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Deciding Bisimulation-Like Equivalences with Finite-State Processes

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TUM-I9805
Februar 98

T E C H N I S C H E U N I V E R S I T Ä T M Ü N C H E N

TUM-INFO-02-I9805-100/1.-FI

Druck: Institut für Informatik der
Technischen Universität München

Deciding Bisimulation-Like Equivalences with Finite-State Processes*

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Abstract

We design a general method for proving decidability of bisimulation-like equivalences between infinite-state processes and finite-state ones. We apply this method to the class of PAD processes, which strictly subsumes PA and pushdown (PDA) processes, showing that a large class of bisimulation-like equivalences (including e.g. strong and weak bisimilarity) is decidable between PAD and finite-state processes. On the other hand, we also demonstrate that no "reasonable" bisimulation-like equivalence is decidable between state-extended PA processes and finite-state ones. Furthermore, weak bisimilarity with finite-state processes is shown to be undecidable even for state-extended BPP (which are also known as 'parallel pushdown processes').

*The first two authors are supported by the Grant Agency of the Czech Republic, grant No. 201/97/0456. The second author is also supported by a Post-Doc grant GA ČR No. 201/98/P046.

1 Introduction

In this paper we study the decidability of bisimulation-like equivalences between infinite-state processes and finite-state ones. First we examine this problem in a general setting, extracting its core in a form of two rather special subproblems (which are naturally not decidable in general). A special variant of this method which works for strong bisimilarity has been described in [JK97]; here we extend and generalize the concept, obtaining a universal mechanism for proving decidability of bisimulation-like equivalences between infinite-state and finite-state processes. Then we apply the designed method to the class of PAD processes (defined in [May97b]), which properly subsumes all PA and pushdown processes. We prove that a large class of bisimulation-like equivalences (including e.g. strong and weak bisimilarity) is decidable between PAD and finite-state processes, utilizing previously established results on decidability of the model-checking problem for EF logic [May97a, May98]. We also provide several undecidability results to complete the picture—we show that any "reasonable" bisimulation-like equivalence is undecidable between state-extended PA processes and finite-state ones. Moreover, even in case of state-extended BPP processes (which form a natural subclass of Petri nets) the weak bisimilarity with finite-state processes is undecidable.

Decidability of bisimulation-like equivalences has been intensively studied for various process classes (see e.g. [Mol96] for a complete survey). The majority of the results are about the decidability of strong bisimilarity, e.g. [BBK93, CHS95, CHM93, Sti96, ČKK97, Kuč97, Jan95].

Strong bisimilarity with finite-state processes is known to be decidable for (labelled) Petri nets [JM95], PA and pushdown processes [JK97]. Another positive result of this kind is presented in [May96], where it is shown that weak bisimilarity is decidable between BPP and finite-state processes. However, weak bisimilarity with finite-state processes is undecidable for Petri nets [JE96]. Thus, in this paper we obtain original positive results for PAD (and hence also PA and PDA) processes, and an undecidability result for state-extended BPP processes. Moreover, all positive results are proved using the same general strategy, which can also be adapted to previously established ones.

2 Definitions

Transition systems are widely accepted as a structure which can exactly define the operational semantics of processes. In the rest of this paper we understand processes as (being associated with) nodes in transition systems of certain types.

Definition 1. *A transition system (TS) $\mathcal{T}$ is a triple $(S, Act, \rightarrow)$ where S is a set of* states*, Act is a finite set of* actions *(or* labels*) and $\rightarrow \subseteq S \times Act \times S$ is a*

transition relation.

We defined Act as a finite set; this is a little bit nonstandard, but we can allow this as all classes of processes we consider generate transition systems of this kind. As usual, we write $s \stackrel{a}{\to} t$ instead of $(s,a,t) \in \to$ and we extend this notation to elements of Act^* in an obvious way (we sometimes write $s \to^* t$ instead of $s \stackrel{w}{\to} t$ if $w \in Act^*$ is irrelevant). A state t is *reachable* from a state s if $s \to^* t$.

Let $Var = \{X, Y, Z, \ldots\}$ be a countably infinite set of *variables*. The class of *process expressions*, denoted $\mathcal{E}$, is defined by the following abstract syntax equation:

$$E \ ::= \ \lambda \mid X \mid E \| E \mid E.E$$

Here X ranges over Var and λ is a constant that denotes the empty expression. In the rest of this paper we do not distinguish between expressions related by *structural congruence* which is the smallest congruence relation over process expressions such that the following laws hold:

- associativity for '.' and '$\|$'
- commutativity for '$\|$'
- 'λ' as a unit for '.' and '$\|$'.

A *process rewrite system* [May97b] is specified by a finite set Δ of *rules* which are of the form $E \stackrel{a}{\to} F$, where E, F are process expressions and a is an element of a finite set Act. Each process rewrite system determines a unique transition system where states are process expressions, Act is the set of labels, and transitions are determined by Δ and the following inference rules (remember that '$\|$' is commutative):

$$\frac{(E \stackrel{a}{\to} F) \in \Delta}{E \stackrel{a}{\to} F} \qquad \frac{E \stackrel{a}{\to} E'}{E.F \stackrel{a}{\to} E'.F} \qquad \frac{E \stackrel{a}{\to} E'}{E \| F \stackrel{a}{\to} E' \| F}$$

The classes of BPA, BPP, PA, and PAD systems are subclasses of process rewrite systems obtained by certain restrictions on the form of the expressions which can appear at the left-hand and the right-hand side of rules. To specify those restrictions, we first define the classes of *sequential* and *parallel* expressions, composed of all process expressions which do not contain the '$\|$' and the '.' operator, respectively. BPA, BPP, and PA allow only a single variable at the left-hand side of rules, and a sequential, parallel, and general process expression at the right-hand side, respectively. Note that each transition $E \stackrel{a}{\to} F$ is due to some rule $X \stackrel{a}{\to} G$ of Δ (i.e. X is rewritten by G within E, yielding the expression F). Generally, there can be more than one rule of Δ with this property—if e.g. $\Delta = \{X \stackrel{a}{\to} X \| Y, Y \stackrel{a}{\to} Y \| Y\}$, then the transition $X \| Y \stackrel{a}{\to} X \| Y \| Y$ can be derived

in one step in two different ways. For each transition $E \stackrel{a}{\rightarrow} F$ we denote the set of all rules of Δ which allow to derive the transition in one step by $Step(E \stackrel{a}{\rightarrow} F)$.

The PA class strictly subsumes BPA and BPP systems; a proper extension of PA is the class of PAD systems (see [May97b]), where sequential expressions are allowed at the left-hand side and general ones at the right-hand side of rules. The PAD class strictly subsumes not only PA but also PDA processes (see below). This fact is demonstrated in [May97b].

Another way how to extend a PA system is to add a finite-state control unit to it. A *state-extended PA system* is a triple (Δ, Q, BT) where Δ is a PA system, Q is a finite set of *states*, and $BT \subseteq \Delta \times Q \times Q$ is a set of *basic transitions*. The transition system generated by a state-extended PA system (Δ, Q, BT) has $Q \times \mathcal{E}$ as the set of states (its elements are called *state-extended PA processes*, or *StExt(PA)* processes for short), Act is the set of labels, and the transition relation is determined by

$$(p, E) \stackrel{a}{\rightarrow} (q, F) \quad \text{iff} \quad E \stackrel{a}{\rightarrow} F \text{ and } (X \stackrel{a}{\rightarrow} G, p, q) \in BT \text{ for some element } X \stackrel{a}{\rightarrow} G \text{ of } Step(E \stackrel{a}{\rightarrow} F)$$

Natural subclasses of *StExt(PA)* systems are *StExt(BPA)* and *StExt(BPP)*, which are also known as pushdown (PDA) and parallel pushdown (PPDA) systems, respectively. Each *StExt(BPA)* system can also be seen as a PAD system; however, the classes of *StExt(BPP)* and PAD systems are semantically incomparable (w.r.t. *strong bisimilarity*, which is defined in the next section—see also [May97b]).

3 A General Method for Bisimulation-Like Equivalences

In this section we design a general method for proving decidability of bisimulation-like equivalences between infinite-state processes and finite-state ones.

Definition 2. *Let $R : Act \rightarrow 2^{Act^*}$ be a (total) function, assigning to each action its corresponding set of* responds. *We say that R is* closed under substitution *if the following conditions hold:*

- $a \in R(a)$ *for each* $a \in Act$
- *If* $b_1 b_2 \ldots b_n \in R(a)$ *and* $w_1 \in R(b_1), w_2 \in R(b_2), \ldots, w_n \in R(b_n)$, *then also* $w_1 w_2 \ldots w_n \in R(a)$.

In order to simplify our notation, we adopt the following conventions in this section:

- $\mathcal{G} = (G, Act, \rightarrow)$ always denotes a (general) transition system.
- $\mathcal{F} = (F, Act, \rightarrow)$ always denotes a finite-state transition system with k states.
- R always denotes a function from Act to 2^{Act^*} which is closed under substitution.
- N always denotes a decidable binary predicate defined for pairs (s, t) of nodes in transition systems (which will be clear from the context). Moreover, N is reflexive, symmetric, and transitive.

Note that $\mathcal{G}$ and $\mathcal{F}$ have the same set of actions Act. All definitions and propositions which are formulated for $\mathcal{G}$ should be considered as general; if we want to state some specific property of finite-state transition systems, we refer to $\mathcal{F}$. We also assume that $\mathcal{G}$, $\mathcal{F}$, R, and N are defined in a 'reasonable' way so that we can allow natural decidability assumptions on them (e.g. it is decidable whether $g \stackrel{a}{\rightarrow} g'$ for any given $g, g' \in G$ and $a \in Act$, or whether $w \in R(a)$ for a given $w \in Act^*$, etc.)

Definition 3. *The* extended *transition relation* $\Rightarrow \subseteq G \times Act \times G$ *is defined as follows:* $s \stackrel{a}{\Rightarrow} t$ *iff* $s \stackrel{w}{\rightarrow} t$ *for some* $w \in R(a)$.

Definition 4. *A relation* $P \subseteq G \times G$ *is an* R-N-bisimulation *if whenever* $(s, t) \in P$, *then* $N(s, t)$ *is true and for each* $a \in Act$*:*

- *If* $s \stackrel{a}{\rightarrow} s'$, *then* $t \stackrel{a}{\Rightarrow} t'$ *for some* $t' \in G$ *such that* $(s', t') \in P$.
- *If* $t \stackrel{a}{\rightarrow} t'$, *then* $s \stackrel{a}{\Rightarrow} s'$ *for some* $s' \in G$ *such that* $(s', t') \in P$.

States $s, t \in G$ *are* R-N-bisimilar*, written* $s \stackrel{RN}{\sim} t$*, if there is an R-N-bisimulation relating them.*

Various special versions of R-N-bisimilarity appeared in the literature, e.g. *strong* and *weak* bisimilarity (see [Par81, Mil89]). The corresponding versions of R (denoted by S and W, respectively) are defined as follows:

- $S(a) = \{a\}$ for each $a \in Act$
- $W(a) = \begin{cases} \{\tau^i \mid i \in \mathbf{N_0}\} & \text{if } a = \tau \\ \{\tau^i a \tau^j \mid i, j \in \mathbf{N_0}\} & \text{otherwise} \end{cases}$

The 'τ' is a special (silent) action, usually used to model an internal communication. As the predicate N is not used in the definitions of strong and weak bisimilarity, we can assume it is always true (we use T to denote this special case of N in the rest of this paper). One can also argue that the N predicate

could be omitted from the definition of R-N-bisimilarity, as it is not employed by any known bisimulation-like equivalence. This is not completely true, as e.g. the version of bisimilarity introduced in [Mol96] uses such a predicate to distinguish between 'terminal' and 'final' states. However, the main reason for introducing the N predicate is our effort to enlarge the class of bisimulation-like equivalences for which we can provide positive decidability results as much as possible.

The concept of R-N-bisimilarity covers many equivalences, which have not been explicitly investigated so far; for example, we can define the function R like this:

- $K(a) = \{a^i \mid i \in \mathbf{N_0}\}$ for each $a \in Act$.
- $L(a) = \{w \in Act^* \mid w \text{ begins with } a\}$.
- $M(a) = \begin{cases} Act^* & \text{if } a = \tau \\ \{w \in Act^* \mid w \text{ contains at least one } a\} & \text{otherwise} \end{cases}$

The predicate N can also have various forms. We have already mentioned the 'T' (always true). Another natural example is the I predicate: $I(s,t)$ is true iff s and t have the same sets of initial actions (the set of initial actions of a state $g \in G$ is $\{a \in Act \mid g \stackrel{a}{\rightarrow} g' \text{ for some } g' \in G\}$). It is easy to see that e.g. $\stackrel{ST}{\sim}$ coincides with $\stackrel{SI}{\sim}$, while $\stackrel{WI}{\sim}$ refines $\stackrel{WT}{\sim}$.

To the best of our knowledge, the only bisimulation-like equivalence which cannot be seen as R-N-bisimilarity is *branching bisimilarity* introduced in [vGW89]. This relation also places requirements on 'intermediate' nodes that extended transitions pass through, and this brings further difficulties. Therefore we do not consider branching bisimilarity in our paper.

R-N-bisimilarity can also be defined in terms of the so-called *R-N-bisimulation game*. Imagine that there are two tokens initially placed in states s and t such that $N(s,t)$ is true. Two players, Al and Ex, now start to play a game consisting of a (possibly infinite) sequence of rounds, where each round is performed as follows:

1. Al chooses one of the two tokens and moves it along an arbitrary (but single!) transition, labelled by some $a \in Act$.

2. Ex has to respond by moving the other token along a finite sequence of transitions in such a way that the corresponding sequence of labels belongs to $R(a)$ and the predicate N is true for the states where the tokens lie after Ex finishes his move.

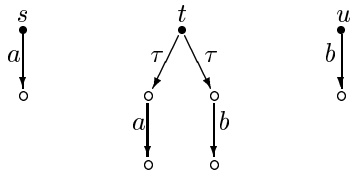
Al wins the R-N-bisimulation game, if after a finite number of rounds Ex cannot respond to Al's final attack. Now it is easy to see that the states s and t are R-N-bisimilar iff Ex has a universal defending strategy (i.e. Ex can play in such a way that Al cannot win).

A natural way how to approximate R-N-bisimilarity is to define the family of relations $\overset{RN}{\sim}_i \subseteq G \times G$ for each $i \in \mathbf{N_0}$ as follows: $s \overset{RN}{\sim}_i t$ iff $N(s,t)$ is true and Ex has a defending strategy within the first i rounds in the R-N-bisimulation game. However, $\overset{RN}{\sim}_i$ does not have to be an equivalence relation. Moreover, it is not necessarily true that $s \overset{RN}{\sim} t \iff s \overset{RN}{\sim}_i t$ for each $i \in \mathbf{N_0}$.

Example 1. *It is a well-known fact that in case of weak bisimilarity (i.e. W-T-bisimilarity) the equivalence*

$$s \overset{WT}{\sim} t \iff s \overset{WT}{\sim}_i t \text{ for each } i \in \mathbf{N_0}$$

does not hold in general ('$\Longleftarrow$' does not have to be valid). Moreover, $\overset{WT}{\sim}_i$ is not transitive for $i \geq 1$. To see this, consider the states s, t, u in the following transition system:



Now $s \overset{WT}{\sim}_1 t$ and $t \overset{WT}{\sim}_1 u$, but $s \not\overset{WT}{\sim}_1 u$.

Now we show how to overcome those drawbacks; to do this, we introduce the *extended* R-N-bisimulation relation:

Definition 5. *A relation $P \subseteq G \times G$ is an* extended *R-N-bisimulation if whenever $(s,t) \in P$, then $N(s,t)$ is true and for each $a \in Act$:*

- *If $s \overset{a}{\Rightarrow} s'$, then $t \overset{a}{\Rightarrow} t'$ for some $t' \in G$ such that $(s',t') \in P$.*
- *If $t \overset{a}{\Rightarrow} t'$, then $s \overset{a}{\Rightarrow} s'$ for some $s' \in G$ such that $(s',t') \in P$.*

States $s, t \in G$ are extended R-N-bisimilar *if there is an extended R-N-bisimulation relating them.*

Naturally, we can also define the extended R-N-bisimilarity by means of the extended R-N-bisimulation game; we simply allow Al to use the 'long' moves (i.e. Al can play the same kind of moves as Ex). Moreover, we can define the family of approximations of extended R-N-bisimilarity in the same way as in case of R-N-bisimilarity—for each $i \in \mathbf{N_0}$ we define the relation $\overset{RN}{\simeq}_i \subseteq G \times G$ as follows: $s \overset{RN}{\simeq}_i t$ iff $N(s,t)$ is true and Ex has a defending strategy within the first i rounds in the extended R-N-bisimulation game where tokens are initially placed in s and t.

Lemma 1. *Two states s, t of $\mathcal{G}$ are R-N-bisimilar iff s and t are extended R-N-bisimilar.*

Proof: Every extended R-N-bisimulation is also an R-N-bisimulation; here we need that $a \in R(a)$ for each $a \in Act$. Conversely, each R-N-bisimulation is also an extended R-N-bisimulation; each extended transition is a finite sequence of transitions, hence we can concatenate 'responses' to those individual transitions, obtaining a valid response to the original extended transition. Here we need the second requirement from Definition 2, that the relation R is closed under substitution. □

Lemma 2. *The following properties hold:*

1. $\overset{RN}{\simeq}_i$ *is an equivalence relation for each* $i \in \mathbf{N_0}$.
2. *Let s, t be states of $\mathcal{G}$. Then $s \overset{RN}{\sim}_i t$ for each $i \in \mathbf{N_0}$ iff $s \overset{RN}{\simeq}_i t$ for each $i \in \mathbf{N_0}$.*

Proof:

1. For the first part, reflexivity and symmetry are obvious. Transitivity follows from the condition that the relation R is closed under substitution.
2. It follows from the definition of $\overset{RN}{\simeq}$ that $s \overset{RN}{\simeq}_i t \implies s \overset{RN}{\sim}_i t$. Hence it suffices to realize that if $s \overset{RN}{\not\simeq}_i t$, then $s \overset{RN}{\not\sim}_j t$ for some $j \in \mathbf{N_0}$—as Al can force his win using i 'long' moves and each of those moves is composed of a finite number of 'short' moves, Al could actually 'decompose' his attacks, playing only (a finite number of) short moves.

□

Remark 1. *For any states s, t of $\mathcal{G}$ and $i \in \mathbf{N_0}$ we have that if $s \overset{RN}{\simeq}_i t$ then also $s \overset{RN}{\sim}_i t$. However, there is no 'reverse correspondence'—it can be easily shown that for arbitrarily large j the implication $s \overset{RN}{\sim}_j t \implies s \overset{RN}{\simeq}_1 t$ is generally invalid (the implication is invalid even in case when t is a state in one-state TS).*

Now we examine some special features of R-N-bisimilarity on finite-state transition systems (remember that $\mathcal{F}$ is a finite-state TS with k states).

Lemma 3. *Two states s, t of $\mathcal{F}$ are R-N-bisimilar iff $s \overset{RN}{\simeq}_{k-1} t$.*

Proof: As $\mathcal{F}$ has k states and $\overset{RN}{\simeq}_{i+1}$ refines $\overset{RN}{\simeq}_i$ for each $i \in \mathbf{N_0}$, we have that $\overset{RN}{\simeq}_{k-1} = \overset{RN}{\simeq}_k$, hence $\overset{RN}{\simeq}_{k-1} = \overset{RN}{\sim}$. □

Theorem 1. *States $g \in G$ and $f \in F$ are R-N-bisimilar iff the following conditions are true:*

1. $g \overset{RN}{\simeq}_k f$
2. *For each state g' reachable from g there is a state $f' \in F$ such that $g' \overset{RN}{\simeq}_k f'$.*

Proof:
'$\Longrightarrow$': Obvious.
'$\Longleftarrow$': We prove that the relation

$$P = \{(g', f') \mid g \to^* g' \text{ and } g' \overset{RN}{\simeq}_k f'\}$$

is an extended R-N-bisimulation. Let $(g', f') \in P$ and let $g' \overset{a}{\Rightarrow} g''$ for some $a \in Act$ (the case when $f' \overset{a}{\Rightarrow} f''$ is handled is the same way). By definition of $\overset{RN}{\simeq}_k$, there is f'' such that $f' \overset{a}{\Rightarrow} f''$ and $g'' \overset{RN}{\simeq}_{k-1} f''$. It suffices to show that $g'' \overset{RN}{\simeq}_k f''$; as $g \to^* g''$, there is a state $\overline{f}$ of $\mathcal{F}$ such that $g'' \overset{RN}{\simeq}_k \overline{f}$. By transitivity of $\overset{RN}{\simeq}_{k-1}$ we have $\overline{f} \overset{RN}{\simeq}_{k-1} f''$, hence $\overline{f} \overset{RN}{\simeq}_k f''$ (due to Lemma 3). Now $g'' \overset{RN}{\simeq}_k \overline{f} \overset{RN}{\simeq}_k f''$ and thus $g'' \overset{RN}{\simeq}_k f''$ as required. Clearly $(g, f) \in P$ and the proof is finished. □

Remark 2. *We have already mentioned that the equivalence*

$$s \overset{RN}{\sim} t \iff s \overset{RN}{\simeq}_i t \text{ for each } i \in \mathbf{N_0}$$

is generally invalid (e.g. in case of weak bisimilarity). However, as soon as we assume that t is a state in a finite-state transition system, the equivalence becomes true. This is an immediate consequence of the previous theorem. Moreover, the second part of Lemma 2 says that we could also use the $\overset{RN}{\sim}_i$ approximations in the right-hand side of the equivalence.

The previous theorem in fact says that one can use the following strategy to decide whether $g \overset{RN}{\sim} f$:

1. Decide whether $g \overset{RN}{\simeq}_k f$ (if not, then $g \overset{RN}{\not\sim} f$).
2. Check whether g can reach a state g' such that $g \overset{RN}{\not\simeq}_k f'$ for *any* state f' of $\mathcal{F}$ (if there is such a g' then $g \overset{RN}{\not\sim} f$; otherwise $g \overset{RN}{\sim} f$).

However, none of these tasks is easy in general. Our aim is to examine both subproblems in detail, keeping the general setting. Thus we cannot expect any 'universal' (semi)decidability result, because even the problems $g \overset{WT}{\simeq}_1 f$ and $g \overset{WT}{\not\simeq}_1 f$ are not semidecidable in general (see Section 5).

As $\mathcal{F}$ has finitely many states, the extended transition relation $\Rightarrow$ is finite and effectively constructible. This allows us to "extract" from $\mathcal{F}$ the information

which is relevant for the first k moves in the extended R-N-bisimulation game by means of branching trees with depth at most k, whose arcs are labelled by elements of Act and nodes are labelled by elements of $F \cup \{\bot\}$, where $\bot \notin F$. The aim of following definition is to describe all such trees up to isomorphism (remember that Act is a *finite* set).

Definition 6. *For each $i \in \mathbf{N_0}$ we define the set of* Trees *with* depth *at most i (denoted $Tree_i$) inductively as follows:*

- *A Tree with depth* 0 *is any tree with no arcs and a single node (the root) which is labelled by an element of $F \cup \{\bot\}$.*
- *A Tree with depth at most $i+1$ is any directed tree with root r whose nodes are labelled by elements of $F \cup \{\bot\}$, arcs are labelled by elements of Act, which satisfies the following conditions:*
  - *If $r \stackrel{a}{\rightarrow} s$, then the subtree rooted by s is a Tree with depth at most i.*
  - *If $r \stackrel{a}{\rightarrow} s$ and $r \stackrel{a}{\rightarrow} s'$, then the subtrees rooted by s and s' are not isomorphic.*

It is clear that the set $Tree_j$ is finite for any $j \in \mathbf{N_0}$. More precisely, its cardinality (denoted $NT(j)$) is given by:

- $NT(0) = k + 1$
- $NT(i+1) = (k+1) \cdot (2^{n \cdot NT(i)} \Leftrightarrow 1)$, where $n = card(Act)$

The set $Tree_j$ is effectively constructible for each $j \in \mathbf{N_0}$. As each Tree can be seen as a transition system, we can also speak about *Tree-processes* which are associated with roots of Trees (we do not distinguish between Trees and Tree-processes in the rest of this paper).

Now we introduce special rules which replace the standard ones whenever we consider an extended R-N-bisimulation game with initial state (g, p), where $g \in G$ and p is a Tree process (formally, this is a *different* game—however, it does not deserve a special name in our opinion).

- Al and Ex are allowed to play only 'short' moves consisting of exactly one transition whenever playing within the Tree process p (transitions of Trees correspond to extended transitions of $\mathcal{F}$).

- The predicate $N(g', p')$, where $g' \in G$ and p' a state of the Tree process p, is evaluated as follows:

$$N(g', p') = \begin{cases} \text{true} & \text{if } \mathit{label}(p') = \bot \text{ and} \\ & N(g', f) = \text{false for any } f \in F \\ \text{false} & \text{if } \mathit{label}(p') = \bot \text{ and} \\ & N(g', f) = \text{true for some } f \in F \\ N(g', \mathit{label}(p')) & \text{otherwise} \end{cases}$$

Whenever we write $g \overset{RN}{\simeq}_i p$, where $g \in G$ and p is a Tree process, we mean that Ex has a defending strategy within the first i rounds in the 'modified' extended R-N-bisimulation game. The importance of Tree processes is clarified by the two lemmas below:

Lemma 4. *Let g be a state of $\mathcal{G}$, $j \in \mathbf{N_0}$. Then $g \overset{RN}{\simeq}_j p$ for some $p \in \mathit{Tree}_j$*

Proof: We proceed by induction on j:

- $\mathbf{j = 0}$: Then p is a Tree with no arcs and just one node labelled by some $f \in F$ such that $N(g, f)$ is true; if there is no such f, then it is labelled by $\bot$. Clearly $g \overset{RN}{\simeq}_0 p$.

- **Induction step:** We need to construct a Tree p such that $g \overset{RN}{\simeq}_{j+1} p$. The Tree p has a root r whose label is the same as in the case when $j = 0$. The successors of r are defined by

$$r \overset{a}{\rightarrow} s \text{ iff } g \overset{a}{\Rightarrow} g' \text{ and } g' \overset{RN}{\simeq}_j s$$

  By induction hypothesis, for each g' there is $p' \in \mathit{Tree}_j$ such that $g' \overset{RN}{\simeq}_j p'$. Thus we have $g \overset{RN}{\simeq}_{j+1} p$ as required. □

Lemma 5. *Let f be a state of $\mathcal{F}$, $j \in \mathbf{N_0}$, and $p \in \mathit{Tree}_j$ such that $f \overset{RN}{\simeq}_j p$. Then for any state g of $\mathcal{G}$ we have that $g \overset{RN}{\simeq}_j f$ iff $g \overset{RN}{\simeq}_j p$.*

Proof:
'$\Longrightarrow$': By induction on j:

- $\mathbf{j = 0}$: As $f \overset{RN}{\simeq}_0 p$ and $g \overset{RN}{\simeq}_0 f$, we have that $N(g, f)$ is true and (the root of) p is labelled by some f' such that $N(f, f')$ is true. Hence $N(g, f')$ is true and $g \overset{RN}{\simeq}_0 p$.

- **Induction step:** Let $f \overset{RN}{\simeq}_{j+1} p$ and $g \overset{RN}{\simeq}_{j+1} f$. We prove that $g \overset{RN}{\simeq}_{j+1} p$. Clearly $N(g, \mathit{label}(p))$ is true (see above). Let $g \overset{a}{\Rightarrow} g'$ (the case when $p \overset{a}{\rightarrow} p'$ can be done similarly). We need to show that $p \overset{a}{\rightarrow} p'$ for some p' with $g' \overset{RN}{\simeq}_j p'$. As $g \overset{RN}{\simeq}_{j+1} f$, there is $f' \in F$ such that $f \overset{a}{\Rightarrow} f'$ and $g' \overset{RN}{\simeq}_j f'$. Furthermore, as $f \overset{RN}{\simeq}_{j+1} p$ and $f \overset{a}{\Rightarrow} f'$, there is p' such that $p \overset{a}{\rightarrow} p'$ and $f' \overset{RN}{\simeq}_j p'$. To sum up, we have $f' \overset{RN}{\simeq}_j p'$ and $g' \overset{RN}{\simeq}_j f'$, hence $g' \overset{RN}{\simeq}_j p'$ by induction hypotheses.

'$\Longleftarrow$': In a similar way. □

Now we can extract the core of both subproblems which appeared in the previously mentioned general strategy in a (hopefully) nice way by defining two new and rather special problems—the Step-problem and the Reach-problem:

The Step-problem
Instance: (g, a, j, p) where g is a state of $\mathcal{G}$, $a \in \mathit{Act}$, $0 \leq j < k$, and $p \in \mathit{Tree}_j$.
Question: Is there a state g' of $\mathcal{G}$ such that $g \overset{a}{\Rightarrow} g'$ and $g' \overset{RN}{\simeq}_j p$?
The decision algorithm may use the oracle which for any state g'' of $\mathcal{G}$ answers whether $g'' \overset{RN}{\simeq}_j p$.

The Reach-problem
Instance: (g, p) where g is a state of $\mathcal{G}$ and p is a Tree-process of depth $\leq k$.
Question: Is there a state g' of $\mathcal{G}$ such that $g \rightarrow^* g'$ and $g' \overset{RN}{\simeq}_k p$?
The decision algorithm may use the oracle which for any state g'' of $\mathcal{G}$ answers whether $g'' \overset{RN}{\simeq}_k p$.

Formally, the transition system $\mathcal{F}$ should also be present in instances of both problems, as it determines the sets Tree_j and the constant k; we prefer the simplified form to make the following proofs more readable.

Theorem 2. *If the Step-problem is decidable (with possible usage of the mentioned oracle), then $\overset{RN}{\simeq}_k$ is decidable between any states g and f of $\mathcal{G}$ and $\mathcal{F}$, respectively.*

Proof: We prove by induction on j that $\overset{RN}{\simeq}_j$ is decidable for any $0 \leq j \leq k$. First, $\overset{RN}{\simeq}_0$ is decidable because the predicate N is decidable. Let us assume that $\overset{RN}{\simeq}_j$ is decidable (hence the mentioned oracle can be used). It remains to prove that if the Step-problem is decidable, then $\overset{RN}{\simeq}_{j+1}$ is decidable as well. We need to introduce two auxiliary finite sets:

- The set of **C**ompatible **S**teps, denoted CS_j^f, is composed exactly of all pairs of the form (a, p) where $a \in Act$ and $p \in Tree_j$, such that $f \stackrel{a}{\Rightarrow} f'$ for some f' with $f' \stackrel{RN}{\simeq}_j p$.

- The set of **INC**ompatible **S**teps, denoted $INCS_j^f$, is a complement of CS_j^f w.r.t. $Act \times Tree_j$.

The sets CS_j^f and $INCS_j^f$ are effectively constructible. By definition, $g \stackrel{RN}{\simeq}_{j+1} f$ iff $N(g, f)$ is true and the following conditions hold:

1. If $f \stackrel{a}{\Rightarrow} f'$, then $g \stackrel{a}{\Rightarrow} g'$ for some g' with $g' \stackrel{RN}{\simeq}_j f'$.

2. If $g \stackrel{a}{\Rightarrow} g'$, then $f \stackrel{a}{\Rightarrow} f'$ for some f' with $g' \stackrel{RN}{\simeq}_j f'$.

The first condition in fact says that (g, a, j, p) is a positive instance of the Step-problem for any $(a, p) \in CS_j^f$ (see Lemma 4 and 5). It can be checked effectively due to the decidability of the Step-problem.

The second condition does *not* hold iff $g \stackrel{a}{\Rightarrow} g'$ for some g' such that $g' \stackrel{RN}{\simeq}_j p$ where (a, p) is an element of $INCS_j^f$ (due to Lemma 4 and 5). This is clearly decidable due to the decidability of the Step-problem again. □

It is worth mentioning that the Step-problem is generally semidecidable (provided it is possible to enumerate all finite paths starting in g). However, it does not suffice for semidecidability of $\stackrel{RN}{\simeq}_i$ or $\stackrel{RN}{\not\simeq}_i$ between states of $\mathcal{G}$ and $\mathcal{F}$.

Theorem 3. *Decidability of the Step-problem and the Reach-problem (with possible usage of the indicated oracles) implies decidability of the problem whether for each g' reachable from a given state g of $\mathcal{G}$ there is a state f' of $\mathcal{F}$ with $g' \stackrel{RN}{\simeq}_k f'$.*

Proof: First, the oracle indicated in the definition of Reach-problem can be used because we already know that decidability of the Step-problem implies decidability of $\stackrel{RN}{\simeq}_k$ between states of $\mathcal{G}$ and $\mathcal{F}$ (see the previous theorem). To finish the proof, we need to define one auxiliary set:

- The set of **INC**ompatible **T**rees, denoted $INCT$, is composed of all $p \in Tree_k$ such that $f \stackrel{RN}{\not\simeq}_k p$ for each state f of $\mathcal{F}$.

The set $INCT$ is finite and effectively constructible. The state g can reach a state g' such that $g' \stackrel{RN}{\not\simeq}_k f$ for any state f of $\mathcal{F}$ (i.e. g is a *negative* instance of the problem specified in the second part of this theorem) iff (g, p) is a positive instance of the Reach problem for some $p \in INCT$ (due to Lemma 4 and 5). □

4 Applications

In this section we show how to apply the previously designed general method to various classes of infinite-state processes. We show that the Step-problem as well as the Reach-problem can be reduced to the model checking problem for the branching-time temporal logic EF. Therefore it is possible to apply decidability results from this area. In this way we elegantly prove that a large class of R-N-bisimulation equivalences is decidable between PAD processes and finite-state ones (the class includes all versions of R-N-bisimulation equivalences we defined in this paper and many others). First we define the logic EF (more exactly an extended version of EF with constraints on sequences of actions). The formulae have the following syntax:

$$\Phi ::= \mathit{true} \;\mid\; \neg\Phi \;\mid\; \Phi_1 \wedge \Phi_2 \;\mid\; \langle a\rangle\Phi \;\mid\; \Diamond_C\Phi$$

where a is an atomic action and C is a unary predicate on sequences of atomic actions. Let $\mathcal{T} = (S, Act, \rightarrow)$ be a transition system. The denotation $[\![\Phi]\!]$ of a formula Φ is a set of states of $\mathcal{T}$, which is defined as follows (sequences of atomic actions are denoted by w):

$$\begin{aligned}
[\![\mathit{true}]\!] &:= S \\
[\![\neg\Phi]\!] &:= S \Leftrightarrow [\![\Phi]\!] \\
[\![\Phi_1 \wedge \Phi_2]\!] &:= [\![\Phi_1]\!] \cap [\![\Phi_2]\!] \\
[\![\langle a\rangle\Phi]\!] &:= \{s \in S \mid \exists s' \in S.\ s \xrightarrow{a} s' \in [\![\Phi]\!]\} \\
[\![\Diamond_C\Phi]\!] &:= \{s \in S \mid \exists w, s'.\ s \xrightarrow{w} s' \ \wedge\ C(w) \ \wedge\ s' \in [\![\Phi]\!]\}
\end{aligned}$$

The predicates C are used to express constraints on sequences of actions. For every R-N-bisimulation we define predicates C_a s.t. for every action a and every sequence w

$$C_a(w) \Longleftrightarrow w \in R(a)$$

Let EF_R be the fragment of EF that contains only constraints C_a for R and the *true* constraint.

An instance of the *model checking problem* is given by a state s in S and an EF_R-formula Φ. The question is whether $s \in [\![\Phi]\!]$. This property is also denoted by $s \models \Phi$.

Let us fix a general TS $\mathcal{G} = (G, Act, \rightarrow)$ and a finite-state TS $\mathcal{F} = (F, Act, \rightarrow)$ with k states in the same way as in the previous section. We show how to encode the Step and the Reach problems by EF_R formulae. The first difficulty is the N predicate. Although it is decidable, we do not know anything about the strategy of the model-checking algorithm, hence this fact is generally of no use. Instead, we restrict our attention to those predicates which can be encoded by EF_R formulae

in the following sense: for each $f \in F$ there is an EF_R formula Ψ_f such that for each $g \in G$ we have that $g \models \Psi_f$ iff $N(g, f)$ is true. In this case we also define the formula $\Psi_\perp := \bigwedge_{f \in F} \neg \Psi_f$.

A concrete example of a predicate which can be encoded by EF_R formulae is e.g. the 'I' predicate defined in the previous section: For every $f \in F$ let $A_f := \{a \in Act \mid \exists f'.\, f \xrightarrow{a} f'\}$. Then

$$\Psi_f := \bigwedge_{a \in A_f} \langle a \rangle true \wedge \bigwedge_{a \in Act - A_f} \neg \langle a \rangle true$$

Now we design the family of $\Phi_{j,p}$ formulae, where $0 \leq j \leq k$ and $p \in Tree_j$, in such a way that for each $g \in G$ the following equivalence holds:

$$g \overset{RN}{\simeq}_j p \iff g \models \Phi_{j,p}$$

Having these formulae, the Step and the Reach problems can be encoded in a rather straightforward way:

- (g, a, j, p) is a positive instance of the Step problem iff $g \models \Diamond_{C_a}(\Phi_{j,p})$
- (g, p) is a positive instance of the Reach problem iff $g \models \Diamond(\Phi_{k,p})$

The family of $\Phi_{j,p}$ formulae is defined inductively on j as follows:

- $\Phi_{0,p} := \Psi_f$, where $f = label(p)$
- $\Phi_{j+1,p} := \Psi_f \wedge \left(\bigwedge_{a \in Act} \bigwedge_{p' \in S(p,a)} \Diamond_{C_a} \Phi_{j,p'} \right) \wedge \left(\bigwedge_{a \in Act} (\neg \Diamond_{C_a} (\bigwedge_{p' \in S(p,a)} \neg \Phi_{j,p'})) \right)$,

  where $f = label(p)$ and $S(p, a) = \{p' \mid p \xrightarrow{a} p'\}$. If the set $S(p, a)$ is empty, any conjunction of the form

  $$\bigwedge_{p' \in S(p,a)} \Theta_{p'}$$

  is replaced by *true*.

The decidability of model checking with the logic EF_R depends on the constraints that correspond to R. It has been shown in [May97a] that model checking PA-processes with the logic EF is decidable for the class of *decomposable constraints*. This result has been generalized to PAD processes in [May98]. These constraints are called decomposable, because they can be decomposed w.r.t. sequential and parallel composition. The formal definition is as follows: A set of decomposable constraints $\mathcal{DC}$ is a finite set of unary predicates on finite sequences of actions that contains the predicates *true* and *false* and satisfies the following conditions.

1. For every $C \in \mathcal{DC}$ there is a finite index set I and a finite set of decomposable constraints $\{C_i^1, C_i^2 \in \mathcal{DC} \mid i \in I\}$ s.t.

$$\forall w, w_1, w_2.\ w_1 w_2 = w \ \Rightarrow \ \left(C(w) \iff \bigvee_{i \in I} C_i^1(w_1) \wedge C_i^2(w_2) \right)$$

2. For every $C \in \mathcal{DC}$ there is a finite index set J and a finite set of decomposable constraints $\{C_i^1, C_i^2 \in \mathcal{DC} \mid i \in J\}$ s.t.

$$\forall w_1, w_2. \left((\exists w \in \mathit{interleave}(w_1, w_2).\, C(w)) \iff \bigvee_{i \in J} (C_i^1(w_1) \wedge C_i^2(w_2)) \right)$$

$w \in \mathit{interleave}(w_1, w_2)$ iff w is an arbitrary interleaving of w_1 and w_2. The formal definition of the function *interleave* is as follows: Let ϵ be the empty sequence.

$$\begin{array}{rcl} \mathit{interleave}(\epsilon, w) & := & \{w\} \\ \mathit{interleave}(w, \epsilon) & := & \{w\} \\ \mathit{interleave}(a_1 w_1, a_2 w_2) & := & \{a_1 w \mid w \in \mathit{interleave}(w_1, a_2 w_2)\} \cup \\ & & \{a_2 w \mid w \in \mathit{interleave}(a_1 w_1, w_2)\} \end{array}$$

It is easy to see that the closure of a set of decomposable constraints under disjunction is again a set of decomposable constraints. All the previously mentioned examples of relations R can be expressed by decomposable constraints. Consider the relation W for weak bisimulation. There we have the following constraints:

$$\begin{array}{rcl} W_\tau(w) & := & (w = \tau^i \text{ for some } i \in \mathbf{N_0}) \\ W_a(w) & := & (w = \tau^i a \tau^j \text{ for some } i, j \in \mathbf{N_0}) \end{array}$$

These constraints can be decomposed w.r.t. sequential and parallel composition. For W_τ this is trivial. For W_a we have

$$\begin{array}{rcl} W_a(w_1 w_2) & \Leftrightarrow & (W_a(w_1) \wedge W_\tau(w_2)) \vee (W_\tau(w_1) \wedge W_a(w_2)) \\ (\exists w \in \mathit{interleave}(w_1, w_2).\, W_a(w)) & \Leftrightarrow & (W_a(w_1) \wedge W_\tau(w_2)) \vee (W_\tau(w_1) \wedge W_a(w_2)) \end{array}$$

Now we show decomposability for some other (nonstandard) relations that were defined on page 6: For the relation K the decomposition is trivial. For the relation L we have the constraint

$$L_a(w) := w \text{ begins with } a$$

The decomposition is

$$\begin{aligned} L_a(w_1 w_2) &\iff L_a(w_1) \\ (\exists w \in \mathit{interleave}(w_1, w_2).\, L_a(w)) &\iff L_a(w_1) \vee L_a(w_2) \end{aligned}$$

For the relation M we have the constraints

$$\begin{aligned} M_\tau(w) &:= \mathit{true} \\ M_a(w) &:= w \text{ contains at least one } a \end{aligned}$$

The decomposition of M_τ is trivial. The decomposition of M_a is

$$\begin{aligned} M_a(w_1 w_2) &\iff M_a(w_1) \vee M_a(w_2) \\ (\exists w \in \mathit{interleave}(w_1, w_2).\, M_a(w)) &\iff M_a(w_1) \vee M_a(w_2) \end{aligned}$$

However, there are also relations R that are closed under substitution, but which yield non-decomposable constraints. For example let $Act = \{a, b\}$ and $R(a) := \{w \mid \#_a w > \#_b w\}$ and $R(b) := \{b\}$, where $\#_a w$ is the number of actions a in w. On the other hand there are decomposable constraints that are not closed under substitution like $R(a) := \{a^i \mid 1 \leq i \leq 5\}$. Now we can formulate a very general decidability theorem:

Theorem 4. *The problem $g \overset{RN}{\sim} f$, where R yields a set of constraints contained in a set $\mathcal{DC}$ of decomposable constraints, N is expressible in EF_R, g is a PAD processes, and f is a finite-state process, is decidable.*

Corollary 1. *Weak bisimilarity between a PAD process and a finite-state system is decidable.*

Let us also mention that decidability of the model checking problem for the EF_R logic in a certain class of transition systems $\mathcal{C}$ is a sufficient but not necessary condition for decidability of R-N-bisimilarity between processes of $\mathcal{C}$ and finite-state ones. For example, model checking the 'pure' EF (without any constraints) is undecidable for Petri nets, but the Step and the Reach problems are decidable for S-T-bisimilarity [JE96]. This is because the Step-problem is trivial for strong bisimulation.

5 Undecidability Results

In this section we provide several negative (undecidability) results which help to clarify the decidability/undecidability border in the area of comparing infinite-state processes with finite-state ones.

Intuitively, any 'nontrivial' equivalence with finite-state processes should be undecidable for a class of processes having 'full Turing power', which can be formally expressed as e.g. the ability to simulate Minsky counter machines:

Definition 7. A counter machine $\mathcal{M}$ *with nonnegative counters* $c_1, c_2, ..., c_m$ *is a sequence of instructions*

$$\begin{array}{ll} 1: & INS_1 \\ 2: & INS_2 \\ \vdots & \\ n \Leftrightarrow 1: & INS_{n-1} \\ n: & HALT \end{array}$$

where each INS_i $(i = 1, 2, ..., n \Leftrightarrow 1)$ *is in one of the following two forms (assuming* $1 \leq k, k_1, k_2 \leq n,\ 1 \leq j \leq m$*)*

- $c_j := c_j + 1;\quad goto\ k$
- *if* $c_j = 0$ *then goto* k_1 *else* $(c_j := c_j \Leftrightarrow 1;\ goto\ k_2)$

The halting problem problem is undecidable even for Minsky machines with two counters initialized to zero values [Min67]. Any such machine $\mathcal{M}$ can be easily 'mimicked' by a *StExt(PA)* process $P(\mathcal{M})$. A construction of the $P(\mathcal{M})$ process is described in [JK97]. The (two) counters are modelled by a simple PA process $(I_1.I_1 \ldots I_1.Z_1) \| (I_2.I_2 \ldots I_2.Z_2)$ where the number of I_i's means the current value of the counter c_i, $i = 1, 2$ (the starting zero point being modelled by $Z_1 \| Z_2$). The control states $(s_1, s_2, \ldots, s_n)$ correspond to the instructions of $\mathcal{M}$; each state determines the unique transition to be performed next with the exception of s_n which is the 'halting state'.

If we label each transition in $P(\mathcal{M})$ by a fixed action a then it is able either to perform the action a boundedly many times and to stop (its behaviour can be defined as a^n for some n) or to do a forever (its behaviour being a^ω); this depends on whether the corresponding counter machine $\mathcal{M}$ halts or not. Notice that a^ω is the behaviour of the 1-state transition system $(\{s\}, \{a\}, \{(s, a, s)\})$. When we declare as *reasonable* any equivalence which distinguishes between (processes with) behaviours a^ω and a^n, we can conclude:

Theorem 5. *Any reasonable equivalence between StExt(PA) processes and finite-state ones is undecidable.*

It is obvious that (almost) any R-N-bisimilarity is reasonable in the above sense, except for some trivial cases. For weak bisimilarity, we can even show that none of the problems $g \overset{WT}{\simeq}_1 f$, $g \overset{WT}{\not\simeq}_1 f$ is semidecidable when g is a *StExt(PA)* process. It suffices to realize that we can label all transitions in $\mathcal{P}(\mathcal{M})$ by τ and add a special a-transition enabled in the (halting) state s_n.

Once seeing that *StExt(PA)* are strong enough to make our equivalences undecidable, it is natural to ask what happens when we add finite-state control parts to processes from subclasses of PA, namely to BPA and BPP.

We have already shown that any R-N-bisimilarity such that R yields decomposable constraints and N is expressible within EF_R is decidable between *StExt(BPA)* (i.e. PDA) processes and finite-state ones. In the case of *StExt(BPP)*, strong bisimilarity with finite-state processes is decidable [JM95]. Here we demonstrate that the problem for weak bisimilarity is undecidable.

We start by recalling the construction of the proof from [JE96] which shows that the weak bisimilarity is undecidable between labelled Petri nets and finite-state systems.

Definition 8. *A* labelled (place/transition) Petri net *over Act is a tuple* $N = (P, T, F, M_0, \ell)$ *where*

- P *and* T *are finite and disjoint sets of* places *and* transitions*, respectively;*
- $F: (P \times T) \cup (T \times P) \to \{0, 1\}$ *determines (by attaching the value 1) the* arcs*;*
- $M_0: P \to \mathbf{N_0}$ *is the* initial marking *of* N *(*$\mathbf{N_0}$ *denoting the set of nonnegative integers);*
- $\ell: T \to Act$ *is a* labelling*, which associates an action to each transition.*

Each net N determines a unique TS; its states are markings, i.e. mappings $M: P \to \mathbf{N_0}$ where M_0 is the distinguished (starting) state, Act is the action set, and $M \xrightarrow{a} M'$ iff there is $t \in T$ s.t. $\ell(t) = a$, $M(p) \geq F(p,t)$ and $M'(p) = M(p) \Leftrightarrow F(p,t) + F(t,p)$ for each $p \in P$.

A place p is *bounded* in N iff there is some $m \in \mathbf{N_0}$ s.t. $M(p) \leq m$ for any M which is reachable from M_0; otherwise p is *unbounded*.

It can be easily shown that a labelled Petri net where each transition t has exactly one input place ($F(p,t) = 1$ exactly for one p) is equivalent to a BPP process (the corresponding transition systems are isomorphic)—cf. e.g. [Esp95]. Similarly, if any transition has at most one unbounded place among its input places, then it is easy to transform the net into an equivalent *StExt(BPP)* process (the marking of bounded places is modelled by finite control states); let us call such nets as *StExt(BPP)*-nets.

The idea of the mentioned construction from [JE96] looks as follows. First, the 7-state transition system of Figure 1 with a distinguished state f is fixed. Then it is shown how to construct a net $N_{\mathcal{M}}$ for any two-counter machine $\mathcal{M}$. The net $N_{\mathcal{M}}$ is sketched in Figure 2. It holds that the net $N_{\mathcal{M}}$ is weakly bisimilar to f iff the machine $\mathcal{M}$ does not halt for the zero input. Therefore, if the net $N_{\mathcal{M}}$ were

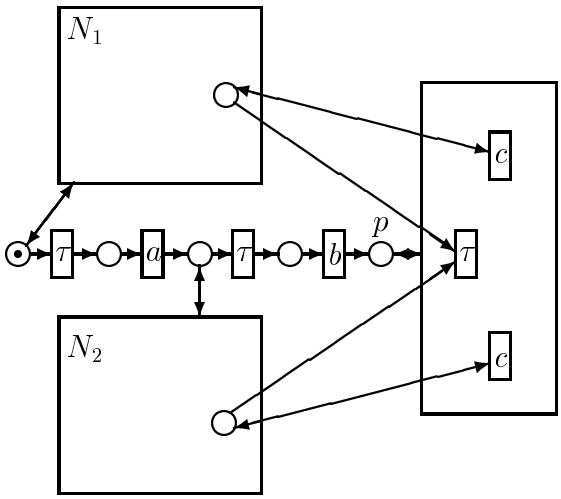


Figure 2

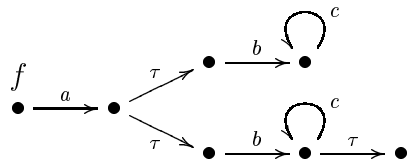


Figure 1

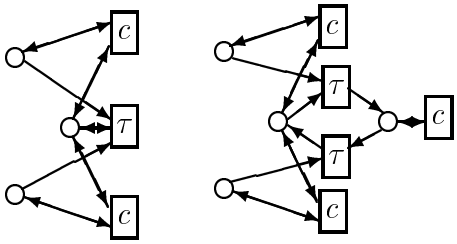


Figure 3

always a *StExt(BPP)*-net, we would be done. In fact, it is not the case but $N_{\mathcal{M}}$ can be suitably transformed. The depicted subnets N_1 and N_2 (with all transitions labelled by τ) can be constructed as in [Jan95] (for showing undecidability of the reachability set equality problem). At most 5 places in N_1 and 5 places in N_2 can be unbounded then; it can be easily verified that each their transition has at most 1 unbounded place among its input places.

The only difficulty are the τ-transitions in the 'right-hand side' of $N_{\mathcal{M}}$ (the places of N_1 correspond bijectively to places of N_2 and there is one such τ-transition for each corresponding pair).

Important here is that all transitions in the right hand side are enabled after the depicted place p is marked (p is bounded and can be marked by 1 at most); p is a 'run-place' for these transitions (it is an input and output place for each of them). At that time, either the action c is enabled forever – when the markings in N_1 and N_2 differ – or a deadlock can be reached (using the τ-transitions) – when the markings in N_1 and N_2 coincide.

For our aims, we can replace each τ-transition which has 2 unbounded input places in the way depicted in Figure 3 (replacing the left-hand side by the right-hand side). Thus we transform $N_{\mathcal{M}}$ into an *StExt(BPP)*-net, leaving it in the same weak bisimilarity class. Therefore we can conclude:

Theorem 6. *Weak bisimilarity is undecidable between StExt(BPP) processes and finite-state ones.*

6 Conclusions, Future Work

We designed a general method for proving decidability of R-N-bisimilarity between infinite-state processes and finite-state ones (Theorem 1) by reducing this problem to two other problems—the Step and the Reach problem (Theorem 2 and 3). We also showed how to encode these special problems by formulae of EF_R logic. As this logic is decidable for PAD (and hence also PA and PDA) processes, we obtained a general decidability theorem (Theorem 4), which says that any R-N-bisimilarity such that R yields decomposable constrains on sequences of actions and N can be expressed by EF_R formulae is decidable between PAD and finite-state processes. This class of R-N-bisimilarities includes all versions of R-N-bisimulation equivalences mentioned in this paper. Examples are the relations $\overset{KI}{\sim}$, $\overset{LT}{\sim}$, $\overset{MI}{\sim}$, or $\overset{WI}{\sim}$, but most importantly $\overset{ST}{\sim}$ and $\overset{WT}{\sim}$ (i.e. the strong and weak bisimilarity).

Then we demonstrated that each "reasonable" R-N-bisimilarity is undecidable between *StExt(PA)* processes and finite-state ones (Theorem 5); this is caused by the fact that *StExt(PA)* processes have full Turing power. Moreover, even if we restrict our attention to *StExt(BPP)*, we get undecidability result for weak bisimilarity (Theorem 6). This proof is obtained by a modification of the one which has been used for Petri nets.

A complete summary of the results on decidability of bisimulation-like equivalences with finite-state processes is given in the table below. As we want to make clear what results have been previously obtained by other researchers, our table contains more rows than it is necessarily needed (e.g., the positive result for PAD and $\overset{RN}{\sim}$, where R and N have the above indicated properties, 'covers' all positive results for BPA, BPP, PA, and PDA). We also add a special column which indicates decidability of the model-checking problem for EF.

	$\overset{ST}{\sim}$	$\overset{WT}{\sim}$	$\overset{RN}{\sim}$	EF
BPA	Yes [CHS95]	**YES**	**YES**	Yes
BPP	Yes [CHM93]	Yes [May96]	**YES**	Yes
PA	Yes [JK97]	**YES**	**YES**	Yes
StExt(BPA), i.e. PDA	Yes [JK97]	**YES**	**YES**	Yes
StExt(BPP), i.e. PPDA	Yes [JM95]	**NO**	**NO**	No
StExt(PA)	No [JK97]	No [JK97]	No [JK97]	No
PAD	**YES**	**YES**	**YES**	Yes
Petri nets	Yes [JM95]	No [JE96]	No [JE96]	No

The results obtained in this paper are in boldface. Note that although model-checking EF logic is undecidable for *StExt(BPP)* processes and Petri nets, strong bisimilarity with finite-state systems is decidable. The original proof in [JM95] in

fact demonstrates decidability of the Reach problem (the Step problem is trivially decidable), hence our general strategy applies in this case, too.

A unifying concept similar to R-N-bisimulation can also be used in case of simulation-like equivalences—we can define the R-N-simulation relation in the very same way as R-N-bisimulation (which can be then seen as a special case of R-N-simulation with the property that its inverse is also an R-N-simulation). The predicate N becomes more important in this context, as it allows to define some of the known and studied simulation-like equivalences (e.g. the ready simulation equivalence). An interesting open problem is whether it is possible to design a general strategy for deciding R-N-simulation equivalence between infinite-state and finite-state processes in a similar way as in case of R-N-bisimilarity. Another set of open problems is decidability of branching bisimilarity with finite-state processes.

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