Chapter 5

Markov processes

Exercise 5.1

Show for an irreducible, aperiodic Markov chain with n states and doubly stochastic transition matrix P, that the limiting probability is given by $v_j = 1/n$ for all j.

Exercise 5.2

Bernoulli trials — We say, that the process is in the state x_i if trials $n-1$ and n resulted in r_i according the the following table.

i	r_i
1	SS
2	SF
3	FS
4	FF

Find the transition probability matrix P and all its powers.

Exercise 5.3

Consider a sequence of Bernoulli trials. For $i \geq 2$ we define $\mathbf{X_i} = x_1$ if trials $i-1$ and i both resulted in success and $\mathbf{X_i} = x_2$ otherwise. Is the sequence $\mathbf{X_2}, \mathbf{X_3}, \ldots$ a Markov process?

Exercise 5.4

Consider a process $X_1, X_2, \ldots$ such, that $X_n = x_j$ if j is the highest result achieved by the first n throws of a dice. Find P^n and verify that

$$p_{m+n}(k \mid j) = \sum_v p_m(v \mid j) p_n(k \mid v).$$

Exercise 5.5

Let us consider a sequence of Bernoulli trials represented by a sequence of success/failure outcomes. This is equivalent to the string of the form $\{S, F\}^*$. This forms a Bernoulli process. We define a new transformed process based on the original Bernoulli process:

We say, that the (transformed) process (in the $(n-1)$th trial) is in the state

x_1 if trials $n-1$ and n of the Bernoulli process resulted in	SS
x_2	SF
x_3	FS
x_4	FF.

Is the transformed process a Markov process? Find the transition probability matrix P and all its powers.

Exercise 5.6

Let us consider a sequence of Bernoulli trials represented by a sequence of success/failure outcomes. This is equivalent to the string of the form $\{S, F\}^*$. This forms a Bernoulli process. We define a new transformed process based on the original Bernoulli process:

We say, that the (transformed) process (in the $(n-1)$th trial) is in the state

x_1 if trials $n-1$ and n of the Bernoulli process resulted in SS
x_2 otherwise

Is the transformed process a Markov process?

Exercise 5.7

Let us consider a process $X_1, X_2, \ldots$ such, that $X_n = x_j$ if j is the highest result achieved in the first n throws of a dice. Find the transition matix P^n and verify that

$$p_{m+n}(k|j) = \sum_v p_m(v|j)p_n(k|v).$$