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Technické řešení této výukové pomůcky je spolufinancováno Evropským sociálním fondem a státním rozpočtem České republiky.

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

CONTENTS

BASIC INFORMATION - I.

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- For lectures I will use: computer slides, overhead projector slides and, sometimes, also the blackboard.

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- Likely, the most efficient use of the lectures is to print materials of each lecture before the lecture and to make your comments into them during the lecture.

BASIC INFORMATION - II.

- Lecture's web page contains also **Appendix** - important very basic facts from the number theory and algebra that you should, but may not, know and you will need - read and learn them carefully.
- Whenever you find an error or misprint in lecture notes, let me know - extra points you get for that.

To your disposal there are also lecture notes called the "Exercises Book" that you can upload from the IS for the lecture IV054, through links "Ucebni materialy –¿ Exercise Book"

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Lecture notes contain selected exercises from the homeworks for the past lectures on Coding, Cryptography and Cryptographic Protocols" with solutions.

LITERATURE

- R. Hill: A first course in coding theory, Claredon Press, 1985
- V. Pless: Introduction to the theory of error-correcting codes, John Willey, 1998
- J. Gruska: Foundations of computing, Thomson International Computer Press, 1997
- J. Gruska: Quantum computing, McGraw-Hill, 1999
- A. Salomaa: Public-key cryptography, Springer, 1990
- D. R. Stinson: Cryptography: theory and practice, CRC Press, 1995
- W. Trappe, L. Washington: Introduction to cryptography with coding theory, 2006
- B. Schneier: Applied cryptography, John Willey and Sons, 1996
- S. Singh: The code book, Anchor Books, 1999
- D. Kahn: The codebreakers. Two story of secret writing. Macmillan, 1996 (An entertaining and informative history of cryptography.)
- Vaudenay: A classical introduction to cryptography, Springer, 2006
- J. Gruska: Coding, Cryptography and Cryptographic Protocols, lecture notes, http://www.fi.muni.cz/usr/gruska/crypto17
- J. Fridrich: Steganography in Digital Media, Cambridge University Press, 2010.
- J.Gruska and collective: Exercises and their solutions for IV054, 2015, FI, MU Brno; http://www.fi.muni.c/ xbohac/crypto/exercice-book.pdf
- A. J. Menezes, P. C. van Oorschot, S. A. Vanstone: The Handbook of Applied Cryptography, 1996

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Cryptography, when broadly understood, is an important tool to achieve such goals.

Part I

Basics of coding theory

PROLOGUE - I.

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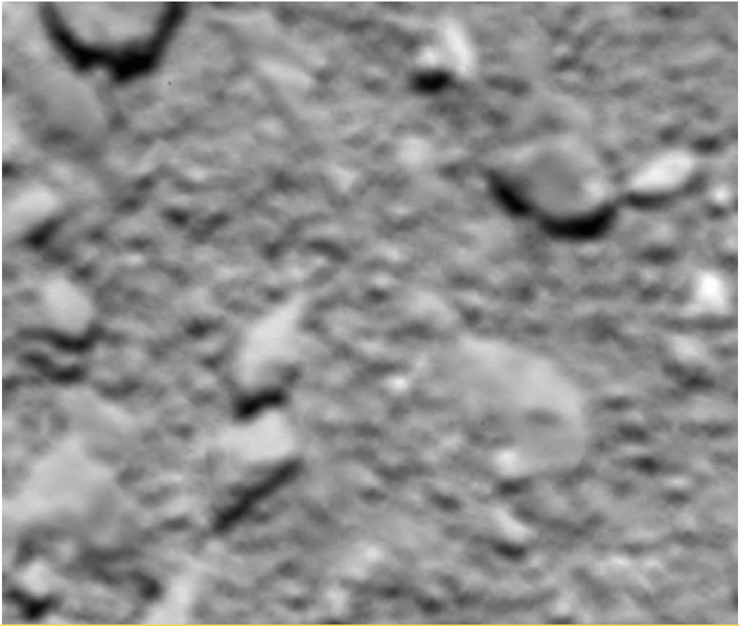
- **Since that time coding theory has made another enormous progress that has allowed, among other things, almost perfect mobile communication and transmission of music in time and space.**

ROSETTA spacecraft

ROSETTA LANDING - VIEW from 21 km -29.9.2016

ROSETTA LANDING - VIEW from 51 m -29.9.2016



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This first chapter presents and illustrates the very basic problems, concepts, methods and results of coding theory.

PROLOGUE - II.

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using mainly classical, but also quantum tools.

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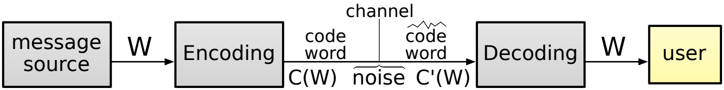
Error-correcting codes are used to correct messages when they are (erroneously) transmitted through noisy channels.



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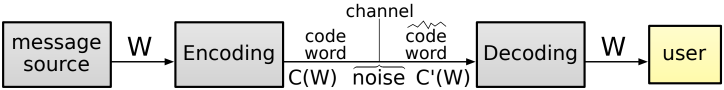

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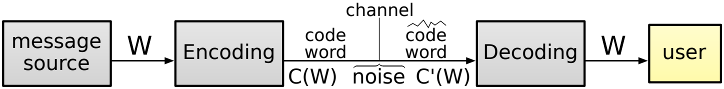

Error correcting framework

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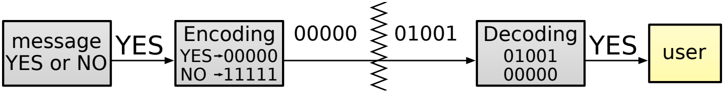

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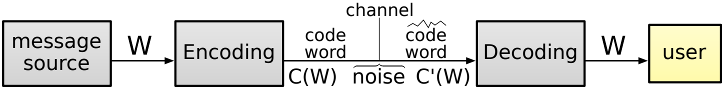

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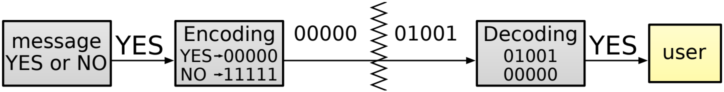

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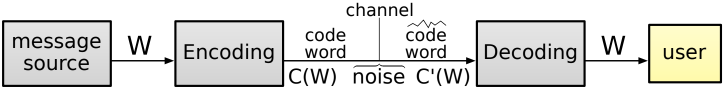

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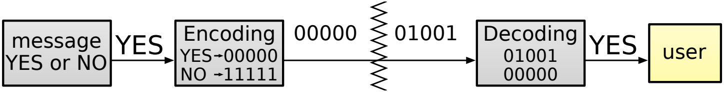

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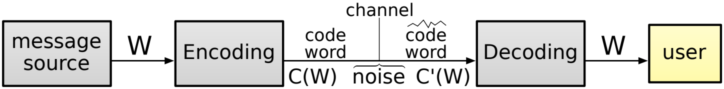

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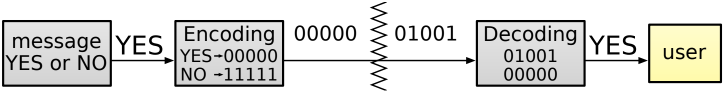

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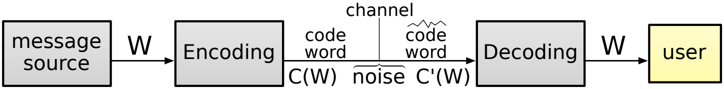

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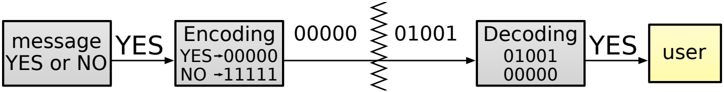

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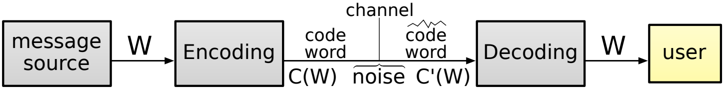

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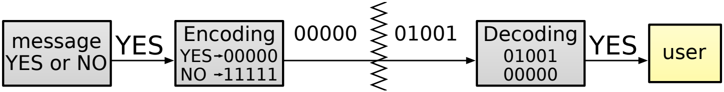

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- **Shannon stochastic (probabilistic) noise model**: $Pr(y|x)$ (probability of the output y if the input is x) is known and the probability of too many errors is low.
- **Hamming adversarial (worst-case) noise model:** Channel acts as an adversary that can arbitrarily corrupt the input codewords subject to a given bound on the number of errors.

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- Σ is an input alphabet
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Summary: The task of a communication channel coding is to encode the information to be sent over the channel in such a way that even in the presence of some channel noise, several (a specific number of) errors can be detected and/or corrected.

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The key idea is that in order to protect a message against a noise, we should encode the message by adding some redundant information to the message.

This should be done in such a way that even if the message is corrupted by a noise, there will be enough redundancy in the encoded message to recover – to decode the message completely.

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then the probability of an erroneous decoding (for the case of 2 or 3 errors) is

$$3p^2(1-p) + p^3 = 3p^2 - 2p^3 < p$$

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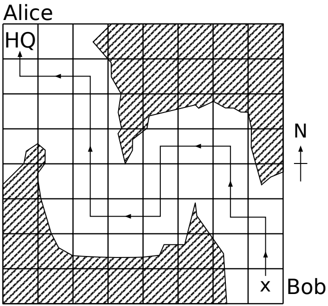

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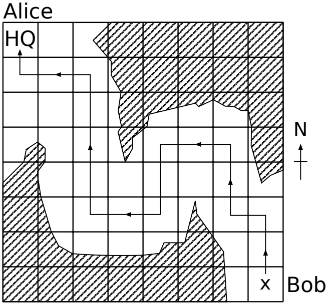

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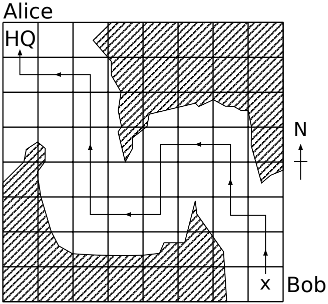

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3. $C3 = \{00000, 01101, 10110, 11011\}$

A single error in decoding each of symbols N, W, S, E can be corrected.

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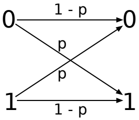

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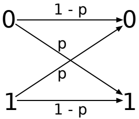

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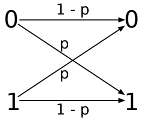

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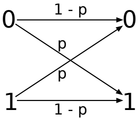

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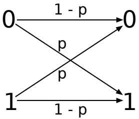

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Corollary One undetected error occurs only once every 2000 days! ($2000 \approx \frac{10^9}{5.5 \times 86400}$).

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Question How much better is two-dimensional encoding than one-dimensional encoding?

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Comment: A good (n, M, d)-code has small n, large M and also large d.

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Decoding was quite simple.

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Code rate (6/32 for Hadamard code), is an important parameter for real implementations, because it shows what fraction of the communication bandwidth is being used to transmit actual data.

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if $k \neq j$ and $x_j \neq x_k$.

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For details about 13-digit ISBN see

`http://www.en.wikipedia.org/Wiki/International_Standard_Book_Number`

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Examples of equivalent codes

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$$(2)\ \begin{Bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{Bmatrix} \begin{Bmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{Bmatrix}$$

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Consequently, equivalent codes have the same parameters (n,M,d) (and correct the same number of errors).

Examples of equivalent codes

$$(1)\ \begin{Bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{Bmatrix} \begin{Bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{Bmatrix}$$

$$(2)\ \begin{Bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{Bmatrix} \begin{Bmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{Bmatrix}$$

Lemma Any q-ary (n, M, d)-code over an alphabet $\{0, 1, \ldots, q-1\}$ is equivalent to an (n, M, d)-code which contains the all-zero codeword $00\ldots0$.

Proof Trivial.

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EXAMPLE

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Example Proof that $A_2(5,3) = 4$.

(a) Code C_3, page (**??**), is a $(5,4,3)$-code, hence $A_2(5,3) \geq 4$.

(b) Let C be a $(5,M,3)$-code with $M = 5$.

- By previous lemma we can assume that $00000 \in C$.
- C has to contain at most one codeword with at least four 1's. (otherwise $d(x,y) \leq 2$ for two such codewords x, y)
- Since $00000 \in C$, there can be no codeword in C with at most one or two 1.
- Since $d = 3$, C cannot contain three codewords with three 1's.
- Since $M \geq 4$, there have to be in C two codewords with three 1's. (say 11100, 00111), the only possible codeword with four or five 1's is then 11011.

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Resulting code is an (n, M, d)-code.

A COROLLARY

Corollary:
If d is odd, then $A_2(n,d) = A_2(n+1, d+1)$.
If d is even, then $A_2(n,d) = A_2(n-1, d-1)$.

Example $A_2(5,3) = 4 \Rightarrow A_2(6,4) = 4$
$(5,4,3)$-code $\Rightarrow$ $(6,4,4)$-code

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{matrix}$$ by adding check.

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Proof Let u be a fixed word in F_q^n. The number of words that differ from u in m positions is

$$\binom{n}{m}(q-1)^m.$$

GENERAL UPPER BOUNDS on CODE PARAMETERS

Theorem **(The sphere-packing (or Hamming) bound)**
If C is a q-nary $(n, M, 2t+1)$-code, then

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Singleton bound: If C is an q-ary (n, M, d) code, then

$$M \leq q^{n-d+1}$$

A GENERAL UPPER BOUND on $A_q(n, d)$

Example An $(7, M, 3)$-code is perfect if

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An example of such a code:

$C4 = \{0000000, 1111111, 1000101, 1100010, 0110001, 1011000, 0101100,$
$0010110, 0001011, 0111010, 0011101, 1001110, 0100111, 1010011, 1101001, 1110100\}$

Table of $A_2(n, d)$ from 1981

n	$d = 3$	$d = 5$	$d = 7$
5	4	2	-
6	8	2	-
7	16	2	2
8	20	4	2
9	40	6	2
10	72-79	12	2
11	144-158	24	4
12	256	32	4
13	512	64	8
14	1024	128	16
15	2048	256	32
16	2560-3276	256-340	36-37

For current best results see http://www.codetables.de

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and therefore

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For example, two main requirements for many telegraphy codes used to be:

- Any two codewords had to have distance at least 2;
- No codeword could be obtained from another codeword by transposition of two adjacent letters.

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To transmit pictures Voyager used the so called Golay code G_{24}.

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Problem: What is the minimal number of bits needed to transmit n values of X?

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$$S(X) = -\sum_x p(x) lg\ p(x)$$

and it is considered to be the information content of X.

In a special case, of a binary variable X which takes on the value 1 with probability p and the value 0 with probability $1-p$, then the information content of X is:

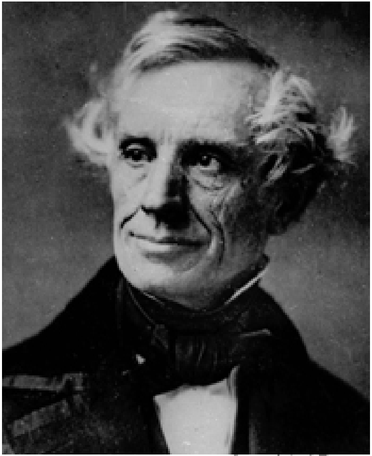
$$S(X) = H(p) = -p\ lg\ p - (1-p) lg(1-p)$$ [1]

Problem: What is the minimal number of bits needed to transmit n values of X?
Basic idea: Encode more (less) probable outputs of X by shorter (longer) binary words.
Example (Moorse code - 1838)

a .-	b -...	c -.-.	d -..	e .	f ..-.	g –.
h	i ..	j .—	k -.-	l .-..	m –	n -.
o —	p .–.	q –.-	r .-.	s ...	t -	u ..-
v ...-	w .–	x -..-	y -.–	z –..		

[1] Notation lg (ln) [log] will be used for binary, natural and decimal logarithms.

Samuel Moorse



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0000	10	0100	010	1000	011	1100	11101
0001	000	0101	11001	1001	11011	1101	111110
0010	001	0110	11010	1010	11100	1110	111101
0011	11000	0111	1111000	1011	111111	1111	1111001

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Observe that this is a **prefix code** - no codeword is a prefix of another codeword.

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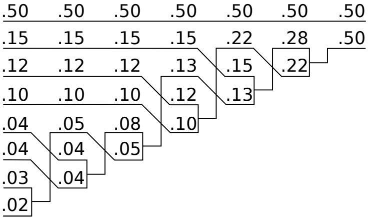

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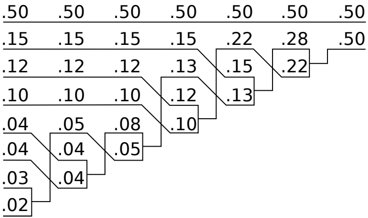

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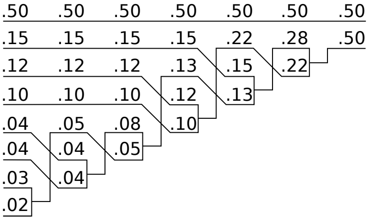

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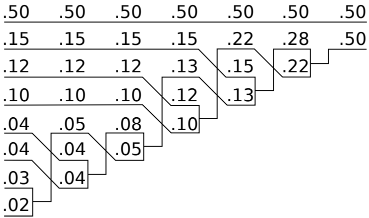

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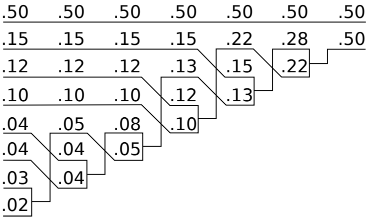

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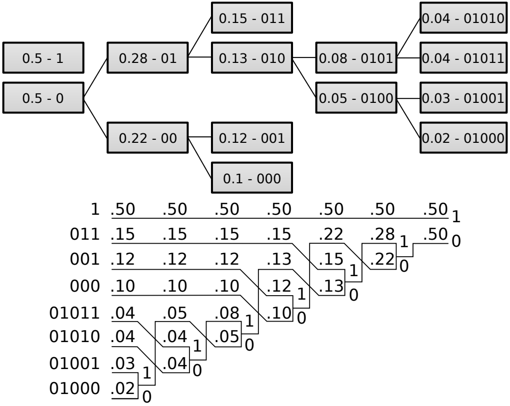

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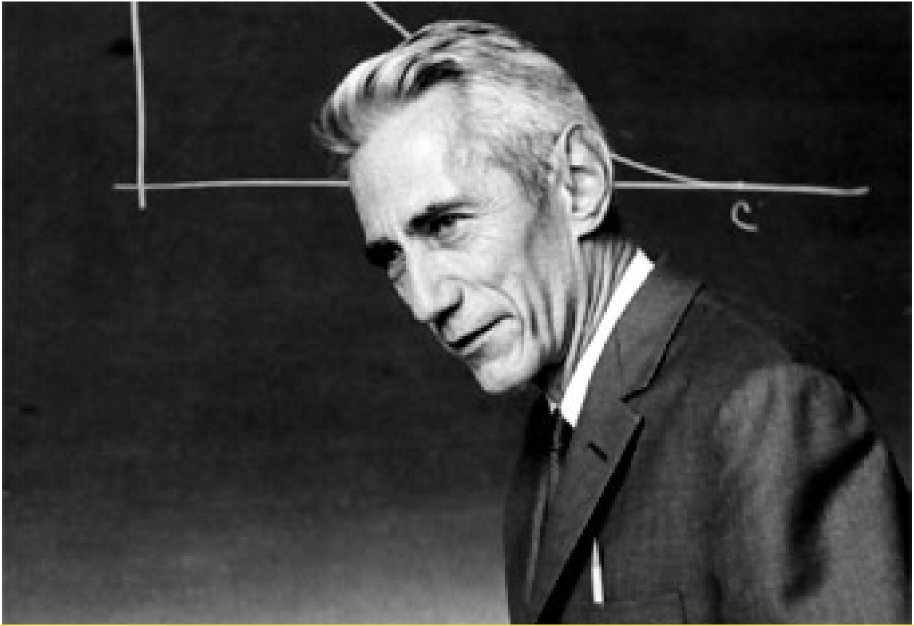
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APPENDIX

HARD VERSUS SOFT DECODING I

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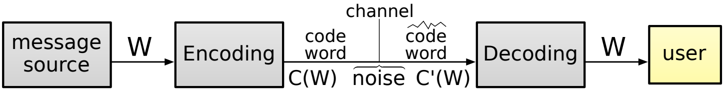

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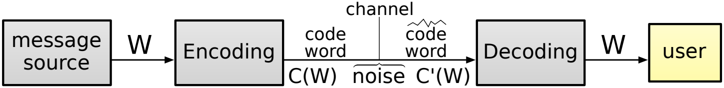

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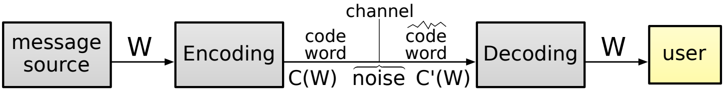

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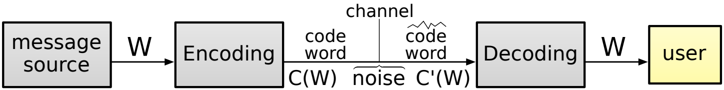

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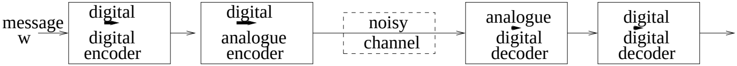

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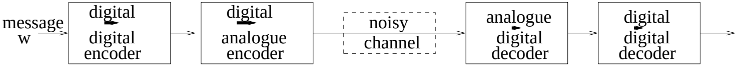

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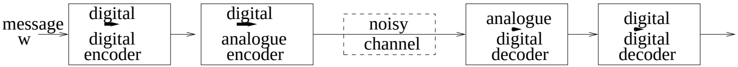

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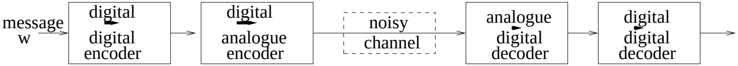

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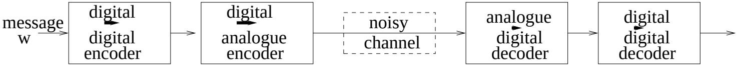

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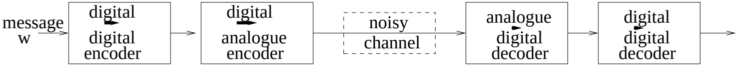

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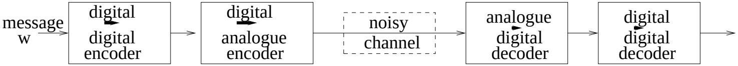

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In case the output of analogous-digital decoding is a pair (p_b, b) where p_b is the probability that the output is the bit b (or a weight of such a binary output (often given by a number from an interval $(-V_{max}, V_{max})$), we talk about a **soft decoding**.

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In order to deal with such a more general model of transmission and soft decoding, it is common to use, instead of the binary symbols 0 and 1 so-called **antipodal binary symbols** $+1$ and -1 that are represented electronically by voltage $+1$ and -1.

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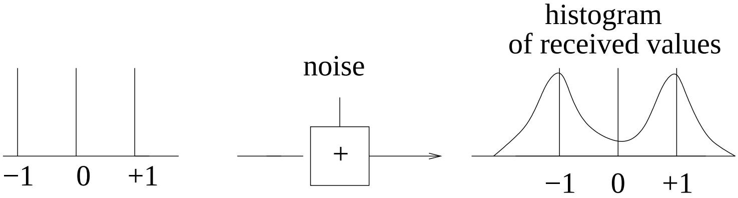

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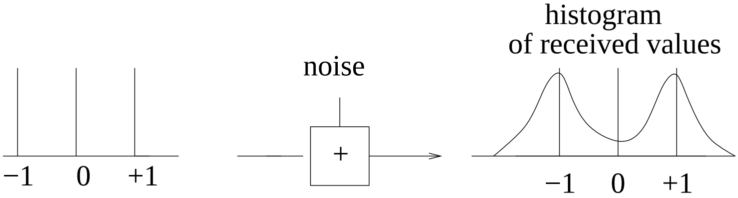

A very important case in practise, especially for space communication, is so-called additive white Gaussian noise (AWGN) and the channel with such a noise is called Gaussian channel.

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For example, in an important practical case of the Gaussian white noise one search at the minimal likelihood decoding for a codeword with minimal Euclidean distance.

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Hard decoding is used mainly for block codes and soft one for stream codes. However, distinctions between these two families of codes are tending to blur.

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For the same code there can be many encoding algorithms that map the same set of datawords into different codewords.

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- The era of Morse telegraph ended on 26.1.2006 when the main telegraph company in US, Western Union, announced cancelation of all telegraph services.

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The binary elements 0 and 1 were first called bits by J. W. Tuckley in 1943.