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A note on multicriteria decision making

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1 Examples I

Distance	
Hotel	distance_value
H1	200
H2	300
H3	400
H4	500
H5	1000

Price	
Hotel	price_value
H1	2500
H2	1000
H3	500
H4	1600
H5	750

YOC	
Hotel	yoc_value
H1	1990
H2	1999
H3	1989
H4	2003
H5	2000

U1_Close	
Hotel	close_score
H1	0.8
H2	0.7
H3	0.6
H4	0.5
H5	0

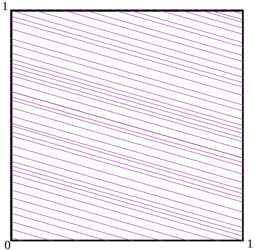
U1_Cheap	
Hotel	cheap_score
H1	0
H2	0.5
H3	0.75
H4	0.2
H5	0.625

U1_New	
Hotel	new_score
H1	0.25
H2	0.7
H3	0.2
H4	0.9
H5	0.75

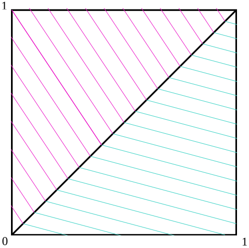
$$\text{U1\_Good\_hotel}(h) = \frac{1}{6}(3 \times \text{h\_Close} + 2 \times \text{h\_Cheap} + \text{h\_New})$$

Various models of combination functions

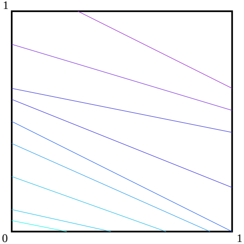
weighted average



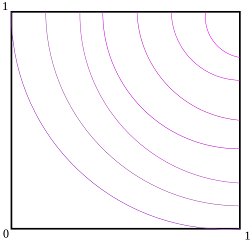
Choquet integral



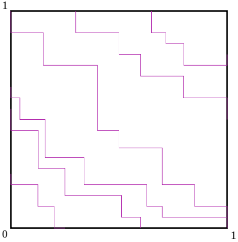
combination of different



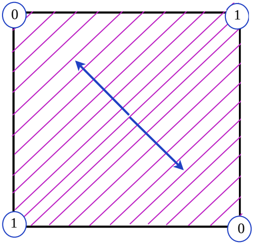
metric, closest to ideal is the best



rules from IGAP



XOR-like, no monotone combin.

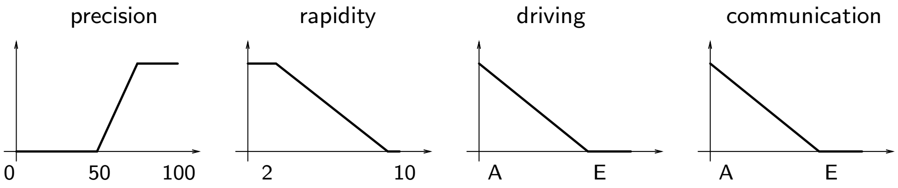


2 Examples II: Grabisch–Roubens

Performances of the different trainees

name	*precision (%)*	*rapidity (tu)*	*driving*	*communication*
Arthur	90	2	B	D
Lancelot	80	4	B	B
Yvain	95	5	C	A
Perceval	60	6	B	B
Erec	65	2	C	B

Scores on the different criteria



Numerical scores on criteria

name	*precision*	*rapidity*	*driving*	*communication*
Arthur	1.000	1.000	0.750	0.250
Lancelot	0.750	0.750	0.750	0.750
Yvain	1.000	0.625	0.500	1.000
Perceval	0.250	0.500	0.750	0.750
Erec	0.375	1.000	0.500	0.750

Ranking of the five trainees

name	*class*	*rank in the class*
Arthur	bad	2
Lancelot	good	1
Yvain	good	2
Perceval	bad	1
Erec	average	1

Mapping from class and rank to $[0,1]$

class	*interval for the global score*
good	[0.75, 1.0]
average	[0.4, 0.75]
bad	[0.0, 0.4]

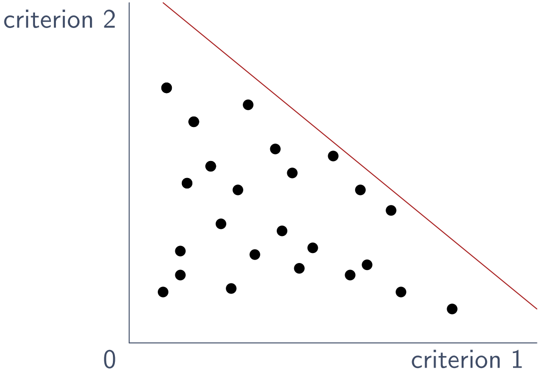
Numerical data on criteria and global performance

name	*precision*	*rapidity*	*driving*	*communication*	global
Arthur	1.000	1.000	0.750	0.250	0.133
Lancelot	0.750	0.750	0.750	0.750	0.917
Yvain	1.000	0.625	0.500	1.000	0.833
Perceval	0.250	0.500	0.750	0.750	0.276
Erec	0.375	1.000	0.500	0.750	0.575

name	*precision*	*rapidity*	*driving*	*communication*	global 2nd
Arthur	1.000	1.000	0.750	0.250	0.3
Lancelot	0.750	0.750	0.750	0.750	0.75
Yvain	1.000	0.625	0.500	1.000	0.7
Perceval	0.250	0.500	0.750	0.750	0.35
Erec	0.375	1.000	0.500	0.750	0.5

3 A bit of geometry

- Mathematical side: **explicit formulation**,
  vague criteria → number scores → a vector in $\boldsymbol{R}^c$.

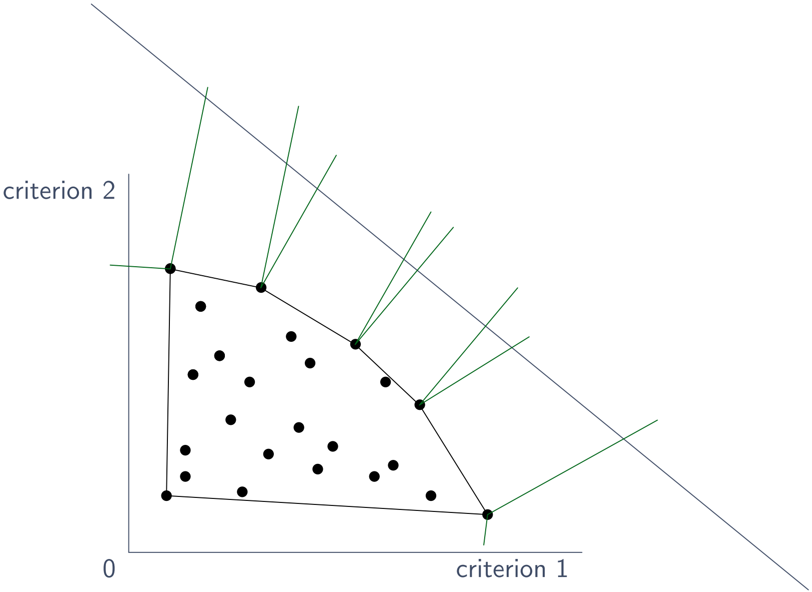


  – We have (relatively) plenty of time for preprocessing. . .

- Computing side: **answering a query**,
  a customer comes with his subjective *cost* (preference) function.
  – This has to be done very fast!

Polyhedral combinatorics approach

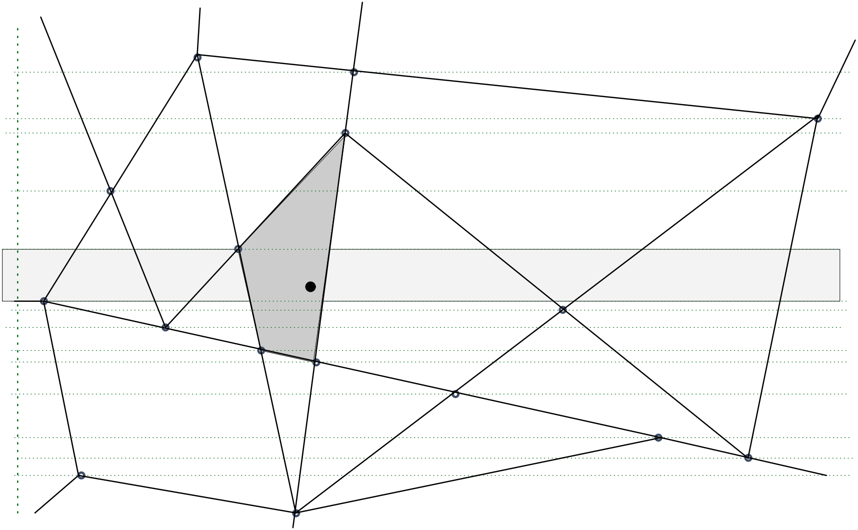
only for linear cost functions (modeled by parallel hyperplanes).



Using "cones of optimality" for each vertex of the convex hull
→ logarithmic binary search!

Extending to 3 criteria

Vectors in 3D → "polar" polyhera in 2D → an involved logarithmic search again:



Ongoing research

- What if the optimal value of a criterion is in the "middle"?

  $\rightarrow$ dividing into fixed qadrants, or user-specified pikes (additional dimensions)...

- Wanting more than one best answer?

  $\rightarrow$ heuristic local search (dual!).

- How to make better geometric structure for k nearest neighbours search (for small k's)?

- What about using Voronoi diagramas to approximate the search? [J. Mareček]