

2011 - Exercises I.

1. Consider the code $C = \{0000001, 1111001, 1100110, 0011110\}$. Determine the values $h(C)$, $s(C)$ and $t(C)$ where $s(C)$ is the number of errors C can detect and $t(C)$ is the number of errors C can correct. Show that C is equivalent to a code which contains the word 0000000.
2. Find a binary $(5, 3, 4)$ -code or show that such does not exist.
3. Find an upper bound U and a lower bound L for $A_2(9, 3)$, such that $U \geq A_2(9, 3) \geq L$ and $3L \geq U$.
4. Construct a quaternary Huffman code for characters A, B, C, D and E with relative frequencies given in the following table.

| character | relative frequency |
|-----------|--------------------|
| A | 0.50 |
| B | 0.28 |
| C | 0.10 |
| D | 0.06 |
| E | 0.06 |

5. Determine d and M for q -ary code

$$C = \{x_1 \dots x_n \mid \sum_{i=1}^n x_i = 0 \pmod{q}\}$$

6. Suppose you want to use a binary channel with the following properties:

- (a)
 - With probability $p_\ell \leq \frac{1}{2}$ the channel rotates a k -bit message being sent to the left (ie. a message $m_1 m_2 \dots m_k$ goes to $m_2 m_3 \dots m_k m_1$);
 - With probability $p_r = p_\ell$ the channel rotates a k -bit message being sent to the right (ie. a message $m_1 m_2 \dots m_k$ goes to $m_k m_1 m_2 \dots m_{k-1}$);
 - With probability $1 - p_\ell - p_r$ a message is not changed.

Design a code with the minimal length that can transmit 8 distinct messages with the probability of erroneous decoding $p_e = 0$. Prove your result.

- (b)
 - With probability p the channel reverts a k -bit message being sent (ie. a message $m_1 m_2 \dots m_k$ goes to $m_k m_{k-1} \dots m_2 m_1$);
 - With probability $1 - p$ a message is not changed.

What is the maximal number of distinct messages you can send with $p_e = 0$ using a 4-bit code? Prove your result.

7. Which of the following codes are possible Huffman codes?

- (a) $C_1 = \{0, 00, 01\}$
- (b) $C_2 = \{0, 10, 11\}$
- (c) $C_3 = \{0, 10\}$
- (d) $C_4 = \{1, 000, 001, 010, 0110\}$

8. Consider an ISBN number $00770x9877$. Determine x and find out which book has this ISBN code.