1. Let $C$ be the binary linear code which has the following parity check matrix:

$$
H = \begin{pmatrix}
1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{pmatrix}.
$$

(a) Find a codeword of minimum weight.
(b) Prove that all codewords have even parity.
(c) Determine $n$, $k$ and $d$.

2. Let $C$ be a code which does not contain any word of even weight. Decide whether the following statements hold. Explain your reasoning.

(a) $C$ is linear,
(b) $h(C) = 5$,
(c) $C \subseteq C^\perp$.

3. Show that the following codes are perfect:

(a) the codes $C = \mathbb{F}_q^n$,
(b) the codes consisting of exactly one codeword,
(c) the binary repetition codes of odd length,
(d) the binary codes of odd length consisting of a vector $c$ and the complementary vector $c'$ (with ones and zeros interchanged),
(e) the $[23, 12, 7]$ binary Golay code.

4. Give the standard form of generator matrix $G$ and parity check matrix $H$ of a $[15, 11, 3]$ binary Hamming code. Using these matrices to encode the message $u = 11111000000$ and decode the message $v = 111000111000111$.

5. (a) Show that in a linear binary code, either all codewords have even weight, or exactly half have even weight and half have odd weight.
(b) Show that in a linear binary code, either all codewords begin with 0, or exactly half begin with 0 and half begin with 1.

6. Answer each of the following questions. Explain your reasoning.

(a) Let $C$ be an $[n, k]$ code over $\mathbb{F}_q$ with a generator matrix $G$ and a parity check matrix $H$. What is the result of the following expressions:
   - $G \cdot H^T$,
   - $H \cdot G^T$.
(b) Let $C$ be an $[n, k]$ self-dual code over $\mathbb{F}_q$.
   - What is the relationship between $q$ and the weights of codewords for $q = 2$ and $q = 3$?
   - Can you generalize this result for an arbitrary $q$?

7. Prove that a perfect $t$-error-correcting linear code of length $n$ has precisely $\binom{n}{t}$ cosets of weight $i$ ($0 \leq i \leq t$) and does not have other cosets.