

2010 - Exercises III.

1. (a) Factorize  $x^6 - 1$  over  $\text{GF}(3)$  into irreducible polynomials.  
 (b) For each cyclic code of length 6 and dimension 2 over  $\text{GF}(3)$  write the generator polynomial and the generator matrix.  
 (c) For each cyclic code of length 6 and dimension 5 over  $\text{GF}(3)$  write the check polynomial and the parity check matrix.
2. Let  $C$  be a binary cyclic code whose codewords have length  $n$ . Let  $c_i$  denote the number of words of weight  $i$  in  $C$ . Show that  $ic_i$  is a multiple of  $n$ .
3. Let  $C$  be a binary cyclic code of odd length. Show that  $C$  contains a codeword of odd weight if and only if it contains the word  $\mathbf{1} = 11 \dots 1$ .
4. Which of the following polynomials are generator polynomials of a binary cyclic code of length 7.  
 (a)  $x^3 + x^2 + x + 1$   
 (b)  $x^3 + x^2 + 1$   
 (c)  $x^2 + x + 1$
5. Let  $C_1$  and  $C_2$  be binary cyclic codes. Decide whether  $C_3$  is cyclic, if  
 (a)  $C_3 = C_1 \cap C_2$ ,  
 (b)  $C_3 = C_1 \cup C_2$ ,  
 (c)  $C_3 = \{c_3 \mid c_3 = c_1 \circ c_2, c_1 \in C_1, c_2 \in C_2\}$ , where  $\circ$  is concatenation operator,  
 (d)  $C_3 = \{c_3 \mid c_3 = c_1 \oplus c_2, c_1 \in C_1, c_2 \in C_2\}$ , where  $\oplus$  is bitwise XOR operation,  
 (e)  $C_3 = \{c_3 \mid c_3 = \neg c_1, c_1 \in C_1\}$ , where  $\neg$  is bitwise negation.
6. Code  $C^{[-1]}$  is the reverse code of a cyclic code  $C$  if

$$c(x) \in C \Leftrightarrow x^{n-1}c(x^{-1}) \in C^{[-1]}.$$

For the polynomial  $p(x)$  of degree  $d$ , its reciprocal polynomial  $p^{[-1]}(x)$  is given by

$$p^{[-1]}(x) = x^d p(x^{-1}).$$

- (a) Consider the smallest cyclic code that contains 000101 and 010010 with generator polynomial  $g(x)$ . Find  $g^{[-1]}(x)$ .
- (b) Prove that if a cyclic code  $C$  is generated by the polynomial  $g(x)$  of minimum degree, then  $g_0^{-1}g^{[-1]}(x)$  generates  $C^{[-1]}$ .