

Entrance Examination - Mathematics

Name and Surname - fill in the field	Application No.	Test Sheet No.
		1

Calculus

1 $\lim_{x \rightarrow \infty} \frac{\frac{1}{x} + 1}{x} =$

- *A 0
- B 1
- C -1
- D ∞
- E e

2 Consider the function

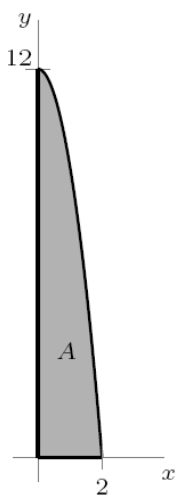
$$f(x) = e^{x^3+x}.$$

What is the value of the derivative of the function f at 1 (i.e. $f'(1)$)?

- A $-e$
- B 0
- C $\frac{e^2}{3}$
- *D $4e^2$
- E $3e^4$

3 Compute the area of a flat shape A , sketched below, which consists of all points (x, y) satisfying the following inequalities:

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\y &\leq -3x^2 + 12.\end{aligned}$$



- A 11
- B 14
- *C 16
- D 18
- E 21

- 4** Let $f(x)$ be a continuous function whose **first derivative is positive** on the interval $[0, 1]$ (i.e., the value $f'(x)$ is defined and is strictly greater than 0 for all $0 \leq x \leq 1$). Which of the following assertions is always true?
- A** $f(x) > 0$ for all $0 \leq x \leq 1$
***B** $f(0) < f(1)$
C $\int_0^1 f(x) dx > 0$
D The second derivative of the function f is also positive on the interval $[0, 1]$.
E The function f is constant on $[0, 1]$.

- 5** Let $f(x), g(x)$ be continuous functions. Which of the following (in)equalities **does not hold** in general?
- A** $\int_{-1}^1 f(x) dx + \int_{-1}^1 g(x) dx = \int_{-1}^1 (f(x) + g(x)) dx$
B $0 \leq \int_{-1}^1 |f(x)| dx$ (where $|f(x)|$ is the absolute value of $f(x)$)
C $\int_{-1}^0 f(x) dx + \int_0^1 f(x) dx = \int_{-1}^1 f(x) dx$
***D** $\int_{-1}^1 f(x) \cdot g(x) dx = (\int_{-1}^1 f(x) dx) \cdot (\int_{-1}^1 g(x) dx)$
E $\int_{-1}^1 -f(x) dx = - \int_{-1}^1 f(x) dx$

Probability

- 6** Consider a random variable X such that $P(X = 0) = \frac{1}{3}$, $P(X = 2) = \frac{1}{2}$ and $P(X = 4) = \frac{1}{6}$. Compute the **variance** of X . (Here $P(X = y)$ denotes the probability of random variable X attaining the value y .)
- A** $\frac{25}{9}$
***B** $\frac{17}{9}$
C $\frac{7}{9}$
D 4
E 2

- 7** Consider an investor who, during each month, loses 100 000 CZK with probability 0.8 and gains 200 000 CZK with probability 0.2. What is the expected value of the investor's wealth after **two** months of trading, provided that the investor starts with assets worth 1 000 000 CZK?
- A** 1 100 000 CZK
B 1 000 000 CZK
C 950 000 CZK
***D** 920 000 CZK
E 860 000 CZK

- 8** Let us throw two (six-sided) dice, a red one and a green one. We assume that these two throws are independent of each other. What is the conditional probability that the total value of both dice (i.e., the sum of values rolled on these dice) is 8 assuming that an even number is rolled on the red die?
- A** 1
B $\frac{1}{2}$
C $\frac{1}{3}$
D $\frac{1}{4}$
***E** $\frac{1}{6}$

- 9** Find the **median** of the following numerical data sample $\{0, 1, 1, 1, 5, 5, 6, 8, 9\}$.
- A** 0
B 1
C 4
***D** 5
E 9

Sets, relations, functions, logic

- 10** Which of the following relations is a **bijective function** on the set $\{a, b, c\}$?

- A** $\{(a, b)\}$
B $\{(a, c), (b, c), (c, c)\}$
C $\{(a, b), (b, b), (b, c), (c, a)\}$
D $\{(a, b), (b, a), (c, a)\}$
***E** $\{(a, b), (b, c), (c, a)\}$

- 11** Let A be any 3-element set and B any 2-element set. What is the cardinality of the power set of the set $A \times B$? (The power set of X is the set of all subsets of X .)

- *A** 2^6
B 6^2
C 2^5
D 30
E $6!$

- 12** Consider the standard inequality relation \neq on the set of all integers. This relation:

- A** is reflexive and symmetric, but not transitive.
B is neither reflexive, nor symmetric, nor transitive.
***C** is symmetric, but is neither reflexive nor transitive.
D is an equivalence relation.
E is reflexive and transitive, but not symmetric.

- 13** Which of the following propositional formulas **is not** satisfiable? (Propositional variables are denoted by A, B, C .)

- A** $(A \wedge B) \vee (\neg A \wedge C)$
B $A \wedge \neg B \wedge \neg C$
C $B \Rightarrow (A \vee \neg A)$
***D** $(\neg A \wedge C) \Leftrightarrow A$
E $(A \vee B) \wedge (\neg A \vee B)$

Incorrect question, not counted into evaluation.

- 14** Which of the following predicate formulas is semantically equivalent to the formula $\neg \exists x \forall y P(x, y)$? (Here P is a binary predicate and x, y are variables.)

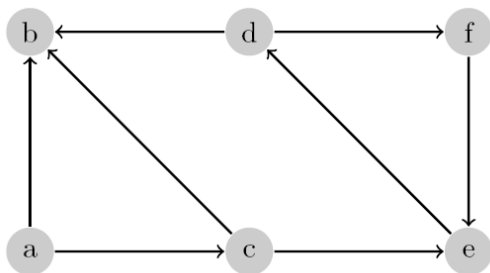
- A** $\forall x \forall y P(x, y)$
B $\exists x \forall y P(x, y)$
C $\forall y \exists x \neg P(x, y)$
***D** $\forall x \exists y \neg P(x, y)$
E $\exists y \forall x \neg P(x, y)$

- 15** Let us assume that the variables x, y, z are interpreted as **integers** and the symbols $<$ and \leq are interpreted as the standard strict and non-strict inequality on integers, respectively. Which of the following formulas **is** true?
- A** $\exists x \forall y \forall z (x \leq y \wedge z \leq x)$
B $\exists x \exists y \forall z (x \leq z \wedge x \leq y)$
C $\forall x \forall y \exists z (x < z \wedge z < y)$
***D** $\forall x \exists y \forall z (z \leq x \vee y \leq z)$
E $\exists x \forall y \forall z (y \leq x \vee z \leq x)$
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Graph theory

- 16** What is the maximal possible number of edges of an **undirected graph without loops** on n vertices?
- A** n^2
***B** $\binom{n}{2}$
C $2n^2$
D $n!$
E 2^n
-

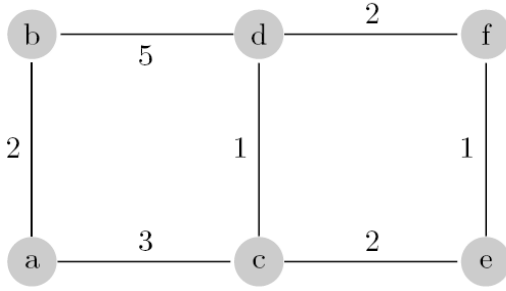
- 17** Consider the following directed graph:



Decide which of the following claims about **depth first** search starting from vertex a holds. (We do not assume any ordering on the vertices, i.e. the order in which the depth first search algorithm visits the vertices is ambiguous.)

- A** Vertex b will always be visited before vertex f .
***B** Vertex e will always be visited before vertex f .
C Vertex f will always be the last visited vertex.
D Vertex b will always be visited before vertex d .
E Vertex b will always be the last visited vertex.
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18 Consider the following undirected, edge-weighted graph:



What is the weight (i.e. the sum of weights of edges) of its minimal spanning tree?

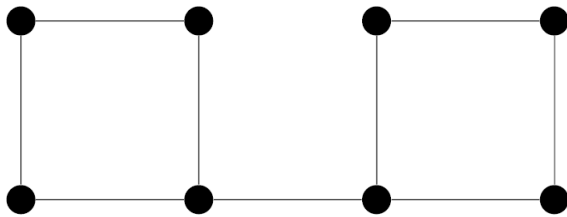
- A 11
- *B 9**
- C 10
- D 8
- E 12

19 Consider a tree on $n \geq 3$ vertices. After adding an edge between any two of its leaves it is true that:

- A The graph may or may not be a tree.
- B Between every two distinct vertices there are two distinct simple paths.
- *C Between at least two distinct vertices there are two distinct simple paths.**
- D The graph has $n + 1$ edges.
- E Each vertex of the graph lies on a cycle.

(A simple path is a path with no repeated vertices. A cycle is a path, containing at least one edge, such that the start vertex and the end vertex of the path are the same and no other vertices appear more than once on it.)

20 Consider the following undirected graph:



How many different spanning trees does it have?

- A 9
- B 1
- *C 16**
- D 8
- E 10

Linear algebra

21 Consider a map $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ which assigns to each vector its mirror image with respect to the line $x = y$. Which of the following is the matrix of this map? (Assume multiplication by a matrix from the left.)

A $\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$

***B** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

C $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

D $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

E The matrix does not exist, because the map is not linear.

22 Compute the determinant of the following matrix:

$$\begin{pmatrix} 3 & 3 & -1 \\ 2 & 1 & 7 \\ 0 & 0 & 2 \end{pmatrix}$$

A 0

B -3

C 18

***D** -6

E 5

23 Consider the following system of equations over \mathbb{R} :

$$-2x + 3y + z = 4$$

$$2x - 3y - z = -4$$

$$4x - 6y - 2z = -8$$

Which of the following claims is true?

A The system has no solution.

B The system has infinitely many solutions and all solutions form a line in \mathbb{R}^3 .

C The system has exactly one solution.

***D** The system has infinitely many solutions and all solutions form a plane in \mathbb{R}^3 .

E All points of \mathbb{R}^3 are solutions of the given system.

24 Which of the following triples of vectors is linearly independent?

A $(1, 0, 1), (1, 1, 0), (-1, -2, 1)$

B $(1, 1, 1), (2, 2, 2), (3, 3, 3)$

C $(1, 2, 3), (3, 2, 1), (1, 1, 1)$

***D** $(1, 1, 0), (0, 1, 1), (1, 0, 1)$

E $(1, 1, 1), (1, 1, 0), (0, 0, 1)$

25 Consider the vector $(2, 4, 6)$ in the standard basis $[(1, 0, 0), (0, 1, 0), (0, 0, 1)]$. Find its coordinates in basis $[(0, 0, 2), (0, 2, 0), (2, 0, 0)]$.

A $(1, 2, 3)$

B $(4, 8, 12)$

C $(-1, -2, -3)$

***D** $(3, 2, 1)$

E Coordinates in the given basis do not exist.