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NOTE

NONDETERMINISTIC MULTICOUNTER MACHINES AND COMPLEMENTATION

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Abstract. It is proved that the family of languages recognized by one-way $f(n)$-time-bounded nondeterministic multicounter machines is not closed under complementation for any polynomial function $f(n) \geq n$. This solves the open problem of Wagner and Wechsung (1986).

1. Introduction

A one-way multicounter nondeterministic machine, 1-multiC-N, is a computing device consisting of a finite state control, an input tape with one-way reading head, and a finite number of counters. We regard a counter as an arithmetic register containing an integer that may be either positive or zero. In one step, a 1-multiC-N machine can enter the state of finite state control, move the head on the input tape one square to the right or keep it stationary, and increase or decrease any counter by 1. The choice of actions of a 1-multiC-N machine is determined by the state of the machine, the input symbol currently scanned and the sign of each counter, positive or zero. The 1-multiC-N machine starts in the initial state with all counters empty, and with the reading head positioned on the first symbol of the input word. It accepts if it reaches the finite state. The formal definition can be found in [7]. Various types of restricted multicounter machines have been considered in [1-6].

We shall consider one-way $f(n)$-time-bounded nondeterministic k-counter machines, 1-kC-NTIME($f(n)$). ($f(n)$ is a function from N to N, N is the set of positive integers). $f(n)$-time-bounded means that the number of steps of each accepting computation of the machine is bounded by $f(n)$, where n is the length of the input word. In particular, we denote by 1-kC-NTIME(id) machines working in real time (i.e., $f(n) = n$).

Let 1-multiC-NTIME($f(n)$) $= \bigcup_{k \in N}$ 1-kC-NTIME($f(n)$). By $\mathscr{L}$(1-kC-NTIME($f(n)$)) and $\mathscr{L}$(1-multiC-NTIME($f(n)$)) we shall denote the family of languages recognized by 1-kC-NTIME($f(n)$) and 1-multiC-NTIME($f(n)$) machines, respectively.

It is known [7] that $\mathscr{L}$(1-multiC-NTIME(n)) is AFL. Among stated problems concerning the closure properties of this language family, the only one remaining open is whether or not it is closed under complementation. This problem is formulated as an open one in [5]. We shall give a negative answer to this question. In fact, we shall prove a stronger result: $\mathscr{L}$(1-multiC-NTIME($f(n)$)) is not closed under complementation for any polynomial function $f(n)$.

2. Results

In [5] it was proved that the family of languages recognized by one-way real-time nondeterministic multicounter machines with constant number of counter reversals is not closed under complementation. The question concerning the language family $\mathscr{L}$(1-multiC-NTIME(id)) was stated as an open problem there. Hromkovič [5] conjectured that the answer is negative and defined the language that should be suitable for solving this problem. We have used another language in our proof.

Theorem 1. *The family of languages* $\mathscr{L}$(1-kC-NTIME(id)) *and* $\mathscr{L}$(1-multiC-NTIME(id)) *are not closed under complementation for any constant* $k \in N$.

Proof. To prove Theorem 1 let us consider the language L

$$L = \{0^{i_1}10^{i_2}1\dots10^{i_r}110^{j_1}10^{j_2}1\dots10^{j_s} \mid r, s, i_1, \dots, i_r, j_1, \dots, j_s \in N, \\ \{i_1, \dots, i_r\} \cap \{j_1, \dots, j_s\} \neq \emptyset\}.$$

We shall show that $L \in \mathscr{L}$(1-1C-NTIME(id)) and $L^c \notin \mathscr{L}$(1-multiC-NTIME(id)).

To see that $L \in \mathscr{L}$(1-1C-NTIME(id)) is not difficult. The machine nondeterministically decides for some i_t, remembers it in its counter and compares with nondeterministically chosen j_p. If $i_t = j_p$, the input word is accepted.

To show $L^c \notin \mathscr{L}$(1-multiC-NTIME(id)) is slightly more difficult. We show it by contradiction.

Let A be a 1-kC-NTIME(id) machine recognizing the language L^c for some $k \in N$. For the word $w = 0^{i_1}10^{i_2}1\dots10^{i_r}110^{j_1}10^{j_2}1\dots10^{j_s} \in L^c$ such that $i_a \neq i_b$, $j_t \neq j_p$ for every $a \neq b$, $t \neq p$, we denote by $R(w)$ and $S(w)$ the sets $\{i_1, \dots, i_r\}$ and $\{j_1, \dots, j_s\}$, respectively. Now, for every constant $m \in N$, we define the subset L_m of L^c as follows:

$$L_m = \{w = 0^{i_1}1\dots10^{i_r}110^{j_1}1\dots10^{j_s} \in L^c \mid i_1 < i_2 < \dots < i_r, j_1 < j_2 < \dots < j_s, \\ R(w) \cap S(w) = \emptyset, R(w) \cup S(w) = \{1, \dots, m\}\}.$$

The cardinality of L_m is $2^m - 2$. We associate one fixed accepting computation of A on w with every word $w \in L_m$. A configuration which the machine A reaches immediately after it has read the prefix $0^{i_1}1\dots10^{i_r}11$ of the word w in this computation will be called a transition configuration of w. Because of the length of the input word w, the content of each counter is at most $\frac{1}{2}(m^2+3m)$. Therefore the number of different transition configurations is at most $q \cdot ((m^2+3m)/2)^k$ (where q is the number of states of the machine A). Since $\lim_{m\to\infty}(q \cdot ((m^2+3m)/2)^k)/(2^m-2)=0$ there have to be two different words $w_1 = u_1 11 v_1$, $w_2 = u_2 11 v_2$ from L_m with the same transition configuration. At least one of the following conditions holds:

(1) $\exists i \quad i \in R(w_1) \,\&\, i \notin R(w_2)$

(2) $\exists j \quad j \notin R(w_1) \,\&\, j \in R(w_2)$.

Let (1) be true. Then the word $z = u_1 11 v_2$ is accepted by A although z does not belong to L^c. In the second case the word $u_2 11 v_1$ is the one that leads to the contradiction. □

Given any polynomial function $f: N \to N$ and constants q, $k \in N$, $\lim_{m\to\infty} q(f((m^2+3m)/2))^k/(2^m-2)=0$. From this fact and from the proof of Theorem 1 we have the following.

Corollary 2. *If f is a polynomial function from N to N and $k \in N$ is a constant, then the language families $\mathscr{L}$(1-multiC-NTIME($f(n)$)) and $\mathscr{L}$(1-kC-NTIME($f(n)$)) are not closed under complementation.*

We note that for the reversal complexity measure, one reversal suffices to recognize L but no number of reversals of polynomial-time-bounded multicounter machines is sufficient for the recognition L^c.

References

[1] T. Chan, Reversal complexity of counter machines, in: *Proc. 1981 IEEE Symp. on Theory of Computing* (1981) 146-157.

[2] P. Ďuriš and Z. Galil, On reversal-bounded counter machines and on pushdown automata with a bound on the size of the pushdown store, *Inform. and Control* **54** (1982) 217-227.

[3] J. Hromkovič, Hierarchy of reversal and zerotesting bounded multicounter machines, in: M.P. Chytil and V. Koubek, eds., *Proc. 11th Conf. on Mathematical Foundations of Computer Science*, Lecture Notes in Computer Science **176** (Springer, Berlin, 1984) 312-321.

[4] J. Hromkovič, Reversal-bounded multicounter machines, *Comput. Artificial Intell.* **4** (1985) 361-366.

[5] J. Hromkovič, Reversal-bounded nondeterministic multicounter machines and complementation, *Theoret. Comput. Sci.* **51** (1987) 325-330.

[6] O.H. Ibarra, Reversal-bounded multicounter machines and their decision problems, *J. ACM* **25** (1978) 116-133.

[7] K. Wagner and G. Wechsung, *Computational Complexity*, Mathematische Monographien **19** (VEB Deutscher Verlag Wissenschaften, Berlin, 1986).