#### Mathematical formulae recognition and logical structure analysis of mathematical papers

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Masakazu Suzuki Kyushu University InftyProject ((http://www/inftyproject.org) Science Accessibility Net (http://www.sciaccess.net)

http://www.inftyproject.org/

## Plan of the talk

#### About InftyProject

#### Making Rich Digital Mathematical Libraries

- Process Flow and Technical Components
- Formulae Recognition
- Adaptive Method
  - Character and Symbol Recognition
  - Logical Structure Analysis

# Section 1 About Infty Project

## InftyProject

#### The beginning:

- Started as a research project to help visually impaired people in scientific fields in 1995.
- Digitization of of mathematical journals, books, etc..

#### Current research subjects :

- Recognition and understanding of math documents,
- User interface and data conversion, etc.

#### Policy:

- Priority in practical system development.

## InftyProject

Main system development
 InftyReader : Math OCR software
 InftyEditor : Editor of math documents
 Data conversion (XML, LaTeX, HTML, PDF, etc.)
 ChattyInfty : InftyEditor + speech output

URL: http;//www.inftyproject.org

<u>Go</u>

### sAccessNet

#### InftyReader is used for

- Helping people with visual handicaps working in scientific fields,
- Digitization of mathematical/scientific Journals in Japan,
  - e.g. J.Math.Soc.Japan, Japanese J. Math., Tokyo J.Math, etc., (11 journals of mathematics pulished in Japan)

by the not-for-profit organization "Science Accessibility Net"

http://www.sciaccess.net/

#### <u>Go</u>

## "InftyReader" OCR software for math documents

#### Demonstration.

#### Original Image

72 YUJI KASAHARA Since we assume  $\varphi_{0}(c, -1) \rightarrow \varphi_{0}(c, -1) \quad (>1),$ there exist positive constant  $\delta$  and N such that (22) $1 + \delta \leq \exp m_n(c), \quad n \geq N,$ Hence,  $m_n(c) \ge r > 0$ ,  $n \ge N$  holds for some suitably chosen constant r(>0). Using (3) again.  $\varphi_n(x, -s) \ge 1 + s \int_a^x m_n(\xi) d\xi \ge 1 + s(x-c), \quad n \ge N.$ (23)Hence, appealing to the dominated convergence theorem,  $\lim_{n\to\infty}\int_{s}^{n}\frac{dx}{\varphi_{n}(x,\,\cdots\,s)^{t}}=\int_{s}^{n}\frac{dx}{\varphi_{0}(x,\,-s)^{t}},\qquad s\!>\!0.$ (24)This proves  $h_n(s) \rightarrow h_n(s)$ . (iii)⇒(i). The family of left-continuous, non-decreasing functions  $\{\arctan m_n(x)\}_{n=1}^{\infty}$ , is uniformly bounded and therefore we can choose an inextensible measure  $dm^*(x)$  and a subsequence  $\{m_n\}$  of an arbitrary subsequence  $\{m_n\} \subset \{m_n\}$  such that (25) m<sub>n</sub>(x)→m<sup>\*</sup>(x) at each continuity point x ∈ [0, ∞) of m<sup>\*</sup>(x). Since we have already proved that (i) implies (iii), it holds that  $h_{w}(s)$  converges to  $h^*(s)$ ,  $h^*(s)$  being the characteristic function of  $m^*(x)$ . Of course  $h^*(s) = h_0(s)$  by the assumption (iii). Hence  $m^*(x) = m_0(x)$  by the uniqueness part of the above Krein's theorem. This completes our theorem. 3. Asymptotic theorems of Krein's correspondence. A function R(x) defined on  $[0, \infty)$  is called regular varying at  $\infty[0]$ with exponent  $\rho$  ( $-\infty < \rho < \infty$ ) if  $\lim_{\substack{x \in \mathcal{I} \\ |x| = 1}} \frac{R(\lambda x)}{R(x)} = \lambda^{p}, \qquad \lambda > 0.$ (26) Especially when L(x) varies regularly with exponent zero, L(x) is said to

vary slowly (cf. [4] p. 275). In case  $\rho$  is finite, R(x) varies regularly with exponent  $\rho$  if and only if there exists a slowly varying function L(x) such ihat  $R(z) = z^* L(x),$ 

(27)

**Recognition Result** 

#### Sample: <u>A sample of Math Journal digitized using InftyReader</u>

#### http://infty.kyushu-u.ac.jp

# Section 2 Toward Rich DML

# **Digitization of Math Journals**

Different levels:

Level 1: Scanned images of papers e.g. GIF, TIFF

- Level 2: Searchable digitized document e.g. PDF with hidden text
- Level 3: Structured document with links e.g. XML, HTML(+MathML), LATEX, ...
- Level 4: (partially) Executable document e.g. Mathematica, Maple
- Level 5: Formally presented document. e.g. Mizar, OMDoc

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Level 1: Bitmap images of printed materials. e.g. GIF, TIFF Infty : Level  $1 \rightarrow$  Level 3 Level 2 e.g. PDF w Level 3: Structured document with links. e.g. XML, HTML(+MathML), LATEX, ... Level 4: (partially) Executable document. e.g. Mathematica, Maple

Level 5: Formally presented document. e.g. Mizar, OMDoc

# **Process Flow of Digitization**



## Layout Analysis



## Layout Analysis



# **Process Flow of Digitization**



#### Line Segmentation (Sample)

so for r large enough,  $J(z) \leq C_2 r^{\varrho}$  for  $\lim_{\substack{r \to \infty \\ \text{and positively homogeneous of order}}} J(rz)/r^{\varrho} \text{ and } j^*(z) = \lim_{\substack{z' \to z \\ \text{of order}}} j(z'), \text{ whi}$ 

 $(S, \cdot)$  for resource k and schedule S. If related, we get the corresponding time-const n  $PS\infty|temp, \overline{d}| \sum \sum c_k^v \varphi_{kt} + c_k^f \Delta^+ \varphi_{kt}$ . An oblem is again called time-optimal.

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#### A Method of Line Segmentation

21. Prove that (2) is equivalent to the pair of relations

$$\lim_{z \to z_0} \operatorname{Re} f(z) = \operatorname{Re} l, \qquad \lim_{z \to z_0} \operatorname{Im} f(z) = \operatorname{Im} l.$$

22. The function  $f(z) = 3(z^2 - 1)/(z - 1)$  is not defined for z = 1, but for all other values of z it is equal to 3(z + 1). Using the definition of the limit, show that  $\lim_{z \to 1} f(z) = 6$ . (Note that the limit is established when some formula is found for  $\delta$  as a function of  $\epsilon$ .)

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# **Process Flow of Digitization**



#### Math/Text Segmentation

Number of characters in Math area is about 20% of all the characters in pure math journals.



#### Math/Text Segmentation

#### Recognition of Ordinary Texts & Math/Text Area Segmentation = Simultaneous Process using DP

- 1. Combination of different OCR engines
- 2. Score using relative position check

Current version:

Infty + Two commercial OCRs (Toshiba + Media Drive)

New version: FineReader engine will be added.



Method 1. Niihama-gun,  $F: N \ddot{u} h a m a - g u n$ , E: Nii harna-gun, I:  $N i i / \iota a m a - g u n$ ,







# **Process Flow of Digitization**



#### Formulae Recognition

- Recognition of Variety of Rare Symbols
- Distinction of Fonts.(*Italic, Bold, Bbb, Caligraphic,etc.*)
- Segmentation of *Touched/Broken* characters in Math Area is still a difficult problem.
- Stable Structure Analysis of math formulae against the miss-recognition of characters.
- Distinction of *Noises* and Small symbols

# **Process Flow of Digitization**



# **Document Structure Analysis**

#### Detection of :

Title, Autor, Section, Subsection, Itemization, BibItem, Theorem, Lemma, etc.

#### - A Naïve method:

Line classification using the combination features such as: *Character size, Font Information (Bold, Italic, Small Capital), Keywords, Indentation, Starting with Numbers or Special pattern (e.g. "[Num]"), etc.* 

- Stronger method is required in actual digitization.
- Hyperlink inside document.

# **Section 3** Formulae Recognition by Infty

#### Formulae recognition

• R. Anderson, Syntax directed recognition of hand-printed two dimensional mathematcs.

Interactive System for Experimental Applied Mathematics, M.Klerer and J. Reinfelds, Eds, Academic Press, 1968, pp. 436-459

- M. Okamoto and H. Twaakyondo, Structure analysis and recognition of mathematical expressions, 3rd ICDAR, 1995, Montreal, (1995), 430--437.
- R. J. Fateman, T. Tokuyasu, B. P. Berman and N.Mitchell, Optical Character Recognition and Parsing of Typeset Mathematics, Journal of Visual Communication and Image Representation vol.7, no.1, (1996), 2--15.
- Y. Eto and M.Suzuki, Mathematical formula recognition using virtual link network, 6<sup>th</sup> ICDAR, 2001, Seattle, IEEE Computer Society Press, 430--437

## Infty OCR engine

- Developed in Suzuki Lab., using more than 1,500,000 sample images of characters and symbols from various math. books/journals.
- Recognizes more than 500 categories
  - Various math symbols
  - Various fonts: Roman, Italic, Calligraphic, Bbb, some German fonts, etc.
  - High speed
    - Three step classification : "rough" classification  $\rightarrow$  "strict" classification

#### 3 step classifications



Recognition result (candidates)

## Voting method



## Voting method



Voting  $\rightarrow$  Normalization of the score of symbol recognition

#### Structure Analysis of Formulae

#### **Output (Tree Structure)**




Collapsing of quotient spaces of 
$$SO(n) \setminus SL(n, \mathbf{R})$$
 at infinity 203  

$$= -C \log \left\{ \Delta_1(x)^{\alpha_1 - \alpha_n} \times \prod_{k=2}^{n-1} \left( \frac{\Delta_k(x)}{\Delta_{k-1}(x)} \right)^{\alpha_k - \alpha_n} \right\}$$

$$= -C \log \Delta_k(x)^{\alpha_k - \alpha_{k+1}}$$

$$= C \log \left( \prod_{k=1}^{n-1} \Delta_k(x)^{\alpha_{k+1} - \alpha_k} \right)$$





Link possibilities :

# $= c_t^*(25)$

Similar characters :

 $z^* \in S^{N-1}$ 



 $\int_{a}^{b} x^{2} dx$ 

#### Virtual link network



 $\int_{a}^{b} x^2 dx$ **Search for correct** spanning tree

#### Virtual link network



#### Link Cost

#### Definitions of : Normalized size (NSize) and Normalized center (NCenter)



x:y:z = 28:51:21 (dafault value)

#### Link Cost



#### Link Cost

#### Distribution map in the (H,D)-plane











-600







- A (c,x,Horizontal,10) $(C,\chi,\text{RSupScript},50)$  $(c,\chi,\text{Horizontal},100)$ (C,x,Horizontal,100)
- B (x,2,RSupScript,10) $(\chi,2,\text{RSupScript},50)$
- C (x,y,Horizontal,10) $(\chi,y,\text{Horizontal},100)$
- D (2,3,Horizontal,10)
- E (y,3,RSupScript,10)







#### Search of spanning tree



and use them at the next step.





# Section 4 Large Volume Recognition

# Large Volume Digitization

- Retro-digitization of journals,
- Reproduction of old book/series of books,
- Translation to different languages,
- Braille transcription, DAISY talking book,



# Large Volume Digitization

#### Adaptive method is efficient:

Get information from the target document:

- Character features,
- Math formula parameters,
- Layout parameters, etc.



After manual checking (Semi-automatic)

#### "InftySystem" for large scald digitization

#### Applications:

- 1. *InftyReader* downloadable from our web site: http://www.sciaccess.net
- 2. InftyReader Pro (professional version)
- 3. BatchInfty
- 4. CharImageManager



4. CharImageManager



4. CharImageManager

#### "InftySystem" for large scale digitization

#### Process Flow using BatchInfty & InftyReader pro

- 1. Noise reduction, centering, etc.
- 2. Trial recognition
- 3. Extraction features:
  - Document style  $\rightarrow$  Logical structure analysis
  - Character cluster images  $\rightarrow$  OCR engine
- 4. Recognition & verification
- 5. PDF output

#### Full automatization of the adaptive method

*From the target documents:* Extraction of character features / layout parameters



Further improvement of character/symbol recognition and structure analysis of math expressions.

- Touched characters, Broken characters in math area
- Low resolution image
- Different type face (Old books, typewriter prints, etc.)
- Bold char detection in math area

Logical Structure Analysis (Automatic detection and manual correction)

- Title, Autor, Section, Subsection, Itemization, BibItem, Theorem, Lemma, etc.
- Hyperlink inside document.

#### Detection/Analysis of Figures and Tables

- Detection of characters in figures
- Table structure analysis
- Graphs  $\rightarrow$  Tables

# Challenge

■ Is it possible to realize:

OCR with higher accuracy than manual imput/correction by human?

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OCR with higher accuracy than manual imput/correction by human?

(I hope a student who challenges to this difficult problem appears in near future!)

# Conclusion

- InftyProject.
  - Research group of math information processing.
- Demo (*InftyReader*).
- A Brief sketch of the methods used in Infty and the current state of the art.
  - There are many problems unsolved, especially in practical sense.
- Proposed some problems to be attacked.


## Thanks you!

Masakazu Suzuki Graduate School/ Faculty of Math. Kyushu University E-mail: suzuki@math.kyushu-u.ac.jp

InftyProject: http://www.inftyproject.org Science Accessibility Net: http://www.sciaccess.net