1. Determine \( d \) and \( M \) for a \( q \)-ary code
\[
C = \{x_1 \ldots x_n | \sum_{i=1}^{n} x_i = 0 \pmod{q}\}.
\]

2. Consider an ISBN number 063201x364. Determine \( x \) and find out which book has this ISBN code.

3. Consider a source that generates symbols 0 and 1 with frequencies 0.9 and 0.1, respectively. These symbols are consequently transferred by a
   a) binary symmetric channel,
   b) Z-channel (binary asymmetric channel)

   with \( p = 0.15 \).

   What is the probability that the symbol 1 was sent provided that the symbol 0 was received?
   (Recall that in Z-channel, 1 \( \rightarrow \) 0 occurs with probability \( p \) whereas 0 \( \rightarrow \) 1 never occurs.)

4. Let \( C = \{111111, 110000, 001100, 000011\} \). Suppose that the codewords are transmitted using a binary symmetric channel with error probability \( p \). Determine the probability that the receiver does not notice that a codeword has been corrupted during a transfer.

5. Let \( q > 0 \). What is the relation (\( \leq \), =, \( \geq \)) between
   a) \( A_2(n, 2d - 1) \) and \( A_q(n + 1, 2d) \),
   b) \( A_q(n, d) \) and \( q^{n-d+1} \),
   c) \( A_q(n, d) \) and \( A_q(n + 2, 2d) \),
   d) \( A_q(n + 1, d) \) and \( A_q(n, d) \).

6. For each of the following pairs of binary codes, prove their equivalence or prove that they are not equivalent:
   a) \( A = \begin{cases} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{cases} \)
   \( B = \begin{cases} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{cases} \),

   b) \( C_i = \begin{cases} \frac{1}{(1 + (-1)^i)/2} \\ 0 \\ 0 \\ \vdots \\ 0 \end{cases} \)

   \( D_i = \begin{cases} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{cases} \).

More on next page >>>>
7. Assume a source $X$ sends messages $A$, $B$, $C$, $D$ with the following probabilities:

<table>
<thead>
<tr>
<th>symbol</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.8</td>
</tr>
<tr>
<td>B</td>
<td>0.1</td>
</tr>
<tr>
<td>C</td>
<td>0.05</td>
</tr>
<tr>
<td>D</td>
<td>0.05</td>
</tr>
</tbody>
</table>

a) Calculate the entropy of the source $X$.

b) Create a Huffman code (binary) for the source $X$. Determine the average number of bits used per symbol.

c) Assume the source sends sequences of thousands of messages in blocks of length 16. Assume that the probability of each symbol occurring is independent of the symbol that have previously occurred.

Find a way to modify the creation of Huffman code so that the average number of bits used per source symbol decreases to a value no greater than 110% of source entropy. Design a code using this modification and determine the average number of bits per symbol achieved.