FO model checking of intersection graphs and twin-width

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Motivation

- While FO model checking of sparse graphs is nowadays well-understood...

Deciding First-Order Properties of Nowhere Dense Graphs

MARTIN GROHE, RWTH Aachen University
STEPHAN KREUTZER and SEBASTIAN SIEBERTZ, Technical University Berlin

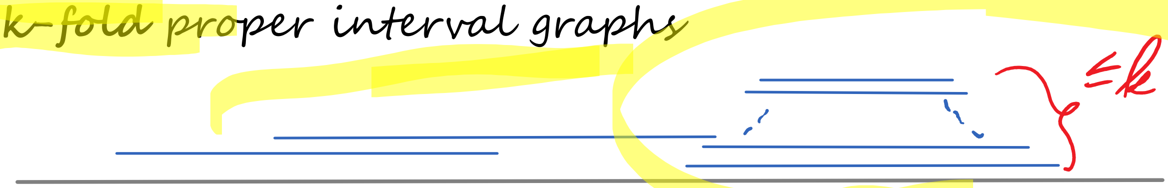
Journal of the ACM, Vol. 64, No. 3, Article 17, Publication date: June 2017.

- For "dense" graphs, we have been starting from scratch in the past decade, such as from rather simple geometric intersection classes.

Examples of FO model checking in FPT

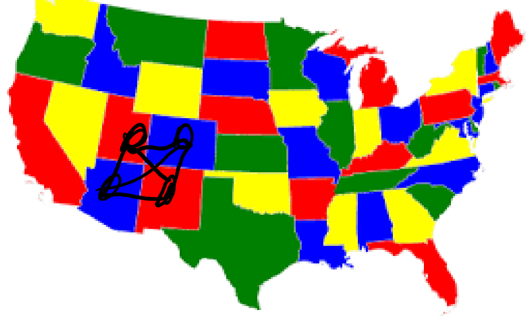
- Interval graphs with a finite set of fixed int. lengths.
  [Ganian, PH, Kráľ, Obdržálek, Schwartz, Teska, 2013/15]

- k-fold proper interval graphs



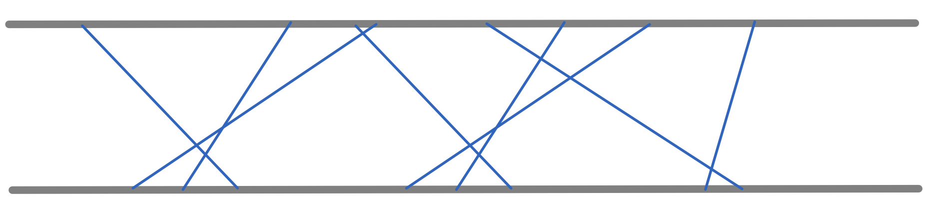
- The **k-fold** case following from a much wider result:
  FO model checking on posets of bounded width in FPT. [Gajarský, PH, Lokshtanov, Obdržálek, Ordyniak, Ramanujan, Saurabh, 2015]

- FO model checking of map graphs in FPT.
  [Eickmeyer and Kawarabayashi, 2017]



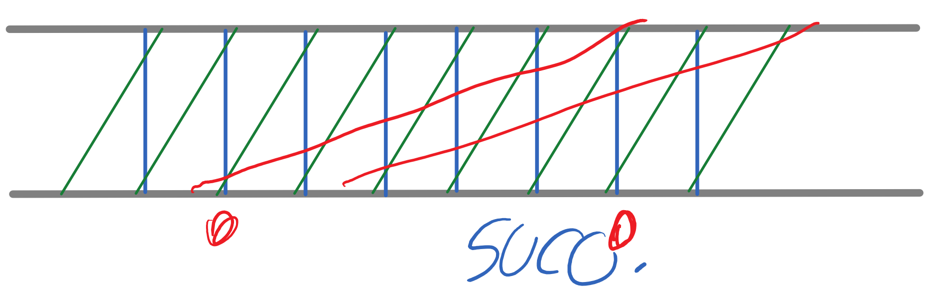
Examples II (even more geometry)

- PERMutation graphs (edges ↔ inversions)



Equivalently, intersection graphs of chords between two parallels.

- **Fact.** All simple graphs interpretable in PERM:



- FO model checking of **PERM** graphs of bounded clique or IS in FPT.

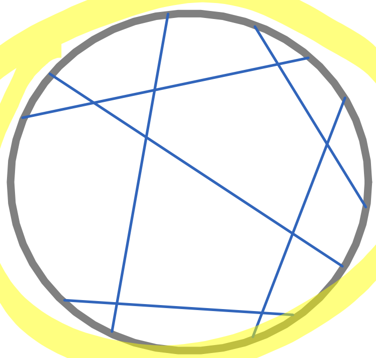
  [PH, Pokrývka and Roy, 2018]

  (Using posets of bounded width.)

- Exact boundary of FO m.c. tractability on PERM?

- **CIR**cle graphs $\supsetneq$ **PERM**

  = intersection graphs of chords in a circle:



- FO model checking of **CIR** graphs of bounded IS (but not clique) in FPT.

  [PH, Pokrývka and Roy, 2018]

  Again, using posets of bounded width.

- What about **CIR** graphs of bounded clique size?

Twin-width – the breakthrough

Twin-width I: tractable FO model checking

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- FO model checking of classes of bounded twin-width is in FPT.

  [Bonnet, Kim, Thomassé, Watrigant, 2020]

  → this includes all previously stated results.

- Poset of width d have twin-width $2^{2^{O(d)}}$.

  [Bonnet, Kim, Thomassé, Watrigant, 2020]

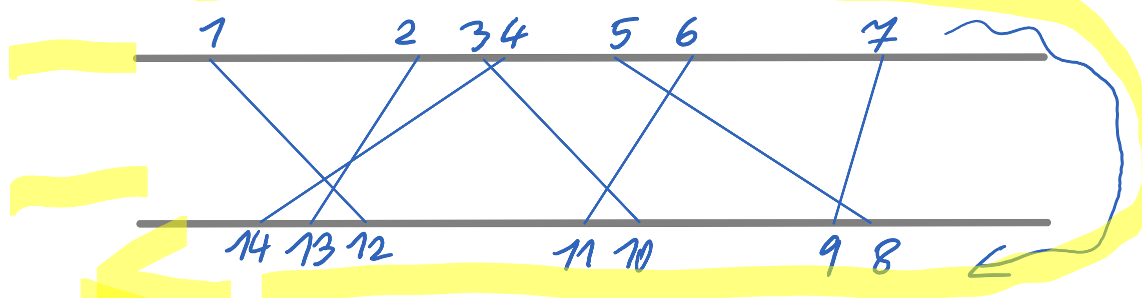
- Poset of width d have twin-width $O(d)$.

  [Balabán and PH, 2021]

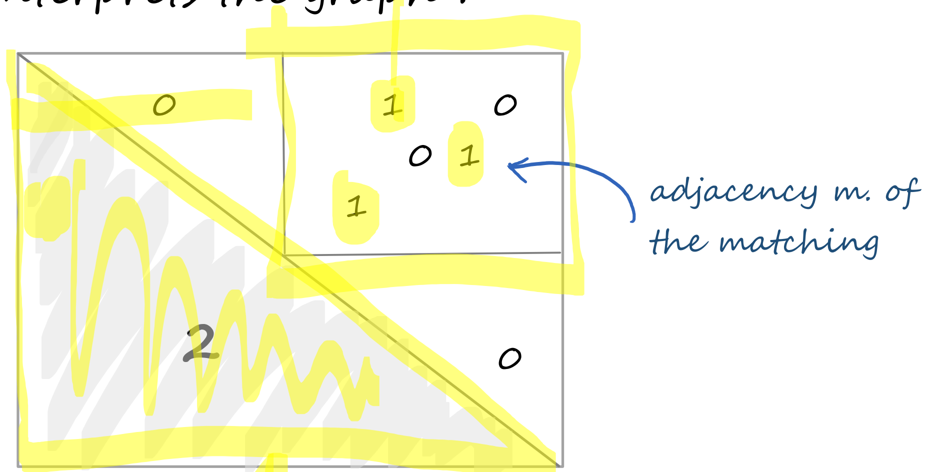
Back to the PERMutation graphs

- A heredit. class of **PERM** graphs has FPT FO model checking iff it excludes some PERM graph.
  [Bonnet, Kim, Thomassé, Watrigant, 2020]

A proof sketch...



- n-vertex PERM g. ↔ a matching on {1,2,...,2n},
- then encoded in an "ordered" matrix which FO interprets the graph :



- Now, bounded **tww.** → tractable FO model c. (note that we use the interpretation of the gr.),
- and unbounded tww. → large mixed minor (in every order, hence also in our order).

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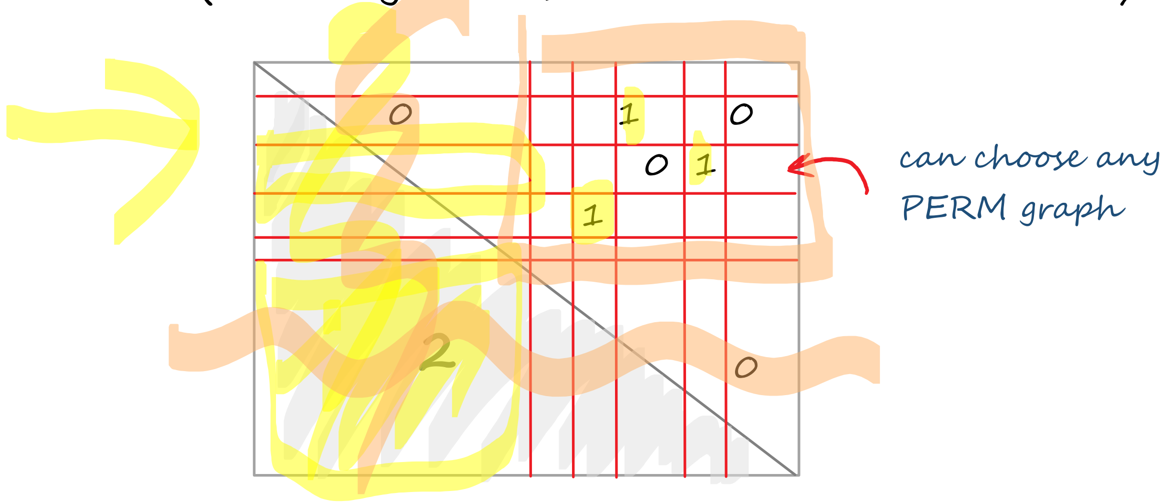

- Hence, unbounded tww. → our class contains all PERM graphs! (and that is not our class)

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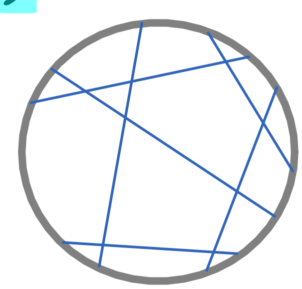
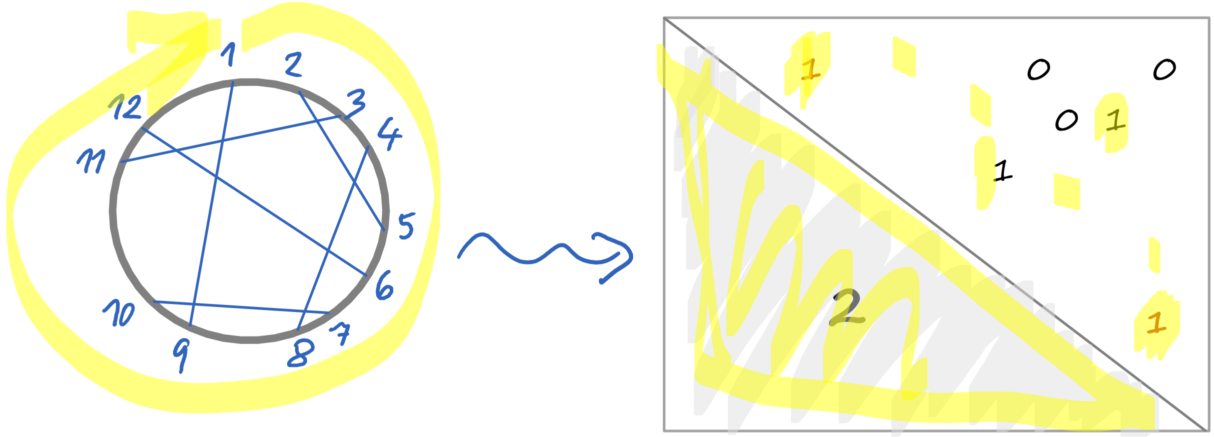
Analogously for CIRcle graphs



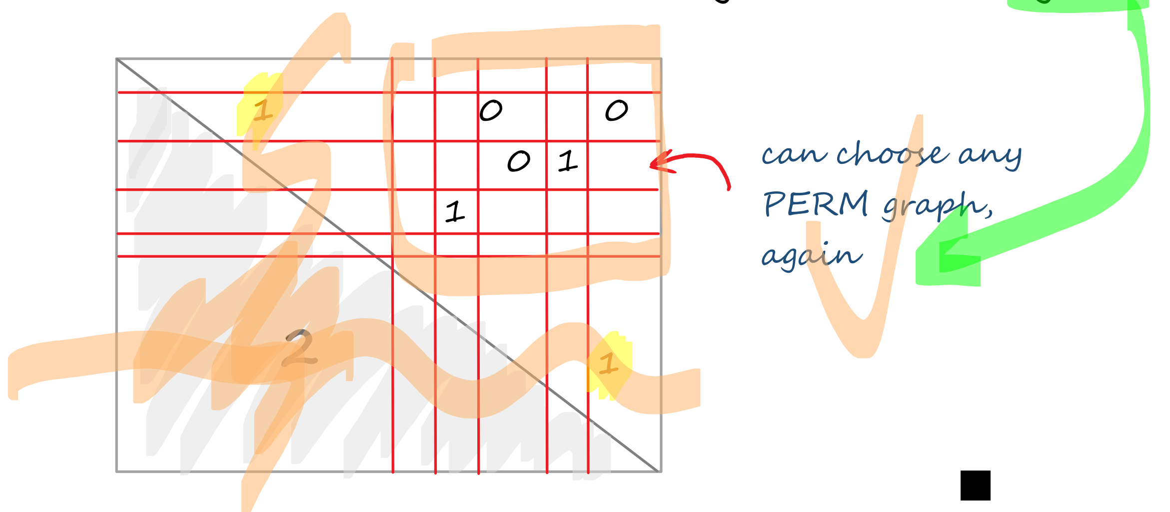
- A hereditary class of **CIR** graphs has FPT FO model checking iff it excludes some PERM graph. [PH and Pokrývka, 2021]
- Note; **CIR** graph representation can be computed in polytime. [Bouchet, 1985], [Spinrad, 1994]

A proof sketch...

- All PERM graphs → intractable FO model c.
- Take again an ordered matrix with a matching:



- Bounded **tww.** → tractable FO model c. again,
- and **unbounded** tww. → large mixed m. again :



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Conclusions

- The question of FO model checking on subclasses of PERM graphs turned out to be the right one…

- Similarly to the case of CIR graphs, can we deduce a sharp boundary between tractable and intractable FO model c. on **interval** graphs ?
    - The literally same approach with ordered matchings and mixed minors can be applied to them, but the outcome is **not satisfactory** (exhibiting perhaps only a big clique).
    - The problem seems to lie in that the order prescribed in the matrix is not the right one to exhibit truly **large tww.**

- ***NEW*** (regarding vertex-minors of Rose)
  Let $\mathcal{B}$ be a hereditary class such that every graph in $\mathcal{B}$ is a p-bounded perturbation of a CIR graph.
  Then $\mathcal{B}$ has bounded twin-width

  iff

  $\mathcal{B}$ excludes some PERM graph.


THANK YOU
FOR YOUR ATTENTION