

Exercises - set 4

Basic techniques.

March 21, 2013, 8:30-9:30 B411

1. Write down some matrix that would represent the outcomes of the rock-paper-scissors game for Alice (rows) and Bob (columns), (for example the winner gets 1 point, loser loses 1 point, if both do the same gesture, they both get 0, but you may choose any other numbers that make sense).
2. Prove that for any matrix $M = (a_{ij})_{i,j=1}^n$ with real numbers as elements the following inequality holds

$$\max_i \min_j a_{ij} \leq \min_j \max_i a_{ij}.$$

3. Consider the two-players game whose outcomes are given by the matrix

$$\begin{pmatrix} 5 & 6 \\ 7 & 4 \end{pmatrix}$$

Find optimal (probabilistic) strategies for both players.

4. Let us consider a directed graph G . Problem is to decide if there exists a vertex with incoming degree 0 (no edges enter the vertex). Find a lower bound on the expected running time of any Las Vegas algorithm solving this problem (hint: use the Yao's principle). The running time may be seen as a number of queries "is there an edge from u to v ?"