

Chapter 4

Random walks and gambler's ruins

Exercise 4.1

A gambler having 15 euro is playing a roulette, betting always one euro to number 1. What amount of money is he expected to own after 15 rounds, given that.

- He is playing American roulette in which there are numbers 0, 00, 1, 2, \dots , 36 and payout is 35.
- He is playing the classical roulette (i.e. the number 00 is missing).

What is the probability that the player will have less than 14 euro after 15 rounds? What is the probability that he will lose all his money in 15 rounds?

Exercise 4.2

Let us suppose that gambler has 10 euro, and adversary has 90 euro. They repeatedly throw a 6-sided die. If numbers 1 or 2 occur, gambler wins one euro, otherwise he loses one euro. What is a probability that gambler wins all adversary's money? What is the expected duration of the game? What if we change rules so that gambler wins if 1 to 3 occur and loses if 2 to 6 occurs? And what if he wins if 1 to 4 occur?

Exercise 4.3

Let us consider games from previous example and let us assume that adversary's funds are infinite. Compute the probabilities that gambler loses all his money and expected durations in all cases.

Exercise 4.4

Consider a drunken person who is just on a half way from a pub to his house. He is trying to go straight to the house, but with probability $49/100$ he loses balance in each step and makes one step back to the pub. What is the probability that he will return to the pub given that he is 100 steps far from his house. How many steps is he expected to do before he enters either the pub or the house?

Exercise 4.5

You are offered to play one of the following games:

- You start with 1 euro and play against the adversary who has 10 euro. Adversary tosses a coin and when head occurs, you win 1 euro. Otherwise, you lose one euro. The game end when you or adversary lose all money.
- You start with 2 euro and play with same adversary, except that now adversary throws 20-sided dice and you win 1 euro if numbers 1 to 9 occur and lose otherwise.
- You start with 2 euro and play with same adversary, except that now adversary throws 20-sided dice and you win 1 euro if numbers 1 to 11 occur and lose otherwise.

Which of the games would you choose?

Exercise 4.6

A drunken person leaves has left the pub. After a while he decided to return. He makes one step towards the pub with probability $6/10$ and he loses balance and goes to opposite direction with probability $4/10$. What is a probability that he will eventually go back to the pub, given that he is only 20 steps away. What is the expected number of steps he will do.

What if the person does remembers the direction to the pub incorrectly (i.e. we switch the two probabilities)?

Exercise 4.7

A gambler having 2 euro plays against adversary having 3 euro. With prob r the gambler loses 1 euro and with prob $1 - r$ he wins one euro. Compute r so that the probability of losing all money in the game is $1/2$.

Exercise 4.8

A gambler makes a series of bets of \$1. He decides to quit betting as soon as his net winnings reach \$25 or his net losses reach \$50. Suppose the probabilities of his winning and losing each bet are both equal to $1/2$.

1. Find the probability that when he quits he will have lost \$50.
2. Find his expected loss
3. Find the expected number of bets he will make before quitting.