# A Tighter Insertion-based Approximation of the Graph Crossing Number

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joint work with **Markus Chimani**

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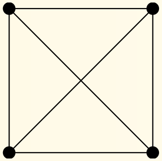
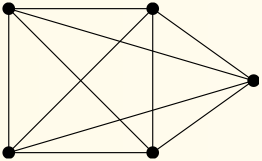
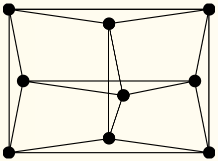
# 1 Graph Crossing Number

**Definition**. *Drawing of a graph* $G$:

- The vertices of $G$ are distinct points, and every edge $e = uv \in E(G)$ is a simple curve joining $u$ to $v$.
- No edge passes through another vertex, and no three edges intersect in a common point.







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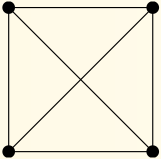

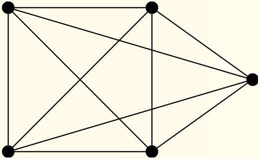

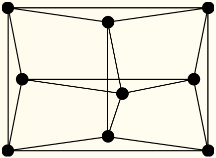

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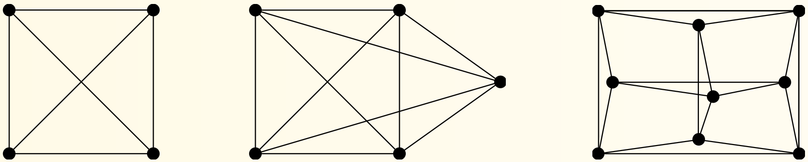

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is the smallest number of edge crossings in a drawing of $G$.

**Warning.** There are slight variations of the definition of crossing number, some giving different numbers! Such as counting *odd-crossing pairs* of edges. [Pelsmajer, Schaeffer, Štefankovič, 2005]...

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- Even fixed *rotation scheme*; [**Pelsmajer, Schaeffer, Štefankovič**, 2007]
- Much worse – hard already for planar graphs plus one edge !
  [**Cabello and Mohar**, 2010]

Can anything be computed efficiently?

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  [**Even, Guha and Schieber**, 2002]
- Constant factors for surface-embedded bounded-degree graphs;
  [**Gitler et al**, 2007], [**PH and Salazar**, 2007], [**PH and Chimani**, 2010]

# 2 Planar Insertion Problems

**Definition**. Given a planar graph $G$ and a set $F$ of additional edges (vert.?).
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**Remark.** Difficulty of insertion problems comes from possible inequivalent embeddings of $G$.

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- Multiple edge insertion $\leftrightarrow$ graph $G+F$ (a very general case)
  - *cr*$(G+F)$ approximated by *ins*$(G,F)$;
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  - however, *ins*$(G,F)$ is NP-complete! (as well as finding $F$)

# 3 Approximating MEI up to Additive Factor

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  Using MEI, a solution to $cr(G+F)$ for given planar $G$ and $F$, with

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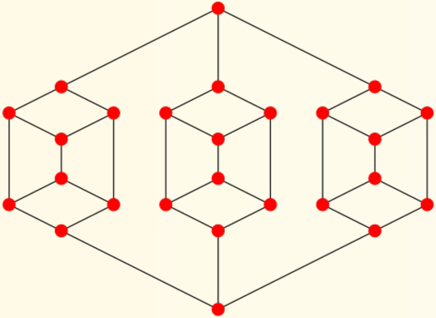
**Theorem 1.** *Given a conn. planar graph $G$ and an edge set $F$, $F \cap E(G) = \emptyset$, Algorithm 2 described below finds, in*

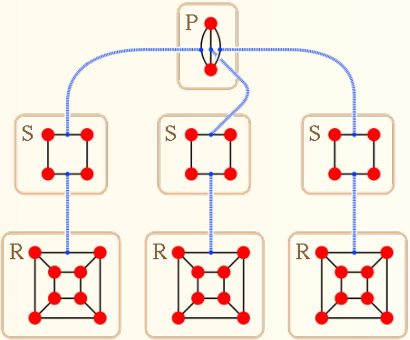
$$O(|F| \cdot |V(G)| + |F|^2) \quad \textit{time,}$$

*an approximate solution to the MEI problem for $G$ and $F$ with*

$$\leq \; ins(G,F) + \left(\lfloor \tfrac{1}{2}\Delta(G) \rfloor + \tfrac{1}{2}\right) \cdot (|F|^2 - |F|) \quad \textit{crossings.}$$

# Gentle introduction to SPQR trees






- Graph broken into the *blocks* first.
- Then, for pairwise gluing on *virtual skeleton edges*, we have got
  - *S-nodes* for serial skeletons,
  - *P-nodes* for parallel skeletons,
  - *R-nodes* for $3$-connected components.

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   and centrally store their *embedding preferences*.
3. Fix an *embedding $\Gamma$ of $G$* by suitably combining the embedding preferences from step 2 (at least "one happy con-chain per node").
4. Independently compute the *insertion paths* for each edge $e \in F$ into the fixed embedding $\Gamma$, as shortest dual paths.

## Proof sketch

A very informal one, neglecting all technical obstacles. . .


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- Two con-chains can split/merge twice, hence $\leq 2\binom{|F|}{2}$ dirty passes.
- Every dirty pass is associated with a $1$- *or* $2$-*cut*, and the inserted edge needs $\leq \lfloor \Delta(G)/2 \rfloor$ crossings to "pass by" it. Altogether

$$\leq \mathit{ins}(G,F) + \left(2\left\lfloor\frac{\Delta(G)}{2}\right\rfloor + 1\right)\cdot\binom{|F|}{2}. \qquad \square$$

# 4 Consequences

**Theorem 3.** *Given a planar graph $G$ and an edge set $F$, $F \cap E(G) = \emptyset$, Algorithm 2 finds an approximate solution to* cr$(G+F)$ *with*

$$\leq \lfloor \tfrac{1}{2}\Delta(G) \rfloor \cdot 2|F| \cdot \mathit{cr}(G+F) + (\lfloor \tfrac{1}{2}\Delta(G) \rfloor + \tfrac{1}{2})(|F|^2 - |F|) \ \ \textit{crossings.}$$

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- Can the MEI $(G, F)$ problem have, say, an FPT algorithm wrt. $|F|$?