Exercises – set 3

PROBABILITY THEORY BASICS I

Shenggen Zheng, March 17, 2013, 8:30–9:30 B411

1. (Cheating game) After lunch one day, Alice suggests to Bob the following method to determine who pays. Alice pulls three six-sided dice from her pocket. These dice are not the standard dice, but have the following numbers on their faces:

die A: -1, 1, 6, 6, 8, 8;

die B: -2, 2, 4, 4, 9, 9;

die C: -3, 3, 5, 5, 7, 7.

The dice are fair, so each side comes up with equal probability. Alice explains that Alice and Bob will each pick up one of the dice. They will each roll their die, and the one who rolls the lowest number loses and will buy lunch. So as to take no advantage, Alice offers Bob the first choice of the dice.

(a) Suppose that Bob chooses die A and Alice chooses die B. Write out all of the possible events and their probabilities, and show that the probability that Alice wins is greater than 1/2.

(b) Suppose that Bob chooses die B and Alice chooses die C. Write out all of the possible events and their probabilities, and show that the probability that Alice wins is greater than 1/2.

(c) Since die A and die B lead to situations in Alice's favor, it would seem that Bob should choose die C. Suppose that Bob chooses die C and Alice chooses die A. Write out all of the possible events and their probabilities, and show that the probability that Alice wins is greater than 1/2.

2. Alice has two coins in her pocket, a fair coin (head on one side and tail on the other side) and a two-headed coin. She picks one at random from her pocket, tosses it and obtains head. What is the probability that she flipped the fair coin?

3*. (Useful proof method to funny questions.)

(a) Give a simple proof that you should change your choice in the "MONTY HALL PARADOX".

(b) Give a simple calculation to the exercise 2.

(c) Give a simple calculation to the "EXAMPLE 2" (Three coins are given - two fair ones and in the third one heads land with probability 2/3, but we do not know which one is not fair one.) in the lecture slides.

(**hints:** Do not use Bayes' rule. If you want to know the secret, please come to the tutorial. I will promise you that the proof method is funny, surprising and very easy to understand!)

4. (Deadly Disease Joke) There is a kind of deadly disease. The probability for a person to get this disease is one out of a millon. But if one has this disease, he/she will die soon for sure. If someone has this disease and goes for a blood test, the probability of the outcome has high accuracy, says providing wrong answer with one out a thousand. One day Alice known about this disease from a book. She thought that it was quite a deadly disease and the blood test had high accuracy. So she went to hospital and had a blood test. Unluckily, the outcome of the blood test was positive. Alice was so sad. She just stayed at home and prepared everything in case that she would die soon. What would you say to Alice in order to console her if you was there with her? Why?

5. Let (S, Pr) be a probability space and let X and Y be two different random variables on S. Let $Z = \min\{X, Y\}$ be the random variable defined by $Z(s) = \min\{X(s), Y(s)\}$ for every $s \in S$. Prove or disprove the following claim:

$$E[Z] = \min\{E[X], E[Y]\}. \tag{1}$$