \leq i \leq n\}$

The class sPA is strictly greater than the union of nBPA and nBPP processes; it suffices to take a parallel composition of two "normed counters" specified by nBPA processes. The resulting sPA process is not bisimilar to any nBPA or nBPP process. It can be easily proved with the help of pumping lemmas for CF languages and for languages generated by nBPP processes—see [6].

**Theorem 28.** *Let $\Phi = \varphi_1 \| \cdots \| \varphi_n$, $\Psi = \psi_1 \| \cdots \| \psi_m$ be sPA processes. It is decidable whether $\Phi \sim \Psi$.*

*Proof.* As each $\varphi_i$, $1 \leq i \leq n$ and $\psi_j$, $1 \leq j \leq m$ can be effectively decomposed into a parallel product of primes, we can also construct the decompositions of $\Phi$ and $\Psi$. If $\Phi \sim \Psi$, then these decompositions must be the same up to bisimilarity (see Remark 5). In other words, there must be a one-to-one mapping between primes forming the two decompostions which preserves bisimilarity. An existence of such a mapping can be checked effectively, because bisimilarity is decidable in the union of nBPA and nBPP processes (see Proposition 3). □

## 5 Conclusions, future work

The main characterization theorem (Theorem 21) says that non-regular nBPA processes which are not prime can be divided into two groups:

1. processes which are bisimilar to a power of some non-regular simple process. It is obvious that each such nBPA process belongs to $\mathcal{S}$*(nBPP)*—see Remark 10.
2. processes which are bisimilar to some process in $DNF(Q)$. It can be proved (with the help of results achieved in [5]) that each such process does *not* belong to $\mathcal{S}$*(nBPP)*.

From this we can observe that our division based on normal forms corresponds to the membership to $\mathcal{S}$*(nBPP)*.

We have also shown that the decomposition of non-prime nBPA processes can be effectively constructed. This algorithm can be interpreted as a construction of the "most parallel" version of a given sequential program. Finally, we proved that bisimilarity is decidable for sPA processes. (see Definition 27).

The first possible generalization of our results could be the replacement of the '$\|$' operator with the parallel operator of CCS which allows synchronizations on complementary actions. This should not be hard, but we can expect more complicated normal forms. Decidability results should be the same.

A natural question is whether our results can be extended to the class of all (not necessarily normed) BPA processes. The answer is no, because there are quite primitive BPA processes which do not have any decomposition at all—a simple example is the process $X = aX$.

Another related open problem is decidability of bisimilarity for normed PA processes. It seems that it should be possible to design at least rich subclasses of normed PA processes where bisimilarity remains decidable.

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