

2010 - Exercises I.

1. Let $q \geq 2$. Decide whether there is a q -ary $(3, q^2, 2)$ -code. Find $A_q(3, 2)$. Explain your reasoning.
2. Does a $(4, 3, 3)$ -code exist? Find it or prove that such does not exist.
3. Let C_1 be an (n, M_1, d_1) code and C_2 be an (n, M_2, d_2) code. What are the maximal and minimal values of n_3, M_3, d_3 of a code C_3 , if
 - (a) $C_3 = C_1 \cap C_2$,
 - (b) $C_3 = C_1 \cup C_2$,
 - (c) $C_3 = \{c_3 \mid c_3 = c_1 \circ c_2, c_1 \in C_1, c_2 \in C_2\}$, where \circ is concatenation operator,
 - (d) $C_3 = \{c_3 \mid c_3 = c_1 \oplus c_2, c_1 \in C_1, c_2 \in C_2\}$, where \oplus is bitwise XOR operation,
 - (e) $C_3 = \{c_3 \mid c_3 = \neg c_1, c_1 \in C_1\}$, where \neg is bitwise negation.
4. Consider an ISBN number $157488x674$. Determine x .
5. (a) Construct a binary and also a ternary Huffman code for characters A, B, C, D, E and F with relative frequencies given in the following table.

character	relative frequency
A	0.25
B	0.20
C	0.20
D	0.15
E	0.10
F	0.10

- (b) Can you find more efficient coding if you know that E can appear only after B? If no, explain why. Otherwise, find that coding and count how many bits will be saved on average when coding 10 letters.
6. Consider the following method for generating a code for a random variable X which takes on m values $1, 2, \dots, m$ with probabilities $p(x) = \Pr(X = x) > 0$.

Let

$$\bar{F}(x) = \sum_{x' < x} p(x') + \frac{1}{2}p(x)$$

where $x \in \{1, 2, \dots, m\}$.

As codeword for x , use the binary expansion of $\bar{F}(x)$ rounded to

$$l(x) = \lceil \log \frac{1}{p(x)} \rceil + 1$$

bits.

(Example of binary expansion: $\frac{2}{3} = 0.10101010 \dots = 0.\overline{10}$ rounded to 3 bits is 0.101)

- (a) Show that $\sum_x p(x)l(x) < H(X) + 2$.
- (b) Show that codes constructed using this method are prefix codes.
- (c) Construct a binary code C for $m = 4$ and the probability distribution given in the following table.

X	$p(x)$
1	1/3
2	1/6
3	1/6
4	1/3

- (d) Compute the average codelength of C and compare it with the average codelength of a Huffman code constructed for the given probability distribution.