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Clique-Width of Point Configurations

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with co-authors

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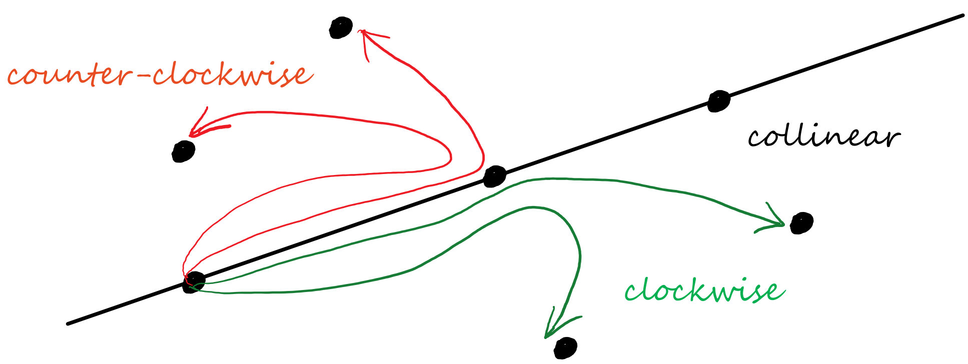
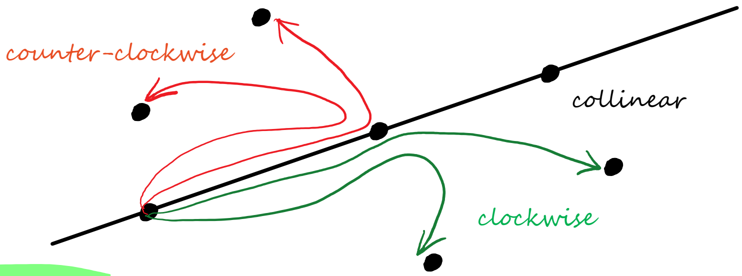
Motivation

- Structural width measures ↪ standard tools of graph algorithms.
- In **computational geometry**?
  "Truly geometric" problems are **not discrete**, but
  many questions only care about, e.g., relative position of objects. . .
- Such as, in points configurations, we care about positions of points with respect to other points and lines spanned by them, but not about precise distances or angles:



Contribution



Considering **point configurations** –
points sets with the relative positions encoded by the so-called "order type"

- give a definition of **clique-width** of the order type;
- study frontiers between bounded and unbounded clique-width for it, in particular, give reasonable examples of bounded-cw configurations;
- list examples of geometric problems which are hard in general, but efficiently solvable with a given bounded clique-width decomposition (via efficient solvability of **all MSO properties** of order types).

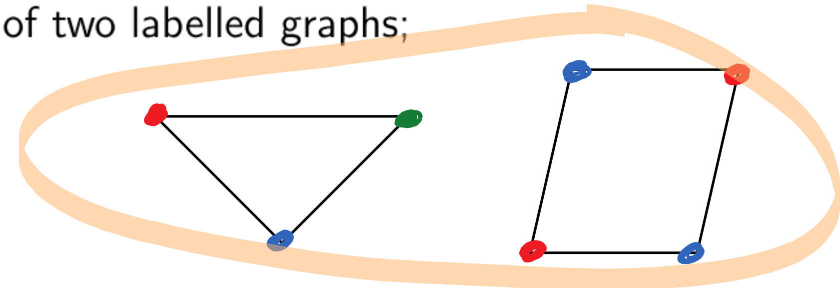
Small detour

Clique-width of a graph

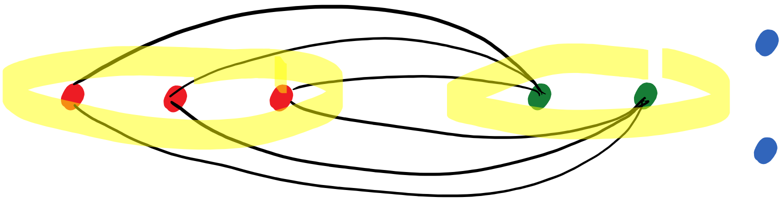
We have the operations:

(u1) create a new vertex with single label i;

(u2) take the disjoint union of two labelled graphs;



(u3) add all edges between the vertices of label i and label j ($i \neq j$); and



(u4) relabel all vertices with label i to label j.

Clique-width := the min # of labels used.

Clique-width in greater generality?

What is more general? **Relational structures** (of finite arity).

As previously...

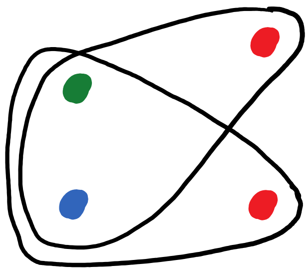
(u1') create new points of the ground set (vertices) with label i;

(u2') take the disjoint union of two labelled structures;

(u4') relabel all entities of label i to label j; and

(u3') add all relational tuples based on the current point labels.

such as, add all blue-green-red triples...



This leads to **unary clique-width**, which does not perform well.

[**Adler and Adler**, 2008]

Multi-ary clique-width!

[**Blumensath**, 2006], [**Blumensath and Courcelle**, 2006]

For relational structures of finite arity:

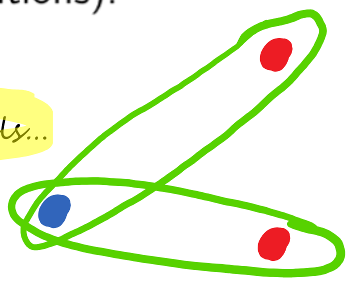
(m1') create new points of the ground set (vertices) with label i;

(m2') take the disjoint union of two labelled structures;

(m4') relabel all entities of label i to label j; and

(m3') create new **multi-ary labels** and/or relational tuples based on the current labels (formally – by quantifier-free operations).

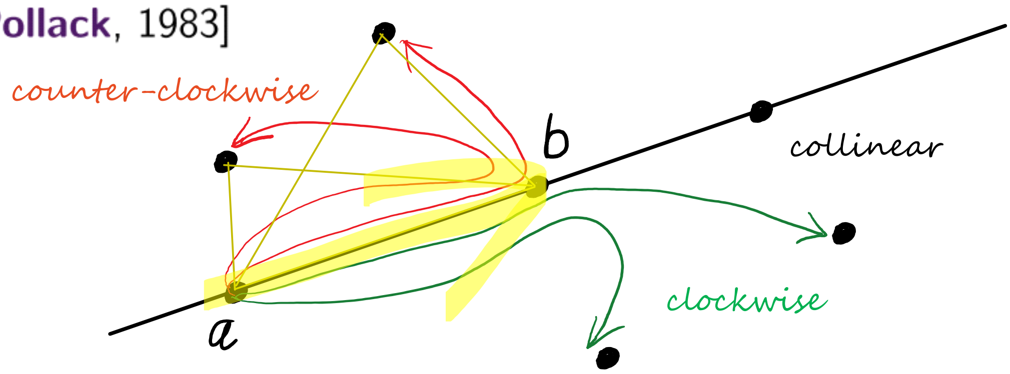
such as, binary green labels...



Three ingredients

Order type of a point set P

[**Goodman and Pollack**, 1983]



Order-type = a ternary structure $\Omega \subseteq P^3$, such that

$(a, b, c) \in \Omega$ iff abc forms a counter-clockwise oriented triangle.

Notes:

- $(a, b, c) \in \Omega$ implies $(b, c, a), (c, a, b) \in \Omega$ (cyclic closure),
- the triple abc forms a clockwise triangle, iff $(b, a, c) \in \Omega$,
- a, b, c are collinear points, iff $(a, b, c), (b, a, c) \notin \Omega$.

Clique-width of a point configuration

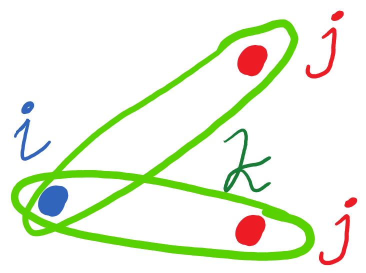
We have the operations:

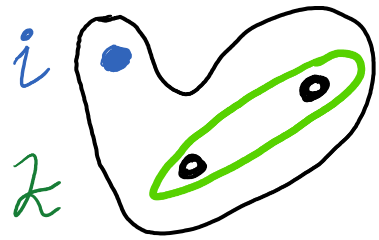
(w1) create a new point with single label i;

(w2) take the disjoint union of two point sets;

(w3) for every two points, point a of label i and point b of label j ($i \neq j$), give the ordered pair (a, b) binary label k;

(w4) for every three pairwise distinct points, a, b and c such that c is of (unary) label i, and the pair (a, b) is of (binary) label k, add to the structure the cyclic closure of the ordered triple (a, b, c);



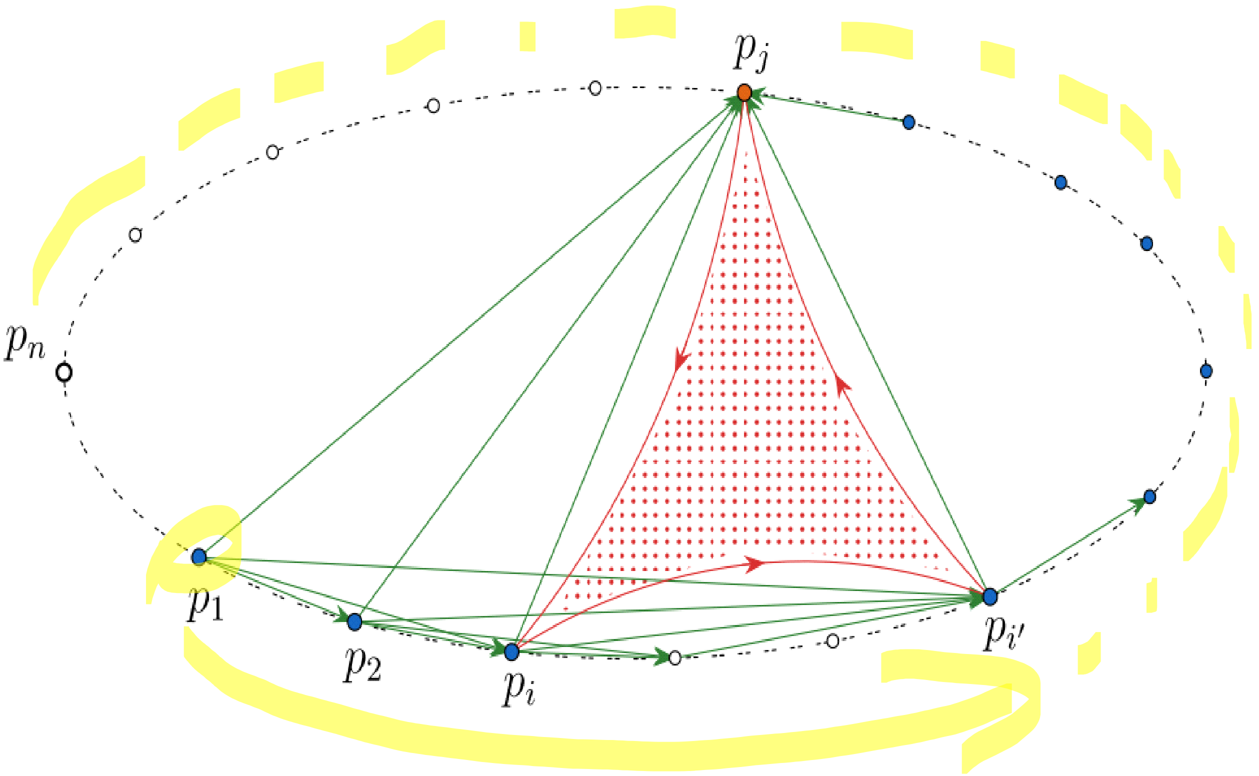


(w4') under the same conditions as in (w4), add the cyclic closure of (b, a, c);

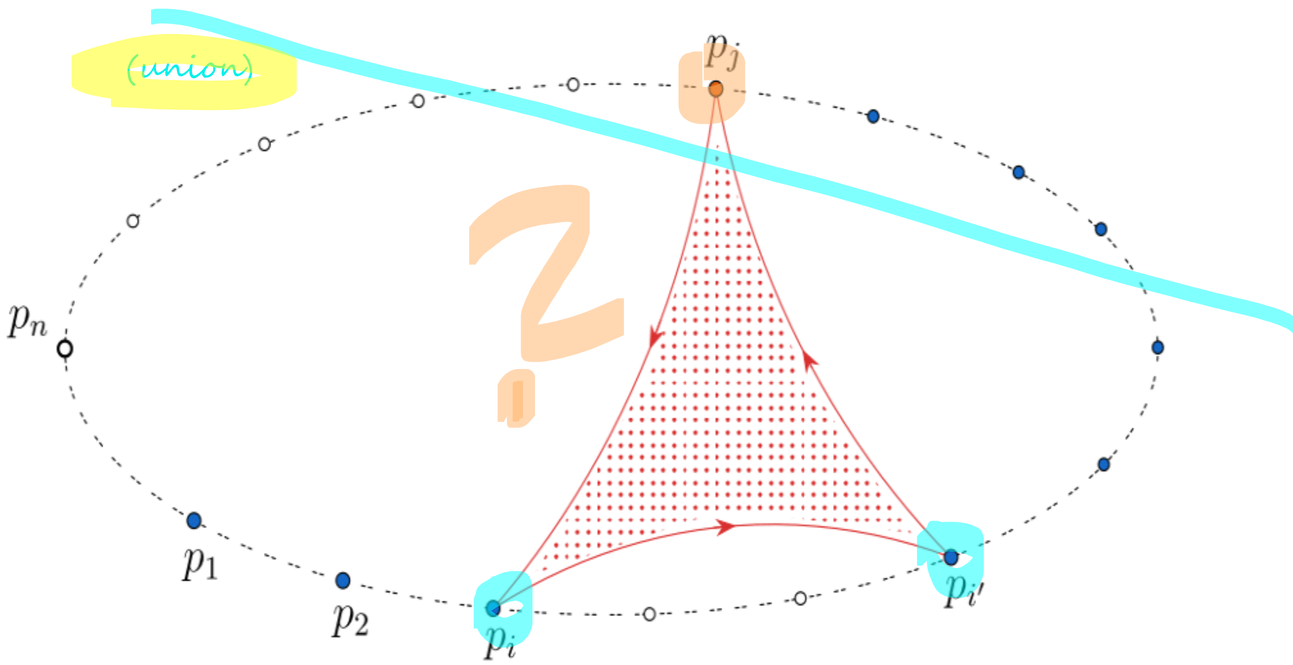
(w5) relabel all tuples with label i to label j of equal arity.

Clique-width := the min sum of arities of labels used.

Example: convex point set has clique-width 4:



Example: convex point set has **unbounded** unary clique-width:



MSO logic of order types

MSO = monadic second-order logic (quantification over points and sets):

- propositional $\neg, \wedge, \vee, \rightarrow$, equality $=$, quantifiers $\exists x$, $\forall x$, $\exists X$, $\forall X$, and
- the predicate *ccw*(x, y, z) with the meaning $(x, y, z) \in \Omega(P)$.

Simple examples

- Points x, y, z are collinear:

$$\neg\, \mathit{ccw}(x, y, z) \wedge \neg\, \mathit{ccw}(y, x, z)$$

- Point y belongs to the convex hull of a set $X \not\ni y$:

$$\forall x, x' \in X \Big[\big(x \neq x' \wedge \forall z \in X \neg\, \mathit{ccw}(x', x, z) \big) \rightarrow \neg\, \mathit{ccw}(x', x, y) \Big]$$

MSO metatheorem(s)

- [**Blumensath and Courcelle**, 2006]

  A class $\mathcal{S}$ of relational structures is of bounded clique-width, iff $\mathcal{S}$ is contained in an MSO transduction of the class of finite trees.

  (Informally, structures of $\mathcal{S}$ can defined by MSO in suitable coloured trees.)

  $\Rightarrow$ If we define, e.g., a big grid in $\mathcal{S}$, then $\mathcal{S}$ has large clique-width.!

- [**Courcelle, Makowsky and Rotics**, 2000]

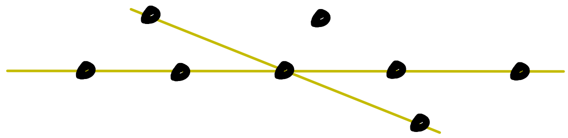
  On graphs of bounded clique-width, one can solve any MSO-definable *decision / enumeration / lin-optimization* property in **FPT**-time.

  $\Rightarrow$ The same holds for any (finite) relational structures.

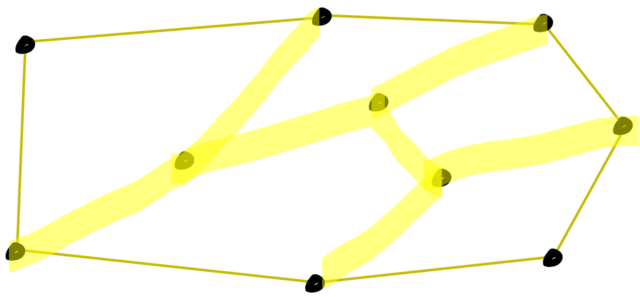
Applications

Examples of NP-hard problems

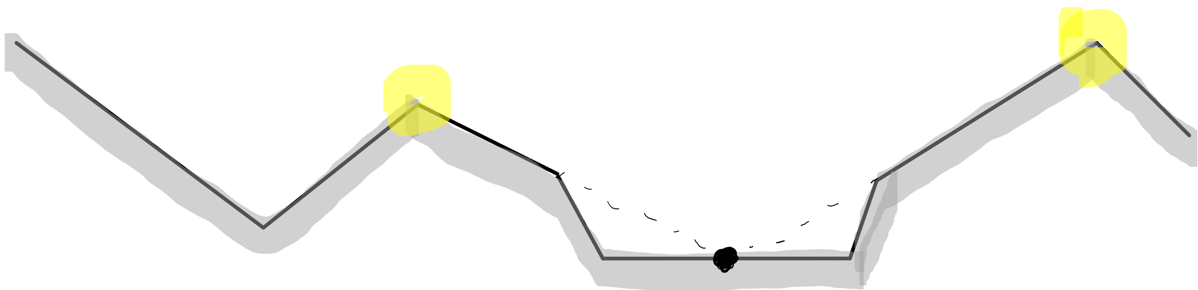
- General position subset - a max subset without collinearity.



- Minimum convex partition into $\leq k$ convex faces.



- Segmented terrain guarding - adjusted classical terr. guarding.



Conclusions

- **New contributions** given:
  - Mathematically sound definition of clique-width of point configurations, based on established concepts from compgeo and logic.
  - Assorted examples showing that the new definition makes good sense in computational geometry.
  - In particular, a new application area for the established algorithmic metatheorem for MSO-definable properties.
- **Future research** proposals:
  - Of course, to provide an FPT algorithm for the new width.
  - Find applications of the new stuff in metric problems on points.
  - Consider clique-width of suitable "restrictions" of order type, e.g., in visibility problems the orientation not inter. for invisible triples.

Thank you for your attention