University Course Timetabling with Soft Constraints

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Abstract. An extension of constraint logic programming that allows for weighted partial satisfaction of soft constraints is described and applied to the development of an automated timetabling system for Purdue University. The soft constraint solver implemented in the proposed solution approach allows constraint propagation for hard constraints together with preference propagation for soft constraints. A new repair search algorithm is proposed to improve upon initially generated (partial) assignments of the problem variables. The model and search methods applied to the solution of the large lecture room component are presented and discussed along with the computational results.

Keywords: university course timetabling, constraint logic programming, weighted constraints, search algorithms.

1 Introduction

This paper describes the design approach and solution techniques being developed for an automated timetabling system at Purdue University. The initial problem considered here is the design of an intelligent system to assist with construction of the large lecture component of the university's master class schedule. The design anticipates expanding the scope of the problem to accommodate a demand-driven approach to timetabling all classes at the University. In *demand-driven timetabling*, student course selections are utilized to construct a timetable that attempts to maximize the number of satisfied course requests. In the initial problem we consider the course demands of almost 29,000 students enrolled in approximately 750 classes, each taught several times a week.

A solution to the timetabling problem is being developed using *constraint logic programming* [28, 18]. CLP is a respected technology for solving hard problems which include many complicated (non-linear) constraints [15]. Its main advantages over other frameworks are the declarative nature of problem descriptions via logical constraints, and a constraint propagation technique for reducing the search space.

Timetabling problems [8, 27] are often over-constrained, which is the case with our problem since it is not possible to satisfy all requests of students for enrollment to specific courses. Preferential requirements for time and room assignment may also lead to the problem being over-constrained. *Soft constraints* [10, 5] can be applied to define these requirements declaratively rather than encapsulating many of them into the control part of the problem solution. In our problem solution, we have applied a weighted CSP [10] approach which considers weights/costs for each constraint and minimizes the weighted sum of unsatisfied constraints.

Our work includes development of a new solver for soft constraints. The solver was implemented as an extension of the CLP(FD) library [6] of SICStus Prolog. This approach is of particular importance for the construction of demand-driven schedules where complete satisfaction of all constraints is not feasible. Soft constraints have also been applied to accommodate the preferences of instructors with respect to time and room assignments for their classes We will also describe a new search algorithm based on backtracking with constraint propagation. This search procedure allows the return of a partial solution even when the problem is over-constrained.

The following section of this paper presents a description of our timetabling problem. Section 3 explains the soft constraint solver that was implemented. The new search algorithm developed for this problem is detailed in Section 4. This is followed by a description of how the problem has been modeled, including the representation of soft and hard constraints. In addition, the methods applied to the search for a feasible solution are discussed. Computational results are discussed in Sec. 6. Comparisons with other approaches to solving demand-driven timetabling problems and applying soft constraints to the problem solution are presented in Sec. 7. The final section reviews the results of our work and looks to future extensions of the problem solution and soft constraint solver improvements.

2 Problem Description

At Purdue University, the timetabling process currently consists of constructing a master class schedule prior to student registration. The timetable for large lecture classes is constructed by a central scheduling office in order to balance the requirements of many departments offering large classes that serve students from across the university. Smaller classes, usually focused on students in a single discipline, are timetabled by “schedule deputies” in the individual departments. This process has been tailored to the political realities of a decentralized university, where faculty can be quite put off by the idea of having a central office tell them when to teach, or even by providing such an office with much information about when they are available to teach.

A natural decomposition of the university timetabling problem has therefore resulted, consisting of a central large lecture timetabling problem and 74 disciplinary problems. Construction of the timetable for large lectures is the primary focus of in this paper. This problem consists of approximately 750 classes having a high density of interaction that must fit within 41 lecture rooms with capacities up to 474 students. Course demands of almost 29,000 students out of a total enrollment of 38,000 must also be considered. The departmental problems range from only a few classes up to almost 700, with an average size of slightly more than 100 organized classes. The largest departmental problems are simplified by having many sections of the same course that are offered at multiple times.

The timetable maps classes (students, instructors) to meeting locations and times. A major objective in developing an automated system is to minimize the number of potential student course conflicts which occur during this process. This requirement substantially influences the automated timetable generation process since there are many specific course requirements in most programs of study offered by the University.

To minimize the potential for time conflicts, Purdue has historically subscribed to a set of standard meeting patterns. With few exceptions, 1 hour x 3 day per week classes meet on Monday, Wednesday, and Friday at the half hour. 1.5 hour x 2 day per week classes meet on Tuesday and Thursday during set time blocks. 2 or 3 hours x 1 day per week classes must also fit within specific blocks, etc. Generally, all meetings of a class should be taught in the same location. Conforming to this set of standard meeting patterns will be seen to have a strong influence on both the problem definition and the solution process, since the meeting patterns defined for each class introduce hard restrictions on the acceptability of any generated solution.

Another important constraint on the problem solution is instructor availability. Balancing instructor time preferences was found to be a critical factor in developing acceptable solutions since an earlier attempt [20] to automate timetabling at Purdue was unsatisfactory largely because solutions heavily favored instructors who imposed the most constraints.

Room availability is a also a major constraint for Purdue. In addition to room capacity, it was necessary to consider specific equipment needs and the suitability of the room's location. Historically, a limited number of classrooms has been used to force a wide distribution class times. Increased enrollments, however, have left the university with little excess room capacity.

Another aspect of the timetabling problem that must be considered here is the need to perform an initial student sectioning. Most of the classes in the large lecture problem (about 75 %) correspond to single-section courses. Here we have exact information about all students who wish to attend a specific class. The remaining courses are divided into multiple sections. In this case, it is necessary to divide the students enrolled to each course into sections that will constitute the classes. Without this initial sectioning it is not easy to measure the desirability or undesirability of having classes overlap in the timetable. Our current approach sections students in lexicographic order before joint enrollments between classes are computed. This gives us the worst case possibility. The university currently processes a precise student schedule after the master class schedule is created, however, which should introduce some improvements. Possible directions for improving this solution are discussed in the final section.

3 Solver for Soft Constraints

Constraint propagation algorithms for the soft constraints are implemented as a part of the soft constraint solver [24]. This constraint solver is built on top of the CLP(*FD*) solver of SICStus Prolog [6] and implemented with the help of attributed variables. An advantage of this implementation is the ability to include both hard constraints from the CLP(*FD*) library and soft constraints from a new *soft constraint solver*.

3.1 Preference Variables and Preference Propagation

The soft constraint solver handles preferences for each value in the domain of the variable which will be called the *preference variable*. Each preference corresponds to a natural number indicating the degree to which any soft constraint dependent on the domain value is violated. An increase of the preference during the computation with soft constraints is called *preference propagation*. Note the difference with constraint propagation, which removes values from the domain of the domain variable during the computation of hard constraints. Removal of domain values may also occur with preference variables. This corresponds to violation of a hard constraint.

We may also set the degree of acceptable violation for any preference variable. If the preference associated with a value in the domain of the preference variable should exceed this limit, it is removed from the domain. This possibility is of particular interest for time (or classroom) variables since all classes (each represented by a preference variable) should be relatively equal in importance.

Zero preference means complete satisfaction of the constraint for the corresponding value in the domain of the variable. Any higher preference expresses a degree of violation that would result from the assignment of this value to variable. All values which are not present in the domain of the preference variable have the infinite preference `sup`. Preferences for each value in the domain of the variable may be initialized with a natural number. This allows us to handle initial preferences of values in the domain of the variable.

Example 1. The unary soft constraint `pref(PA, [7-5, 8-0, 10-0], ...)` creates the preference variable `PA` with initial domain containing values `7`, `8`, and `10` and preferences `5`, `0`, and `0`, resp. It means that the value `7` is discouraged wrt. other values. Preferences for remaining values are assumed as infinite, indicating complete unsatisfaction.

3.2 Binary Soft Constraints

Two binary soft constraints have been implemented in the problem solution.

```
soft_different( PA, PB, Cost )
soft_disjunctive( PStart1, Duration1, PStart2, Duration2, Cost )
```

The `soft_different` constraint expresses that the two preference variables `PA`, `PB` should have different values. The constant `Cost` gives us the cost for violation of this constraint. The `soft_disjunctive` constraint asks for the non-overlapping of the two tasks specified by the preference variables `PStart1`, `PStart2` and the constant durations `Duration1`, `Duration2`. Again the `Cost` is the weight of this constraint.

Algorithms for both constraints are based on the partial forward checking algorithm and inconsistency counts [10] which are stored in the preferences for each value of the preference variable. Let us take a look at the `soft_different` constraint. Once the first preference variable is instantiated to some value `X`, the inconsistency count for the second variable and the value `X` should be increased by `Cost`, i.e., the preference propagation is processed for this variable and value. The `soft_disjunctive` would process preference propagation for all values in the interval `X..(X+Duration-1)`.

3.3 Cost Function

For each preference variable, the soft constraint solver maintains an additional domain variable (*cost variable*) having the current best preference of the preference variable as its lower bound. The initial upper bound is set to infinity. Any preference propagation results in an increase of the current best preference, with the lower bound of the cost variable being increased accordingly.

Preferences for each value of the preference variables are used to store initial preferences and any changes in the inconsistency counts for these values. This information is reflected in the cost variables. The sum of all cost variables gives us the total cost of the solution (*cost function*). The cost function can be then applied during labeling and optimization.

For efficiency, only the bound consistency is processed for all cost variables. This means that only changes to the lower and upper bounds are maintained. Any change in the current best preference is stored in the lower bound and, during optimization, the upper bound may be used to prune the search space.

4 Limited Assignment Number Search Algorithm

The aim of our problem solution is to be able to search for the complete assignment of the preference variables giving the best possible satisfaction of all soft constraints. Since the evaluation of the assignment is given by the sum of the corresponding cost variables, it may seem possible to apply a classical branch & bound algorithm. Unfortunately it is not easy to find such a complete assignment. Mistakes in the assignment of some variable(s) may lead to a time consuming exploration of the search space with no complete solution. A complete assignment may not even exist due to conflicts among the hard constraints.

Because of these disadvantages, we have proposed a new non-systematic iterative search algorithm based on chronological backtracking — *limited assignment number (LAN) search* algorithm [29]. It attempts to find some initial *partial assignment* of the variables and subsequently *repair* it such that all, or at least most, of the variables are assigned a value.

For each variable, the LAN search algorithm maintains a count of how many times a value has been assigned. A *limit* is set on this count. If the limit is exceeded, the variable is left unassigned and the search continues with the other variables. Labeling of unassigned variables is not processed even during backtracking. As a result of this search, a partial assignment of variables is obtained together with the set of the remaining *unassigned variables*.

Limiting the number of attempts to assign a value to each variable ensures the finiteness of this incomplete search. The current limit is set to the maximal domain size d of any labelled variable. As each of n variables can be tried d times, one iteration of the LAN search is of *linear complexity* $\mathcal{O}(dn)$.

Results of the LAN search process are used in subsequent iterations of the search. The following variable and value ordering heuristics are developed based on the previous iteration:

– values of successfully assigned variables are used as initial assignments in the next iteration — once a suitable value for a variable has been found, it remains a promising assignment;
– unsuccessfully attempted values for any variable left unassigned are demoted in the ordering so that they will be tried last in the subsequent iteration — a suitable value is more likely to remain among those that have not been tried;
– any variable left unassigned is labeled first in the subsequent iteration — it may be difficult to assign a value to the variable, therefore, it should be given preference in labeling.

In the first iteration of the algorithm (*initial search*) we have a choice of using either problem-specific heuristics or standard heuristics, such as first-fail [28], for variable ordering. The most promising values are used for value ordering. In successive iterations (*repair searches*), heuristics based on the previous iteration are primarily used. Any ties are broken in favor of the initial search heuristics.

The user may also manually modify the results after each iteration to influence the behavior of the heuristics, relax constraints to eliminate contradictory requirements, or change the problem definition. The options available for continuing the search are as follows:

1. processing the automated search as proposed above;
2. defining other values to be tried first or last based on user input, and process the repair search directed by the updated value ordering heuristics;
3. relaxing some hard constraints based on user input, and processing the initial search or the repair search;
4. addition or deletion of variables or constraints based on user input, and applying the repair search to reuse results of the former solution.

The first possibility is aimed at automated generation of a better assignment. Approach 2 allows the user to direct the search into parts of the search space where a complete assignment of variables might more easily be found. Step 3 can be useful if the user discovers a conflict among some hard constraints based on a partially generated output. The repair search can reuse most of the results from the former search and permute only some parts of the last partial assignment. Let us note that there is often an advantage to a user directed search in timetabling problems, since the user may be able to detect inconsistencies or propose a suitable assignment based on a partially generated timetable. The last possibility introduces a new direction for the development of the algorithm aimed at incorporating changes to the problem input. This will be studied in detail as a part of our future work.

5 Problem Solving

We would like to describe a model for the timetabling problem which consists of variables for the time and room assignments of each class and of both hard and soft constraints, applying an approach described in the previous section. We also explore the control portion of the solution, which consists of the application of the proposed initial and repair searches.

5.1 Time and Classroom Variables

The domain of the time variables is represented by the natural numbers `0..104`, corresponding to `5` days of `21` half-hours. The domain of the classroom variables is represented by the natural numbers `1..Number_Of_Classrooms`.

Each class consists of between one and five meetings per week (typically two or three). All meetings have the same duration and are typically taught at the same time of day. Valid combinations of the number of meetings and the duration are called *meeting patterns*. Each meeting pattern (e.g., 1 hour x 3 meetings) has a defined set of days on which the meetings may be scheduled (e.g., Monday, Wednesday, Friday for 1 hour x 3 meetings). Interestingly, the start time of the first meeting of a class differs from the start times of the following meetings by a constant factor for most combinations (see Table 1). Excepting the MF (Monday and Friday) combination for 2 meetings per week and the 4 meeting patterns (includes less than 1% of classes), one time variable is sufficient to contain the complete information about the start time of classes. This is a preference variable indicating the start time of the first meeting (`T1`). It will be referred to as the *time preference variable*. The starting times of all remaining meetings (`T2, ..., Tn`) are domain variables only, and may be related to the time preference variable by the simple constraint

$$\mathtt{Ti} \#= \mathtt{T1} + \mathtt{Constant} * (\mathtt{i} - \mathtt{1}) \ . \quad (1)$$

Preferences associated with each value in the domain of the time preference variable allow us to express the degree to which any time assignment for a class

Table 1. Maximal sets of the possible combinations of days for class with given number of meetings per week (e.g., TTh means that course can have its meetings on Tuesday and Thursday).

Number of meetings	Possible combination of days
1	M or T or W or Th or F
2	MW or TTh or WF or MF
3	MWF
4	TWThF or MWThF or MTThF or MTWF or MTWTh
5	MTWThF

is preferred or discouraged. The remaining domain variables may be referenced in the hard constraints (e.g., `serialized`), but they do not require the more expensive processing by the soft constraint solver.

Since all class meetings should be taught in the same room, we suffice with only one common classroom variable for all meetings (called the *classroom preference variable*). As a preference variable, it associates a preference with each classroom expressing how desirable or undesirable it is for a given class.

5.2 Hard Constraints

Let us summarize the requirements which are implemented in the system using hard constraints:

1. meeting pattern specification;
2. prohibited or required times for classes;
3. class requires room with sufficient seating capacity;
4. class requires or prohibits some building(s) or room(s);
5. class requires or prohibits classroom of a specified generic type (computer, computer projection, audio recording, document camera, ...);
6. classes taught by the same instructor do not overlap;
7. sections of the same course do not overlap;
8. additional constraints over selected sets of classes: classes must be taught at the same times, on the same days, in the same classrooms, ...

Meeting pattern constraints relate the domain variables for all class meetings as was described in Eqn. 1. In addition, the domain of the time preference variable is reduced such that all invalid values are removed.

Example 2. A 1.5 hour x 2 meetings class is represented by the two variables `T1`, `T2`. The first of these is the time preference variable with the initial domain (`0..104`) reduced to the values `21,24,...,39` because the TTh combination is valid only. The second domain variable is related with `T1` by the constraint `T2 #= T1 + (21*2)*1`. The constant separating start times here is 21 periods x 2 days.

A 2 hours x 2 meetings class has MW, TTh, and WF as valid meeting day combinations. It is represented by the two variables `T1`, `T2` related by the same constraint as before. The reduced domain of `T1` corresponds to the values `0,4,8,12,16, 21,25,29,33,37, 42,46,50,54,58`.

Requirements 2–5 are implemented by domain reduction in the corresponding domains of the time and classroom preference variables. Requirements 6 and 7 are included with help of the constraint `serialized` which constrains tasks, each with a start time and duration, so that no tasks ever overlap. Built-in constraints of CLP(*FD*) library of SICStus Prolog are used to implement various requirements over selected sets of classes as mentioned in the item 8.

Additional hard constraints must be posted to assure that each class is assigned to just one suitable classroom. This requirement could be implemented via the `disjoint2` constraint, which ensures non-overlapping of a set of rectangles. In our case, the rectangle is defined by the start time variable (`Time`) and the duration (`Duration`) of each meeting, and by the classroom variable (`Classroom`) for the corresponding class:

```
disjoint2( [ rectangle(Time, Duration, Classroom, 1) | _ ] ) .
```

The number `1` represents the requirement of *one* classroom for each meeting.

A different type of propagation among the time variables is achieved via the `cumulative` constraint. It ensures that a resource can run several tasks in parallel, provided that the discrete resource capacity is not exceeded. If there are `N` tasks, each starting at a certain time (`StartI`), having a certain duration (`DurationI`) and consuming a certain amount of resource (`ResourceI`), then the sum of resource usage of all the tasks must not exceed resource limit (`ResourceLimit`) at any time:

```
cumulative([Start1,...,StartN], [Duration1,...,DurationN],
           [Resource1,...,ResourceN], ResourceLimit) .
```

The `cumulative` constraint helps to assign a classroom of sufficient size to each meeting while allowing smaller classes to be assigned to larger classrooms.

Example 3. Let us imagine a small example with 2 rooms for 40 students, 3 rooms for 20 students, and 1 room for 10 students. The set of `cumulative` constraints follows

```
cumulative(Time_meetings_with_size_40, Dur_40, ListOf1, 2),
cumulative(Time_meetings_with_size_20_40, Dur_20_40, ListOf1, 5),
cumulative(Time_all_meetings, Dur_all, ListOf1, 6).
```

The first constraint ensures that the largest classes are accommodated into the largest rooms, the second constraint allows medium-sized classes to be placed into rooms for 20 students, and also into rooms for 40 students if they are not already asked for by the first constraint. The third constraint allows movement of small classes between all rooms, subject to the condition that they are not occupied by any larger classes at the same time.

More precisely, we can post one `cumulative(Starts, Durations, ListOf1, Limit)` constraint for each possible size of classroom denoted by `Size`. The constant `ListOf1` denotes a list of 1 representing a unit resource requirement (one classroom) by each course. `Durations` represents the durations of classes with the start time `Starts`. Variables `Starts` and `Limit` should satisfy the following properties

$$\texttt{Starts} = \{\texttt{Start|meeting(Start,Duration,Capacity)} \wedge \texttt{Capacity} \geq \texttt{Size}\}$$
$$\texttt{Limit} = \mathrm{card}\{\texttt{Id|classroom(Id,Capacity)} \wedge \texttt{Capacity} \geq \texttt{Size}\}$$

Actually, it is sufficient to post this constraint only for some specific sizes of classrooms. Classrooms of similar size are grouped together to achieve better efficiency.

Another possibility for taking `cumulative` constraints into account consists of splitting classrooms into *groups* by size. Each class would be included in the group of corresponding size only. Such division can be useful if we do not want to put smaller classes into larger classrooms of other group at any time (e.g., smaller classes must be in the classrooms with a capacity smaller than 400 students).

5.3 Soft Constraints

Three types of soft constraints are currently handled by the system which will be discussed in this section:

1. unary constraints on time variables — faculty time preferences;
2. unary constraints on classroom variables — faculty preferences on the classroom selection for classes;
3. binary constraints for each joint enrollment between two classes.

Instructors may specify preferences for the days, hours, or parts of days they wish to encourage or discourage. This specification is transformed into a list of integer preferences corresponding to the possible start time of each class. We have seen that the initial selection of start times for each class is determined by its meeting pattern. The domain size of this set of start times can differ greatly among meeting patterns (it ranges from 5 to 50 possible values). This causes the relative effect of any given preference to vary greatly among the meeting patterns. To compensate for this effect, the number of preference points associated with instructor time preferences differ based on the meeting pattern.

Each class is associated with a time preference variable with preferences initialized either as specified by the instructor or to a set of default preferences. These default preferences are very important — their exclusion would lead to the construction of timetables which discriminate against classes for which no preferences have been provided. Many such classes would be placed in undesirable times, which no human timetabler would want to do.

Instructors may also specify positive or negative preferences towards the room selection for each class. It is possible to prefer or discourage particular classrooms, buildings, or properties of the room (e.g., “I prefer classrooms with a computer." or "I discourage classrooms without windows."). Each value in the domain of the classroom preference variable has either the specified preference or the neutral preference specification.

Any two classes potentially have a number of students who are enrolled to both at the same time. We seek to control their degree of overlap in the timetable by a generalization of the soft disjunctive constraint (see `soft_disjunctive` in Sec. 3.2). Such binary soft constraints include two time preference variables for corresponding classes, with the cost given by the number of students enrolled in both. Since each class may have several meetings, such a generalized disjunction needs to propagate preferences to the all values of the uninstantiated preference time variable which could be affected by the overlap of any of the meetings.

5.4 Cost Functions

There are two types of cost functions in our problem formulation. The first is related to the assignment of time preference variables. The second is dependent on the classroom preference variables.

Preferences associated with the time preference variables are influenced by the times faculty wish to teach or not teach, and by the soft constraints on joint enrollments. Time preferences are initialized based on faculty input. Constraints on joint enrollments propagate (increase) preferences during computation of the cost function. The sum of the cost variables (see Sec. 3.3) for the time preference variables gives this cost function, i.e., the solution cost wrt. time assignment. As a consequence, we need to balance the number of student joint enrollments from the third constraint with the number of preference points assigned by the first constraint, (e.g., the sum of preference points associated with an instructor for one class corresponds to an overlapping of 20 students). The relationship between preference points and joint enrollments is specified as part of the input data.

The sum of the cost variables related with the classroom preference variables represents the second cost function. Its value is dependent on the satisfaction of faculty preferences on classroom selection.

Both cost functions are independent of each other and introduce two different criteria in our problem. Minimizing student conflicts and accommodating the time preferences of classes were judged to be a more critical aspect of the problem than meeting preferences for classrooms.

5.5 Labeling

Labeling consists of two parts. Time preference variables are processed first, followed by room preference variables. This ordering corresponds to the relative importance of the cost functions defined over particular sets of variables (see Sec. 5.4).

One iteration of the LAN search is applied to find a partial assignment of time variables. A branch & bound algorithm is then used to find an optimal solution over classroom variables. If no solution for classroom variables is found within the time limit, one iteration of the LAN search over the classroom variables is processed. Once a partial assignment is generated by this process, the search for a complete solution may continue by repeating these steps. The user may also provide input to influence the behavior of the search (see Sec. 4).

Initial and repair searches using the LAN algorithm over time variables are processed as described in Sec. 4. The initial search is always processed for classroom variables however (i.e., the limit on the number of assignments is set but new heuristics are not developed). Information about an unassigned classroom variable reflects back upon the corresponding time preference variable. Any classroom variable non-assignment is the result of no classroom being available for the corresponding time. To reflect this fact, the value assigned to a time variable with a corresponding unassigned classroom variable is discouraged in the subsequent search.

Different initial heuristics were used for time and classroom variable labeling but the main idea remains always the same: a variation of first-fail was applied for variable ordering and we have chosen the most preferred values.

Our first-fail heuristics to determine the ordering of classes for time assignment selects the most highly constrained variables with respect to both hard and soft constraints. First, we select among the variables having the smallest domain. Ties are broken based on the greatest number of soft constraints related to this variable. If not selected early, the domain of such a variable may become too small to select a sufficiently preferred value, or it may even become empty and cause a backtracking. Early propagation of soft constraints is also encouraged, so as not to discover mistakes too late. A specific class time assignment was selected among the most preferred values, i.e., we have chosen an optimistic approach for the value selection.

Just the first-fail approach was used to choose a class to be placed into a classroom. The most preferred classroom was selected and ties were broken by the selection of the smallest available classroom so as not to waste available resources.

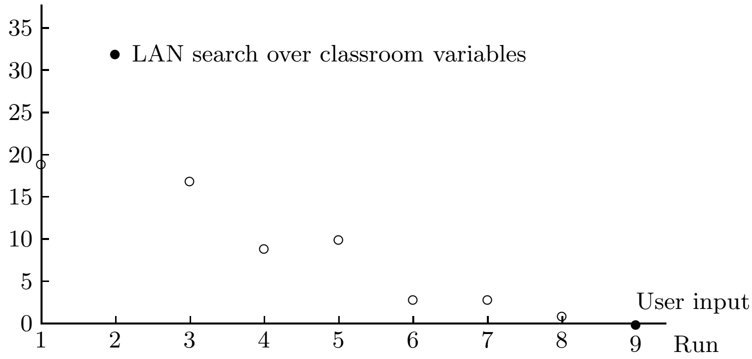
6 Computational Results

Our data set from fall semester 2001 includes 747 classes to be placed into 41 classrooms. The classes included represent 81,328 course requirements for 28,994 students. A complete data set in the form of Prolog facts can be downloaded from `http://www.fi.muni.cz/~hanka/purdue_data`. In the future, we inted to add data from other semesters.

The results presented here were computed by SICStus Prolog 3.9.1 on a PC with AMD Athlon/850 MHz processor and with 128 MB of memory.

Figure 1 illustrates the number of classes with either time or classroom variables left assigned during subsequent iterations. It can bee seen that the automated repair search was able to substantially improve on the initial solution. Only one class remained unassigned after eight iterations. Assignment of this class was successfully completed with the help of user input. The increase in the

Fig. 1. Change in unassigned classes during the subsequent iterations.



number of unassigned classes during the second iteration occurs as a result of the branch & bound search over classroom variables being replaced by the initial iteration of the LAN search.

Table 2 shows the computational results for the initial search and the subsequent automated repair searches. *Satisfied time* gives the percentage of how many encouraged times for classes were selected. *Unsatisfied time* refers to the percentage of discouraged times that were selected for classes. *Student conflicts* estimates the percentage of unsatisfied requirements for courses by students. *Preferred classrooms* measures the percentage of classes for which encouraged classrooms were selected. The current data set does not include any preferential requirements which discouraged specific classrooms.

Table 2. Results of particular iterations.

Run	1	2	3	4	5	6	7	8	9
Satisfied time (%)	83.8	81.8	79.9	82.2	80.7	80.7	80.6	80.7	80.7
Unsatisfied time (%)	4.2	4.6	4.4	4.5	4.5	4.5	4.2	4.2	4.5
Student conflicts (%)	1.5	1.6	2.0	1.9	2.1	2.2	2.3	2.3	2.3
Preferred classrooms (%)	74.2	45.3	52.1	47.0	52.7	56.3	49.7	52.3	52.3

We can see that most of preferential requirements of instructors were satisfied during assignment of time variables. The unsatisfied time percentage mostly illustrates that a number of classes must be taught at unpopular times due to the limited number of rooms available. Let us also note that these results include default preferences for classes with no preferred times.

One iteration took about 2-3 minutes for time labeling. One step of the branch and bound search for classroom variables took 1-2 seconds. The time labeling takes longer due to the preference propagation and more complex constraints (e.g., `cumulative`) posted on time variables.

Originally we started to solve the problem using a built-in backtracking search algorithm with a variety of variable and value ordering heuristics. This attempt did not lead to any solution after 10 hours of computations however. Too many failed computations were repeated exploring parts of search tree with no solution.

One of the more important lessons learned here is that the `disjoint2` and `cumulative` constraints must be used together in a redundant manner to find an acceptable solution (for description of both constraints see Sec. 5.2). The `cumulative` constraints were able to introduce additional constraint propagation for the time variables by informing them of the available room resources. Neither `cumulative` nor `disjoint2` constraints alone were able to find an acceptable solution. Results using only the `cumulative` constraints left about 20 unassigned classes. Using only the `disjoint2` constraint resulted in 50 unassigned classes.

7 Related Work

Currently the timetable for Purdue University is constructed by a manual process. An earlier approach examined for automating construction of the Purdue University timetable modeled the room-time assignment problem as a multiple choice quadratic vertex packing and utilized a tabu search algorithm [20]. This approach was further developed into a prototype system used to create a schedule for large lecture classes in spring 1994, but was never adopted by university schedulers due to inadequacies in the way it handled instructor time preferences and student conflicts.

7.1 Soft Constraints

Various approaches to soft constraints have been introduced and studied [23]. These include basic frameworks over particular types of preferences (e.g., weighted [10], fuzzy CSPs [9]) and also meta-frameworks (e.g., partial [10], semiring-based CSPs [5]). Solving algorithms for soft constraints include extensions of the branch& bound algorithm [10, 19] and local search methods, e.g., tabu search [12]. However, there are still few tools [4, 13] available for soft constraint solving. To these authors' knowledge, no library can be used together with an existing CLP(*FD*) solver [6, 17]. This paper presents the main ideas behind our proposal for a new soft constraint solver built on top of the SICStus Prolog CLP(*FD*) solver [6]. We have further extended this work. A detailed description can be found in [24].

Soft constraints are often applied to solve timetabling problems with the help of constraint satisfaction. Unfortunately they are mostly applied via the standard constraint satisfaction method, which offers no special support for more effective resolution of the soft constraints. Golz et al. [14] applies the typical solution — the given unary soft constraints, with priorities, are integrated into the solution search through value and variable ordering heuristics. An optimization constraint was applied for solving medium-sized problems [16]. Support for soft constraints is included in Abdennadher et al. [1]. They describe the solution of a department-sized problem, including soft constraint solver, for a cost-based approach implemented with Constraint Handling Rules [11].

7.2 Constraint Programming and Demand-Driven Timetabling

There have not been many attempts [25, 3] to apply constraint programming to the solution of demand-driven timetabling problems where it is not possible to satisfy all requests of students. Such over-constrained problems require enhancements to the classical constraint satisfaction approach. Once these are developed, we can apply all of the advantages of constraint programming, including a declarative description of the problem together with strong propagation techniques.

We have previously constructed a demand-driven schedule for the Faculty of Informatics at Masaryk University [25] having 270 classes and about 1250 students. Conflicts of students between classes were controlled using a similar cost function as in our current approach. Constraint logic programming allowed implementation of a variety of constraints available in ILOG Scheduler [21]. Soft constraints were implemented with the help of special variable and value ordering heuristics defined by the preferences of particular variables in the constraints. The timetable constructed was able to satisfy 94 % of the demands of students and more than 90 % of the preferential requirements of teachers. Unfortunately, it was not easy to extend this implementation to larger problems due to the bound consistency algorithms in ILOG Scheduler. Since these algorithms only propagate changes over the bounds of the domain variables, both constraint and preference propagations were much weaker than there are now.

A solution of the section assignment sub-problem is included in Banks, van Beek, and Meisels [3] via iterative addition of constraints into a CSP representation. Inconsistent constraints are not included in the final CSP representation, which allows solution of an over-constrained problem. This implementation, including its own constraint satisfaction solver, was verified using random timetabling problems based on problems from high schools in Edmonton, Alberta, Canada. The largest data set included requirements of 2,000 students and 200 courses. They were able to satisfy 98 % of student demand on more than half of the experiments. The solution presented is influenced by a special set of times that must be assigned to each course. It conforms well to high schools, but is rather different from the situation in university course timetabling. University class meeting patterns are not as strict, which results in a problem with a variety of additional requirements and preferences. Their inclusion into the problem solution can be more easily accomplished with the help of the embedded CLP system rather than implementing new procedures.

7.3 Other Approaches for Demand-Driven Timetabling

The comprehensive university timetabling system described by Carter [7] is characterized by problem decomposition with respect to both type and size of final sub-problems. They were able to solve the problem for 20,000 students and 3,000 course sections. The system was used for 15 years at the University of Waterloo.

Aubin&Ferland [2] propose an iterative heuristic method to solve the problem which alternately assigns times and students to course sections until no further improvements to the solution can be found. The system was tested on data from a High School in Montreal including demands of 3,300 students and 1,000 courses.

Robert&Hertz [22] decompose the problem into a series of easier sub-problems corresponding to time, section, and classroom assignments and solve them via tabu search methods. The method presented is able to generate an initial solution which can be incrementally improved after problem redefinition (negotiation on initial constraints with teachers and students). The initial solution for about 500 students and 340 course sections satisfied about 80 % of the preferential requirements of teachers. Forty-five students were involved in overlapping situations.

The local search heuristic procedure of Sampson, Freeland, and Elliot [26] solves a problem having a smaller solution space with 89 course sections and 230 students. They were able to meet 94 % of the student scheduling requirements at the Graduate School of Business Administration at the University of Virginia.

8 Conclusion

We have proposed and implemented a solution to a large scale university timetabling problem and have constructed a demand-driven schedule which is able to reflect diverse requirements of students during course enrollment. Our solution is able to satisfy the course requests of 98 % of students. About 80 % of preferential requirements on time variables were also met with only a small number of classes taught at discouraged times (about 4 %). The automated search was able to find suitable times and classrooms for almost all classes. One remaining class was assigned with the help of the built-in support for user input.

Our proposal included a new solver for soft constraints which is of particular interest for timetabling problems where the costs in the problem are directly related to the present values for time and room assignments of classes. We have proposed a new search algorithm which allows us to find a solution to the problem (even when it is over-constrained). We have also discussed a special set of `cumulative` constraints which, together with the `disjoint2` constraint, processes stronger constraint propagation.

Our future research will include an extension of the problem solution together with improvements to the soft constraint solver and search algorithm that have been described. Also, our approach must be validated using data sets from other semesters.

We are working on improvements to the search algorithm originally proposed for Purdue timetabling. First results, including experiments on random problems, can be found in [29].

Our current work also concerns pattern-oriented heuristics. They are aimed at improving the solution with the help of a pattern matching mechanism based on the sets of meeting patterns in the problem.

A new approach for making initial student section assignments for courses with multiple sections is currently under development. The proposal and implementation is based on Carter's [7] homogeneous sectioning, which tends to result in fewer classes having joint enrollments with others. This simplifies the task of finding non-conflicting assignments and appears to be an accurate representation when a final sectioning process will take place after construction of the timetable.

Purdue University currently relies on a completely manual process for constructing its timetable. A detailed comparison of results between the approach described in this paper and the manual process for the full large lecture problem is one of the next steps in our work. However, timetables are under continual revision by the timetablers who try to detect any possible problems in the generated solution. This allows us to extend the system and adjust it to be accepted by the community.

The primary focus of our future work will be support for making changes to the generated timetable. Once timetables are published they require many changes based on additional input. These changes should be incorporated into the problem solution with minimal impact on any previously generated solution.

We feel that the solution methods used for the large lecture problem should be directly applicable to construction of the 74 academic unit timetables. Some solution refinements may be necessary to simplify time assignments for introductory courses with large numbers of sections. Additional system architecture work will also be necessary to allow unit timetablers to use local preference files, and to work cooperatively if there is a high degree of interrelationship between classes offered by the units. The administration of the university feels this work is very promising and has funded continuing work based on this approach.

Acknowledgements

This work is partially supported by the Grant Agency of Czech Republic under the contract 201/01/0942 and by Purdue University.

We would like to thank our students Vlastimil Křápek, Aleš Prokopec, and Kamil Veřmiřovský, who are assisting with the solution of this problem, and Purdue staff who have helped in many ways.

Our thanks also go to the Supercomputer Center Brno where experiments with the search algorithm were conducted.

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