Exercises - set 1

Basic concepts and examples of randomized algorithms.

February 28, 2013, 8:30-9:30 B410

1. Consider an alternative definition of class ZPP as the class of languages which have Las Vegas algorithms (gives correct answer with probability 1, the time to compute it may vary) deciding whether $x \in L$ running in expected polynomial time. Show that the equality $ZPP = RP \cap co - RP$ holds with this definition of ZPP.

2. Consider the randomized algorithm for finding min-cut from the lecture. Suppose that at each step instead of choosing a random edge for contraction we choose two vertices at random, identify these two vertices into one vertex and remove any selfloops that appear by the identification. Find examples of graphs on which the probability that this modified algorithm finds a min-cut is exponentially small.

3. Let ϵ, δ be two positive real numbers such that $0 < \epsilon < \delta < 1$. Let A be a randomized algorithm that computes a function F with

$$Prob(A(x) = F(x)) \geq \epsilon,$$

$$Prob(A(x) = \text{"?"}) = 1 - Prob(A(x) - F(x)).$$

Let A_k be the randomized algorithm that for any input x executes k runs of A (for any positive integer $k \geq 2$). The output of A_k is "?" only if all k runs ended with result "?". In other cases A_k computes the right result $F(x)$. Estimate the smallest k such that $Prob(A_k(x) = F(x)) \geq \delta$.