M A C E K   1.2.+

Petr Hliněný

hlineny@fi.muni.cz, hlineny@member.ams.org

http://www.mcs.vuw.ac.nz/research/macek/

http://www.fi.muni.cz/ hlineny/MACEK/

Acknowledgement

Please cite Macek properly (via web page) in all your work where you use it.

Table of Contents

1 Overview of Macek

The *Macek Project* has been developed primarily for math researchers in matroid theory. (If you do not know what matroid theory is, then the package is likely not for you.) This project is intended both to help with usual tiresome matroid routines, and to allow for long exhaustive computations over matroid classes. We suggest potential users to read the book [J.G. Oxley, Matroid Theory, Oxford University Press 1992]. The project main web page with recent updates can be found at `http://www.mcs.vuw.ac.nz/research/macek/`, or a current page `http://www.fi.muni.cz/~hlineny/MACEK/`.

The Macek package deals mainly with matroids represented by reduced matrices over finite fields and partial fields. (I.e. matrix representations in Macek are always stripped of the leading identity submatrix! See Section 2.5 [Represmatr], page 8.) Many common matroids are distributed with the program, and new ones may be easily entered. There are various tools for handling matroids, their matrices, and sets of matroids. One may pivot matrices, delete or contract matroid elements, and generate 3-connected extensions for matroid representations. Structural tests for minors, equivalence, connectivity, branch-width, girth, etc, are also provided in the package. From 1.1.9, limited capabilities for computation with "abstract" matroid properties, like isomorphism, flats, aut group, and representability over other fields, are added.

You may read about the theoretical background of the *Macek Project*, and about its use in [P. Hlineny: Using Computer in Matroid Theory Research, Acta Math. Univ. M.Belii 11 (2004), 27-44, `http://actamath.savbb.sk`]. Other papers dealing with Macek are, for example, [P. Hlineny: Equivalence-Free Exhaustive Generation of Matroid Representations, Discrete Appl. Math. 154 (2006), 1210-1222, `http://dx.doi.org/10.1016/j.dam.2005.12.001`], [P. Hlineny: On the Excluded Minors for Matroids of Branch-Width Three, Electr. J. of Combinatorics 9 (2002), R32, `http://www.combinatorics.org`], or [P. Hlineny: Combinatorial Generation of Matroid Representations: Theory and Practice, Acta Math. Univ. M.Belii 12 (2005), 31-41, `http://actamath.savbb.sk`].

However, there are certain limitations to computing power of some Macek functions — following from the fact that *represented matroids* are handled in the program. For example, one matroid may have many inequivalent representations over larger fields, and they behave differently with respect to extensions. So be careful and read the documentation thoroughly.

Citations!   Please cite the Macek project in all your papers in which you use results and/or examples obtained with Macek. In the bibliography reference please provide the author's and program names and exact Macek version, and the web address http://www.mcs.vuw.ac.nz/research/macek/. Keep in mind that versions with odd minor number (the number after the first dot, like 1.1.x – a usual GNU convention) are unstable development releases intended for testing purposes, not for serious computations. Please let me know by email if you have successfully used Macek in your scientific research. Thank you.

More functions are (still) planned for the future...

1.1 Using Macek

The Macek package can be used on (almost) any computer platform with the GNU C compiler, relevant C libraries and development environment, and the basic set of GNU utilities. All these required programs are free for anybody – see `http://www.gnu.org`. More about supported platforms is in Section 2.1 [Installing], page 5. (No official support for M\$ Windows is provided; however, there are no major design reasons why the program would not compile and work there after few adjustments. The program has been reported to work under Cygwin.)

When considering the Macek program, do not expect anything like a graphical user-interface. The program provides only a rich command-line (non-interactive) interface, with some basic internal scripting capabilities. (Given the nature of the program, and also that of matroids, a graphical interface would not be of much use, anyway.) So forget your mouse and use the keyboard! For more complicated batch-computations, you should call the Macek program within a suitable external scripting language, like a unix shell.

Many of the computations in Macek are exponential, even in their nature, and so they may take quite long time. (And some other computations take long time simply because the author was too lazy to implement them efficiently.) However, usually matroids on up to about 20 elements can be handled in reasonable computing time on a usual modern PC computer. The memory requirements of the program also grow exponentially for certain computations, but this should not cause serious problems on modern computers with 128MB or more.

1.2 About Version 1.2

The Macek project started during 2001, and the first stable public release was with version 0.8 in March 2002. The minor version 0.8.2 is the last one in that branch. The following development branch 0.9.x have brought many updates and speed-ups to the program, and have resulted in a new stable branch 1.0 in August 2002.

The main improvements in version 1.0 are these:

- More information about matroid properties, now also including connectivity. Section 6.2 [Printing], page 34.

- Cleaner and faster algorithms are now used for various structural functions, mainly for minor and equivalence testing. Section 6.6 [Structural], page 37.

- Better organized canonical check in extension generating, which makes extensions of large matrices much faster. Section 6.8 [Generating], page 40.

- Some limited program-flow capabilities for internal scripting in Macek. Section 6.10 [Com-Flow], page 44.

- Confusing atomic matrix-entry expressions like $a - 1$ in '`-a-1`' are no longer scanned from the input. That means, '`-a-1`' is read really as $-a - 1 = -(a + 1)$. Unfortunately, this may cause some problems when reading matrices written by an earlier version of Macek over partial fields like $NREG$ or $GMEAN$. Section 4.2 [Matrix-Entry], page 20.

After having released the last minor version 1.0.2 in the beginning of 2003, work on a new development line 1.1.x had started. The 1.1.x development was frozen in July 2003 with version 1.1.9, and a turn into the stable version 1.2 happened in February 2005, after long testing and bug-fixing.

The main changes in version 1.2 are these:

- Command-line options have been added to set file search paths, extensions, etc. Moreover, a switch for a "safe-execution" version has been added. Section 3.1 [Command-line], page 13.

- There are two important new functions testing abstract matroid isomorphism (even between matroids represented over different fields), and generating matroid representations over other fields. These functions are still implemented quite inefficiently, but they are reasonably fast for practical use. Section 6.7 [Isomorphism], page 39.

- More structural information can be printed out about a matroid (like the aut group), and also written to a matroid file. Section 6.2 [Printing], page 34; Section 6.3 [FWriting], page 35.

- Printed output of many commands has been adjusted to be more practical. In some situations, less is printed out, but one may obtain additional information by using '`!verbose`'.

- Listings of all bases or all circuits in a matroid can be obtained now. Section 6.2 [Printing], page 34.

- There are few minor changes in the commands generating matrix extensions. Section 6.8 [Generating], page 40. In particular, one may now generate only those extensions starting with specified entries, for the case when there are too many extensions even in a single step.

Already since 2005 some new enhancements have been added, in subversions 1.2.09 till 1.2.11. The main addition is a possibility to handle lower-than-3-connected matroids in extension generation. The related options are '`@ext-connected`', '`@ext-cosimple`', '`@ext-simple`', '`@ext-3connected`', and new supplementary connectivity-testing commands are like '`!issimple`', '`!iscosimple`'. These additions are not yet included in this documentation, but their use is fairly trivial. See, for instance, [P. Vymola: Computer-assisted enumeration of small matroids (in Czech only), MSc. Thesis, FI MU Brno, `http://is.muni.cz/th/60663/fi_m/thesis-final.pdf`] for use of these extensions, and Section 8.1 [Reliability], page 61 for nice consequences.

2 Quick-Start

This chapter contains installation instructions for the Macek package, and a brief introduction to using the program. It is intended for those who want quickly see the program in action, without reading the long manual first. Consult also the file 'doc/QUICKSTART' for up-to-date information. We provide examples demonstrating the basic parts and concepts of the program. However, if you want to understand these examples (and the program itself) in depth, then there is no other way than to read the whole manual...

2.1 Installation of Macek

We briefly describe how to install the Macek package on your computer. This description is prepared mainly for skilled computer users. If you have problems following these instructions or obtaining the required GNU programs, better ask your computer specialist. No knowledge of matroids is necessary to install the package.

The Macek package can be used on almost any computer platform with the GNU C (`gcc`) compiler, relevant C libraries and development environment, and the basic set of GNU utilities (at least `gmake`, `flex`). All these required programs are free for anybody – get them from `http://www.gnu.org` or other mirrors. No C++ compiler/development is necessary. In particular, Linux and other unix-clones, with GNU C development and GNU utils installed, would work fine. The list of the current "officially" supported platforms can be found in the file 'doc/SUPPORTED' of the Macek package.

First create a subdirectory './macek', copy the package archive to it, and unpack the archive with tar / gzip. To obtain a current documentation, read the files in the 'doc' subdirectory, or type 'gmake info' in the top directory. Then compile the program with 'gmake compile' in the top directory. You may also run 'gmake all' and 'gmake xall' in the 'src' subdirectory, with the same effect. Keep in mind that you *really have* to use the GNU version of `make` – `gmake`, not the ordinary version.

If the compilation is successful, then the resulting two executables './macek' and 'macek.nodebug' appear in the 'src' subdirectory. You may also look at the file 'src/Make.local' which contain local modifications to the project makefiles. Various useful things may be set/modified there, but these require detailed knowledge of your computing platform. When you get into troubles, Section 8.2 [Troubleshooting], page 62 may help...

2.2 Run the Program

Run the ./macek executable from the 'exe' subdirectory of the package, where it is symlinked from. We also suggest to run the program on a terminal with at least 100 columns, and better with 132 columns. Try first to call './macek -h' to see a simple online help. Follow the instructions to obtain more online help.

The presented Macek examples are called from the `bash` shell. However, similar shells like `zsh`, `ksh` should work in the same way. If you want to use `csh` or others, you may need to modify or escape active characters in the commands. (In particular, it looks like '!' is active in `tcsh` even when it is quoted. So if you do not find a way around such a syntactical problem, use `bash` as your shell.) Notice that you need to have the current directory in the shell search path, or you have to call the program as ./.macek ....

The following call

```
    bash$ ./macek -g-2 -pGF3 '!print' U24
```
produces an output similar to this sample:
```
    567~          Printing output of the command "!print ((t))   ..[1]":
    567~   Printing matrix of the frame [U24]: "the matroid U_2,4 uniform"
       ~      ------------------------------------------------------------
       ~      matrix 0x80fcb48 [U24], r=2, c=2, tr=0, ref=(nil)
       ~              '-1')   '-2')
       ~
       ~      '1')        1       1
       ~      '2')        1       2
       ~      ------------------------------------------------------------
```
The option '-g-2' suppresses usual debugging messages during program run. The option '-pGF3' selects the finite field $GF(3)$ for the computation. Then there is the program command !print followed by the command parameter – the matroid U24. (Note the quotas around the command since ! is an active shell character.) The meaning of !print command is pretty obvious — it prints the matrix representing the matroid $U24$ over $GF(3)$. Numbers on the left of the output mean current time in seconds (modulo 1000), which may be useful to see in longer computations.

Next, try to run
```
    bash$ ./macek -g+1 -pGF3 '!print' U24
```
with the option '-g+1' (or even higher values) instead of '-g-2'. In this way, you get some debugging messages that show what the program does. For example, among other output lines, you may see:
```
    [emflexsu:frame_doinput_()89  ~926] Calling to scan a list of frames...
    [emflexsu:frame_doinput_()99  ~926] Input frame - scanning "!print":
    [emflexsu:frame_doinput_()99  ~926] Input frame - scanning "U24":
    [emflex.l:   frame_flex()520 ~926] Including from 'U24' (->'U24')
```
The prefix of each debugging line point to the source file, function and line that generated the message, and then follows the message itself. The debugging messages are mainly for those, who want to follow the program computation in the source files, and for catching possible bugs in further development. If you do not want the messages at all, you may run the macek.nodebug executable instead. However, note that the latter version also skips all internal consistency checks in the program.

2.3 Command Shortcuts

We have seen a simple example of the command '!print' above. (Every command in Macek starts with the character '!'.) Since Macek commands are quite complex, we provide several simplified shortcuts for the most common tasks, Section 6.12 [Procedures], page 46. Those are intended mainly as a fast introduction to the rich Macek capabilities.

The following examples print basic and extended information about some (distributed) matroids.
```
    bash$ ./macek print R10
    bash$ ./macek print grK33 grK5
    bash$ ./macek prints grK5
    bash$ ./macek prints R10
```

The matrices to print may be simply given on a command line; with semicolon-separated lines. (Note the space starting each entry!)

```
bash$ ./macek print ' 1 1 0; 0 1 1; 1 1 1'
bash$ ./macek prints ' 1 1 0; 0 1 1; 1 1 1'
```

By default, Macek considers matrices over $GF(2)$, but that may be easily changed with the '`-p`' option.

```
bash$ ./macek -pGF3 print U24
bash$ ./macek -pGF4 print U35
bash$ ./macek -pGF3 prints F7-
```

Some more involved shortcuts are presented next.

```
bash$ ./macek connect R10 R12
bash$ ./macek minor R10 grK33
bash$ ./macek -pGF3 minor F7- U24
bash$ ./macek isomorph R10 'R10;!dual'
bash$ ./macek isomorph grK33 grK5 R10 grK5 R10
bash$ ./macek represent-gf5 R10
bash$ ./macek represent-gf5 F7
```

2.4 Matrices and Frames

In order to use the Macek program, one needs to input the *matrices representing matroids*. The program works with matrices in the standard reduced form, i.e. without the leading identity matrix. (So to read off a representation of a matroid by vectors, prefix your matrix from Macek by an identity matrix, and then read the element vectors in columns.) Some common matroid representations are distributed with this package in the '`exe/Matrices`' subdirectory. You may easily create your own matroid files in a similar fashion, with space-separated entries, line by line. Each matrix line should start with a space. Comment the files by lines starting with '`# `'. Any bracketed math expression may appear as a matrix entry.

It is also possible to give a matrix directly on the command line like these:

```
bash$ ./macek -pGF5 '!print' ' 1 2; 3 2+2'
bash$ ./macek -pGF2 '!print' ' 1 1; 0 0 1; 0 1 0 1'
bash$ ./macek -pGF4 '!print' ' w w^2; w^3'
bash$ ./macek -pGF4 '!print' ' w w^2; (w^3+w)*(w+1)'
```

Here '`;`' replaces line-ends. Notice that, for example, inputting an entry '`w`' in $GF(3)$ or '`2`' in $GF(2)$ cause an error.

In fact, the basic input entity in the program is called a *frame*; See Chapter 4 [Frames], page 19. One frame usually holds one matrix, but it may also hold frame- *commands* and *options*; See Chapter 6 [Commands], page 33, See Chapter 5 [Options], page 29. All given command-line arguments that do not start with '`-`' are read as frames. These result in a tree-structure of frames, with the first argument as the root.

The tree structure can be printed with a command:

```
bash$ ./macek -pGF4 '!prtree' U24 '{ U25 U35 F7 }'
...
~820~   Printing the subtree of the frame 0x81552b8 [noname]:
  ~       (1.1)fr  [noname]    ""
  ~         (2.1)fr  [U24] m2x2  "the matroid U_2,4 uniform"
  ~         (2.2)fr  [noname-2]    ""
  ~           (3.1)fr  [U25] m2x3  "the matroid U_2,5 uniform"
  ~           (3.2)fr  [U35] m3x2  "the matroid U_3,5 uniform"
  ~           (3.3)fr  [F7] m3x4  "the matroid F_7 Fano"
~820~   ---------------------------------
```

The command !prtree (with no matrix) forms the root frame, the next two arguments
form its descendant frames, and the included matroids $U25, U35, F7$ form the descendants
of the second son of the root.

Another example is the command !move that manipulates the frames in the tree (moves,
copies, or deletes them). To understand this command better, you need to learn about
addressing command parameters Section 4.5 [Addressing], page 23. (Nodes of the tree are
addressed by bracketed expressions in the natural way; 'T' picks a node, 'S' picks all sons
of a node, the lower-case letters 't','s' also erase the selected nodes afterwards in some
commands.) Run the next examples, and see the action:

```
bash$ ./macek '!prtree;!move ((t));!prtree' W3 '{ W4 R10 R12 }'
bash$ ./macek '!prtree;!move (()(t));!prtree' W3 '{ W4 R10 R12 }'
bash$ ./macek '!prtree;!move ((t)(()(t)));!prtree' W3 '{ W4 R10 R12 }'

bash$ ./macek '!prtree;!move ((T)) >(()(t));!prtree' W3 '{ W4 R10 R12 }'
bash$ ./macek '!prtree;!move ((T)) >((2)(t));!prtree' W3 '{ W4 R10 R12 }'
bash$ ./macek '!prtree;!move ((t)) >(()(t));!prtree' W3 '{ W4 R10 R12 }'
bash$ ./macek '!prtree;!move ((T)) >(((t)));!prtree' W3 '{ W4 R10 R12 }'

bash$ ./macek '!move (()(S)) >(((s)));!prtree' W3 '{ W4 R10 R12 }'
bash$ ./macek '!move (()(S)) >(((t(t(t)))));!prtree' W3 '{ W4 R10 R12 }'
```

2.5 Matroid Representations

Although we assume the reader of this manual is familiar with matroid theory, it may be
helpful to include a brief overview of matroid representations over fields, so that the reader
gets familiar also with our notation and Macek's terms.

A representation of a matroid M is a matrix A over a field F whose columns correspond
to the elements of M, and linearly independent subsets of columns form the independent
sets of M. Clearly, the matroid of A is unchanged when columns are scaled by non-zero
elements of F. So we may alternatively view the matrix A as a point configuration in a
projective space over F.

A matroid M is regular if M is representable by a totally-unimodular matrix. A regular
matroid is then representable over all fields. We remark that cycle matroids of graphs are
regular. A matroid M is binary, or ternary, if M is representable over the fields $GF(2)$, or
$GF(3)$, respectively. Not all matroids are representable over a field F, some of them are even

representable over no field at all. One also has to consider the problem that representable matroids typically do not have "unique" representations.

Another issue, which has to be particularly considered in the context of exhaustive matroid generation (See Section 6.8 [Generating], page 40.), is the one of labeled vs.~unlabeled objects: We are interested in generating unlabeled objects to avoid unnecessary duplicities, while the objects generated by a computer are (usually) implicitly labeled.

In practice, it is much better to work only with the reduced matrix A' instead of the full one $A = [I|A']$, i.e. to strip the matrix reprersentation of a matroid by the leading identity submatrix I. We say that such A' displays a basis B of M where B is formed by the labels of I. Then actually the rows of A' correspond to the elements of B. We note that the transpose of A' is a reduced matrix of the dual matroid $M*$, and that removing a column / a row of A' means deleting / contracting the corresponding element in the matroid M. (Eventually, an element can be pivoted to get it as a row / column when needed.)

That is, roughly, the way Macek works with matroid representations. So the rows of a reduced matrix A' in Macek label elements of a displayed basis of the matroid, and the columns label the remaining elements. The matroid rank equals the number of matrix rows. We, however, note that the element labels in Macek are there mainly for information purposes, and most of Macek computations actually work with matroids as unlabeled combinatorial objects.

2.6 Examples of Use

In this section, we show several more examples demonstrating the use of some Macek commands. (Recall that you get an online overview of all commands with '`./macek -HHc`'.)

One may easily pivot matrices like in the next example:

```
bash$ ./macek -pReg '!print;!pivot 1 2;!print' R10
```

Matroid elements are deleted or contracted in the following way:

```
bash$ ./macek -pReg '!print;!delete -3;!print' R10
bash$ ./macek -pReg '!print;!delete 2;!print' R10
bash$ ./macek -pReg '!print;!contract 2;!print' R10
bash$ ./macek -pReg '!print;!contract -5;!print' R10
```

Minor-testing (up to inequivalence of representations!) is demonstrated in the next commands:

```
bash$ ./macek -pReg '!minor' R12 R10
bash$ ./macek -pReg '!minor' R12 grK33
```

Extensions or coextensions of represented matroids are generated as follows (all 3-connected, matrix-equivalence factored-out):

```
bash$ ./macek -pBinary '!extend c;!prtree' W3
bash$ ./macek -pReg '!extend c;!prtree' W4
bash$ ./macek -pReg '!extend r;!prtree' R12
bash$ ./macek -pReg '!extend b;!prtree' R10
```

Representability and isomorphism over different fields are tested here:

```
bash$ ./macek -pBinary '!repres GF3;!repres GF4' F7
bash$ ./macek -pBinary '!isomorph' F7 '@inputpf GF3;P7'
```

Some more involved chains of commands are demonstrated in the following examples:

```
bash$ ./macek -pReg '!deleach;!prtree;!filt-minor;!prtree' R12 grK33
bash$ ./macek -pReg '!deleach;!prtree;!filx-minor;!prtree' R12 grK33
bash$ ./macek -pdyadic '!extend r;!prtree;!minor' F7- 'F7-;!dual'
```

Finally, one may print out many interesting structural properties of a matroid using the following command:

```
bash$ ./macek -pGF3 '!verbose 2;!printmore' P7
...
~420~            Output of the command "!printmore ((t))   [1]":
    ~    ----------------------------------------------------
    ~    matrix 0x817eef0 [P7], r=3, c=4, tr=0, ref=(nil)
    ~            '-1')   '-2')   '-3')   '-4')
    ~    '1')        1       o       1       1
    ~    '2')        1       1       o       1
    ~    '3')        2       1       1       o
    ~    ----------------------------------------------------
~420~    Number of matroid [P7] bases:  30
~420~    Aut group orbits of [P7] are (via first elem id):
             (1, 1, 3,  1, 1, 1, 1)

~420~    There are -NO- (nontrivial) flats in [P7] of rank 0.
~420~    There are -NO- (nontrivial) flats in [P7] of rank 1.
~420~    Listing all (nontrivial) flats in [P7] of rank 2:
    ~    - rank-2 flat (1)      {  -1, -2, -3 }
    ~    - rank-2 flat (2)      { 3,  -1, -4 }
    ~    - rank-2 flat (3)      { 1, 3,  -3 }
    ~    - rank-2 flat (4)      { 1, 2,  -4 }
    ~    - rank-2 flat (5)      { 2, 3,  -2 }

~420~    There are -NO- exact separations in [P7] of lambda 1.
~420~    There are -NO- exact separations in [P7] of lambda 2.
~420~    Listing all exact separations in [P7] of lambda 3:
    ~    - 3-separation (1)     (  -1, -2, -3, )
    ~    - 3-separation (2)     ( 1, 2,  -4, )
    ~    - 3-separation (3)     ( 1, 3,  -3, )
    ~    - 3-separation (4)     ( 2, 3,  -2, )
    ~    - 3-separation (5)     ( 3,  -1, -4, )

~420~    Matroid [P7] connectivity is 3.
~420~    Matroid [P7] girth (shortest cycle) is 3.
~420~    Matroid [P7] representability: -GF(2)- +GF(3)+
             +GF(4)+ +GF(5)+ +GF(7)+ +GF(8)+ +GF(9)+
    ~    ----------------------------------------------------
```

```
bash$ ./macek -pGF3 '!prbases' P7
...
~966~   Number of matroid [P7] bases:  30
    ~      - base (1)      { 1, 2, 3 }
    ~      - base (2)      { -1, 2, 3 }
    ~      - base (3)      { -3, 2, 3 }
    ~      - base (4)      { -4, 2, 3 }
...
    ~      - base (26)     { 1, -4, -2 }
    ~      - base (27)     { 1, -4, -3 }
    ~      - base (28)     { 1, 2, -1 }
    ~      - base (29)     { 1, 2, -2 }
    ~      - base (30)     { 1, 2, -3 }

bash$ ./macek -pGF3 '!prcircuits' P7
...
~053~   Number of matroid [P7] bases:  30
    ~      - circuit (1)  { 1, 2, 3, -1 }        len 4,
    ~      - circuit (2)  { 2, 3, -2 }           len 3,
    ~      - circuit (3)  { 1, 3, -3 }           len 3,
    ~      - circuit (4)  { 1, 2, -4 }           len 3,
    ~      - circuit (5)  { -1, 2, 3, -3 }       len 4,
...
    ~      - circuit (18) { -1, 2, -2, 1 }       len 4,
    ~      - circuit (19) { -1, 2, -3, 1 }       len 4,
    ~      - circuit (20) { -2, 2, -3, 1 }       len 4,
```

3 The Macek Program

First, read the installation instructions in See Section 2.1 [Installing], page 5. After installing the Macek program successfully, you find the executable(s) in the 'src' subdirectory of the package. However, we suggest to run the executable from the 'exe' subdirectory of the package, where it is symlinked from.

Since the Macek program has only command-line interface, we suggest to run it within a suitable (comfortable) command-shell, like unix shells bash, zsh, new ksh or similar. If you want to use csh-clones, you would probably have to adjust the provided examples. (In particular, it looks like '!' is active in tcsh even when it is quoted. So if you do not find a way around such a syntactical problem, use bash as your shell.) Moreover, to get the program output neatly organized, we suggest to use a terminal of 100 or more (up to 132) characters wide.

The program has two executables — ./macek and macek.nodebug. Usually you would run the first one. The second executable, macek.nodebug, is, however, faster since it skips most of the internal consistency checks and all debugging messages. So it is suitable for long computations when you are already sure that your script computes the right results correctly.

3.1 Command-line Arguments

We list all command-line options of the Macek program (i.e. all recognized arguments starting with '-'):

* -gN (or --debug=N) Adjust the debugging level in the program by N — how much is printed during program run See Section 3.3 [Messages], page 14. (Not applicable to macek.nodebug.)

* -h (or --help) Print a simple program help.

* -H[H][pfco] Print more help on specified topics (partial fields, frames, commands, options), and even more with -HH. See Chapter 4 [Frames], page 19.

* -pF (or --pfield==F) Set the default partial field in the program to F. See Section 3.6 [P-Fields], page 16.

* -r dir (or --readpathapp=dir) Append given 'dir' at the end of the file search path for reading. See Section 3.5 [Environment], page 15.

* -R dir (or --readpathins=dir) Insert given 'dir' as the first one in the file search path for reading. See Section 3.5 [Environment], page 15.

* -w dir (or --writedir=dir) Insert given 'dir' as the first one in the file search path for writing, i.e. to use 'dir' for writing all data files if possible. See Section 3.5 [Environment], page 15.

* -t pref (or --temps=pref) Change the prefix ('/tmp/.macek/') added to all temporarily and automatically writen data files in the program. (Use -t- to disable all autosaving.)

* -s (or --safe) Run the Macek program in a safe mode – not allowing shell execution. See Section 6.10 [Com-Flow], page 44.

* -S (or --safest) Run the Macek program in a very safe mode – not allowing shell execution, no further changes to file paths, and restricting file names only inside the current

writing subdirectory (forbidding `/fname` and `x/../fname`). See Section 3.5 [Environment], page 15, Section 6.3 [FWriting], page 35.

* `-T[smhq]` (or `--time=[smhq]`) Print (system) time in each output line in seconds, minutes, or hours (default in seconds). Use `-Tq` to disable printing time in output lines.

* `-v` (or `--version`) Print the program version.

* `-x ext` (or `--extension=ext`) Use '.ext' as the usual extension for frame files. (The extension is prefered over a plain file name when writing to disk.) See Section 3.5 [Environment], page 15.

3.2 Frame Input

All other command-line arguments of the Macek program (that do not start with '`-`') are read as *frames*; they form the program input. A frame is the basic data-entity in the program. See Chapter 4 [Frames], page 19.

As explained later in details, the frames form a tree-structure in the program. The first frame-arguments forms the root of the tree, and all others are its sons. Moreover, some frames may include other subframes that are then stored as their sons, and so on. . .

When giving frames as arguments to the program, do not forget to quote them, as they may contain spaces and active shell characters inside.

3.3 Program Output

There are two categories of messages printed from the Macek program — the regular *output*, and the *debugging messages*. (Debugging messages are used to trace the program execution, and to provide additional profiling information.)

Typical command output in the program looks like the following:

```
705~  Printing the subtree of the frame 0x80e9a38 [noname]:
 ~    (1.1)fr  [noname] ""
 ~       (2.1)fr  [U25] "the matroid U_2,5 uniform"
 ~          (3.1)fr  [U25_r1] "mat #1 row co-exten to 'U25'"
```

For profiling purposes, each output line starts with the current time `~nnn~` in seconds modulo 1000. Then the output itself follows.

Typical debugging message in the program looks like the following:

```
[gener.c:gener_extensions()368 ~520] >>extension #1 of [U25]: 1,w+1,w
```

The starting bracketed information contain the source file name, the function, and the line of the message, and the current time in seconds modulo 1000. These information allow to easily track the program computation in the program source. The next text message then tells you what the program currently does. You may control the amount of printed debugging messages with Section 3.1 [Command-line], page 13. No debugging messages are printed from the `macek.nodebug` executable.

3.4 Error Reporting

If anything unusual or problematic happens in the Macek program, then an *error message* is issued. Similarly as in debugging messages Section 3.3 [Messages], page 14, an error message first tells you what function in what source file generated this message, and then

the text description follows. Fatal errors immediately terminate the program, while the program run continues for other errors. (However, this may cause subsequent induced errors.) Additionally, the programs prints a warning at the end when an error happened during the computation.

A usual error message looks like the next example

```
*** ERROR (by emflexsup.c, in yyincl() 412) in "NOxxx" l.1: ***
  Cannot open include file 'NOxxx'!
```

or

```
*** ERROR: (by frameop.c, in frame_getparamfr_r() l.706) ***
  Missing requested subframe of 0x80ea5f8 in '(t|', depth 1.
```

Such messages usually tell you that there was something wrong with the program input or commands. You have to correct the input to get your computation right...

Moreover, there is another more serious type of error messages, called *program errors*, which start with 'Program ERROR [bad:-(]: '. These indicate that something very wrong happened inside the program. Such a program error may happen after a usual error. If a program error is issued for a correct program input, then it indicates a bug in the program. Please report such incidents to the author at http://www.mcs.vuw.ac.nz/research/macek/ or hlineny@fi.muni.cz.

To make the Macek program more reliable, there is number of internal checks implemented at various places in ./macek. (These checks usually watch consistency of data structures, or provide alternative ways of computing the same results.) Uncovered inconsistencies result in program errors. Learn more about them in the program sources. However, such internal checks take time, and so they are often only randomized to make things faster. The provided alternative fast executable macek.nodebug skips all these internal checks.

3.5 Program Environment and Files

This section describes how the Macek program interacts with its computing environment; including reading / writing files, search paths, using environment variables, etc...

The program input is taken from one input stream that starts with the command-line argument, but then it may include input from arbitrary files. File names follow unix conventions, and they are case-sensitive (of course, if supported by the system). A file name may contain spaces and other strange characters, but then it must be quoted. Be reasonable, and use "normal" file names, though.

If a file name starts with the slash '/', then it is taken as an absolute path from the filesystem root. Similarly file names starting with './' or '../' are searched from within the current directory. Otherwise, the files is searched on the input search path, which may be listed by calling './macek -Hf', and changed using '-r dir' or '-R dir' on the command line. See Section 3.1 [Command-line], page 13. The output search path (which is different!) is used when writing files, and the prefered output directory may be set using '-w dir' on the command line.

A special file name handling applies in the safest running mode '-S', Section 3.1 [Command-line], page 13. Then no names starting with '/' or containing '../' are allowed, and all such dangerous characters are replaced with underscores. This applies both to reading and writing files. Moreover, the write path is restricted to the first entry (which

is supposed to be set by the user with '`-wdir`'), and no other dirctories allow for overwriting files.

When reading, the given file name is first tried without additional extension, and then the default extension is appended. When writing to a file, the default extension is automatically appended unless the file already has an extension. The program implements also an output dir path for writing files — a default entry of the path is used only when the directory already exists. However, when a user inserts a new directory to the path with '`-w dir`', then this directory is created before writing. By default, Macek tries to write output files to subdirectories '`out`' or '`temp`' if they exist, or to the current directory.

To avoid an accidental loss of computing data, the Macek program in some situations automatically saves the whole frame tree to the '`/tmp/.macek/`' directory. If the program computation is long (in about minutes), then the frame is saved to the file '`/tmp/.macek/macek-out-NN.mck`'. Similarly, if an error happens, then the current frame tree is saved to '`/tmp/.macek/macek-err-NN.mck`'. A message is printed out after such automatic save. To avoid automatic saving of such files completly, give '`-t-`' on the command line. See Section 3.1 [Command-line], page 13.

Handling environment variables may be added later......

3.6 Partial Fields

The Macek program can work with matrices over so called *partial fields*. A partial field is an extension of a usual field which allows the sum to be a partial operation (i.e. not all results are defined). A typical example are regular (also known as totally-unimodular) matrices in which only the numbers $-1, 0, 1$ are used, and the sums $1 + 1$ or $-1 - 1$ are undefined. You do not need to know about partial fields to use this program, just consider usual finite fields instead.

The program works only with finite partial fields, which means those in which the equation $x = y + 1$ has finitely many solutions. This includes all finite fields. To obtain the list of all partial fields currently implemented in the Macek program, run '`./macek -HHp`'. You set the partial field for the program as with `-pPF` in Section 3.1 [Command-line], page 13. If you want to add more partial fields to the program, you have to update the program source and recompile it. (See the file '`src/pfield/pfdef-more.inc`'.)

Each partial field in the program is represented by the generators of the multiplicative subgroup. Specifically, each number is given by a sign (with values 0,-1,1), and a list of integral exponents corresponding to the generators. Multiplication is implemented in the obvious way. Addition is implemented via multiplication and a table of *fundamental elements* — those x for which $x - 1$ is defined. If some input expression (sum) is not defined in the partial field, then an error is reported. Division by zero results in a zero value, but no error is reported.

A matrix over a partial field is *proper* if all of its subdeterminants are defined. Then also all subsequent matrix operations in the program will be defined. When reading input, proper matrices are checked by a randomized test (unless this feature is switched off), and possible errors are reported. One may also thoroughly test proper matrices with an explicit command (which is quite slow), Section 6.6 [Structural], page 37. If it still happens that an improper matrix gets into the program, then lots of arithmetic-error messages may be reported during program execution.

One may also change between partial fields during program execution, and to translate matrices from one partial field to another. The list of all supported partial field translations, run '`./macek -HHp`'. Read more in Section 6.9 [Diff-fields], page 43...

Remark.   Partial fields naturally (and often implicitly) appear in matroid-representation theory, like for totally-unimodular or dyadic matrices. They were formalized by Charles Semple and Geoff Whittle in [Partial Fields and Matroid Representation, Advances in Applied Mathematics 17 (1996), 184-208]. We follow their formalization here.

4 Frames – Data Handling

As noted above, the basic data entity in the Macek program is called a *frame*. One frame usually holds a matrix representation (only one), but it may also hold an arbitrary number of frame- *commands* and *options*; Chapter 6 [Commands], page 33, Chapter 5 [Options], page 29. Each frame can be identified by its name Section 5.2 [Naming], page 29.

One frame may also refer to several subframes, called its *sons*. In this way, frames in the program form a rooted tree-structure; the frames are the nodes of the tree. (In general, the structure of frames could form an arbitrary directed graph, but the program allows only one rooted tree as the frame structure.)

4.1 General Input Syntax

Frames are given to the Macek program in a text format. The frame input is organized by lines which may have several different meanings. A line starting with '#' or '%' is a *comment line*, and it is ignored. A line starting with space(s) or '=' is a *matrix line* — its expressions define the entries of the matrix in this frame. For example, let the following be the file 'U24x':

```
# The uniform matroid U_2,4 represented over GF(3) or GF(5).
   1      1
   1      2
```

A line starting with '<' or the keyword 'include' is an *include line*. Each subsequent word on such a line gives a file-name to be included into the input stream. If the file-names contain special characters, like a space, enclose them into quotas like '< "long file name.mck"'. Note that include lines have nothing to do with subframes. We continue the previous example with the file 'U35x', using an include:

```
# The uniform matroid U_3,5 represented over GF(5).
< U24x
   1      3
```

Lines starting options, commands, or subframes are described in details later. For now, we just give a simple introduction. A *command* is given on a line starting with '!' or the keyword 'command'. The next call shows the command for printing a matrix.

```
bash$ ./macek -pGF5 '!print' U25
```

An *option* is given on a line starting with '@' or the keyword 'option'. One useful basic option tells the program to consider the previous entered matrix entries in the matrix-transposed way. This example shows an easy way to obtain a file for the matroid $U25$ from $U35$ (cf. the previous paragraph).

```
# The uniform matroid U_2,5 represented over GF(5),
#  obtained by transposing U_3,5.
< U35x
@transpose
```

As already noted, input frames are given to the program as command-line arguments Section 3.2 [Frame-arg], page 14. To make the life easier, there are several shortcuts used for an input text taken from the command-line: Before reading the frame, all occurrences of the character ';' on the command-line are replaced by newlines, and all occurrences of ','

are replaced by spaces. This does not apply to quoted strings inside the input. Moreover, any resulting input line that starts with a digit is taken for a matrix line, and any line starting with a letter or one of '`./~`' is taken for an include line. In particular, an input '`U24`' given as an argument to the program means to read the file '`U24`' (the same as the "full syntax" '`< U24`').

Using these shortcuts, one may give the whole input matrix on a line. Moreover, one may combine the matrix input with commands or options, and even with include lines (the matrix is then a concatenation of all input matrix lines):

```
bash$ ./macek -pGF5 '!print; 1 2; 3 4'
bash$ ./macek -pGF5 '1,2;!print;3,4'
bash$ ./macek -pGF5 'U24; 0 2; 3 4 ;!print'
```

In this context we remark that one cannot input a list of matroids like '`U24; U25; U35`' since this would result in concatenation of all three matrices. On the other hand, another shortcut described later allows to simply input a list of matroids as subframes. Section 4.4 [Subframes], page 23.

```
bash$ ./macek -pGF5 '!prtree;!print ((S))' '{U24 U25 U35}'
```

4.2 Matrices and their Entries

The program works with matrices representing matroids in the standard reduced form, i.e. without the leading unit submatrix. In this way the rows of the matrix correspond to the elements of a selected basis, and the columns to the remaining matroid elements. Before reading on here, it is good to understand (partial) fields; See Section 3.6 [P-Fields], page 16. We provide a more theoretical overview of matrix representations and their (in)equivalence in the next section.

This part provides a detailed description of matrix entries in frames. In general, the expressions on the n-th matrix line of the frame input give the entries of the n-th row of the matrix. (Modulo possible use of the '`@transpose`' option.) On one matrix line, the k-th expression gives the entry in the k-th column. The total number of rows of the resulting matrix is equal to the number of input matrix lines, and the total number of columns is equal to the maximal number of expressions over all matrix lines. Entries that are not directly given (like the rest of a short matrix line) are filled with zeros.

```
bash$ ./macek '!print' ' 1 0 1; 0 0 0 0 1'
bash$ ./macek ' 0;!print; 1 0 1; 1 0 0 0 1'
bash$ ./macek '!print; 0; 1 0 1; 1 0 0 0 1;@transpose'
bash$ ./macek '!print' ' 1 0 1 1;@transpose; 0 0 1 0 1'
```

An *expression* on the matrix line is a sequence of characters with no spaces. Spaces separate one expression from another. The *atomic expressions* describe generators of the partial field, and depend on its selection Section 3.6 [P-Fields], page 16. Moreover, one may use a special symbol '`o`' for zero. An expression is then built up from atomic expressions using parentheses '`()`' and symbols '`+-`', '`*/`', and '`^`' for arithmetic operations in the natural way.

It is best to illustrate matrix expressions by several examples. Keep in mind that not all expressions are defined in partial fields, so they may result in an error message.

```
bash$ ./macek -pGF4  '!print' ' 1+w^2  w*(w+w^3)  1+w+w*w'
bash$ ./macek -pGF5  '!print' ' 1+1+1  2^2+3^3+4^4'
bash$ ./macek -pNREG '!print' ' (a^4-a^3)^2  ((a-1)^2+a-1)*(a^3-a^2)'
```

Notice that some atomic expressions may look similarly like arithmetic operations. A good example is the generator $a-1$ of the near-regular partial field. In such a case we use brackets '[a-1]' for the generator. (This is an important change from the pre-1.0 versions of Macek which scanned 'a-1' as an atom, creating confusion in expressions like $-a-1$.) However, an input like '(a-1)^2' is still correct – this is evaluated as arithmetic subtraction and power. If you are still confused, then use more parentheses.

Since we want to use one input file to represent the same matroid over different partial fields, we need a way to replace "transcendental elements" in the general representation matrix by specific field elements. We also need to ensure that the current partial field has necessary algebraic properties to represent the matroid. See Section 5.3 [Substitution], page 30.

There is an option '@replace' that accomplishes the first task. It is used as '@replace X (a-1)' to replace all further occurrences of the symbol 'X' on the matrix input with the expression '(a-1)'. (We suggest to use upper-case letters for transcendentals, and parentheses around replaced expressions to avoid confusion after the replacement.) Analogously, an option like '@repl-PF X (a^2)' replaces the symbol 'X' only in one specific partial field PF. This pfield-specific replacement has priority. The symbol replacement is fully recursive, and it may be used only in matrices. One may prevent a symbol from a recursive replacement by prefixing it with the underscore '_', which is otherwise silently ignored on matrix lines.

Another option '@require' checks necessary algebraic properties of the current partial field. Use it as '@require a+1 [.01]', where the version with one expression followed by a dot checks whether the expression is defined over the current partial field, and the version with the second parameter as '0' or '1' check whether the expression value is zero or nonzero.

As an example we show the Fano matroid with a requirement of characteristic 2.

```
@comment "the matroid F_7 Fano"
@require 1+1 0
        1       1       1       0
        1       1       0       1
        1       0       1       1
```

Another example shows use of the symbolic replacement to give a representation of the matroid $U24$ in various finite fields. (Notice that if -1 was substituted for X over $GF(4)$, then the second requirement would fail.)

```
@replace    X     -1
@repl-GF(4) X      w
@require    (X)   1
@require    (X)-1 1
        1       1
        1       (X)
```

The next calls show the use of '_' in preventing recursive replacements. (The first call obviously results in an error.)

```
bash$ ./macek -pGF4 '@replace w w+1;!print' U24
bash$ ./macek -pGF4 '@replace w _w+1;!print' U24
```

Some common matroid representations are distributed with this package in the 'exe/Matrices' subdirectory. We do not list them here since more matroids are added frequently. Look at the file listing of 'exe/Matrices/ *' to see all of them. Each distributed matroid file is commented. You may easily create your own matroid files in a similar fashion. We suggest to enter new matroid representations (if possible) as highly symmetric matrices – see the command '!selfmap'. A symmetric matrix may help some algorithms to run faster.

If you create new matroid files that may be interesting and useful to others, please send them to the author.

4.3 Matroid Representations

In general, a matroid representation is a matrix whose columns represent the matroid elements, and usual linear dependency determines the dependent sets. However, it is better to work with a representation in the so called *standard reduced form*, obtained as follows: Choose a basis of the matroid and display it as a (maximal) unit submatrix. Then forget possible remaining zero rows and the columns of the unit submatrix. Finally, the rows of this standard-form matrix correspond to the elements of the selected basis, and the columns correspond to the remaining matroid elements. In this manual, we simply say a *matrix* instead of a "standard-form matrix".

There is no way to give names to the matroid elements – matrix lines, but the lines initially receive number labels as follows: The rows (elements of the selected basis) are numbered from 1 to R, and the columns (remaining elements) are numbered from -1 to $-C$. If a command later changes the matrix in a way that the lines move elsewhere, the labels are moved along with them. (See, for example, in the commands !dual or !pivot.)

When giving a matrix over a partial field, it is important to ensure that the matrix is proper — that means all subdeterminants are defined over this partial field Section 3.6 [P-Fields], page 16. Only then it is guaranteed that no arithmetic error occurs during program execution, and that the results are correct. See Section 6.6 [Structural], page 37.

Moreover, some commands in the program require connectivity of the given matroid to compute the result correctly. Some commands even need the input matroid to be 3-connected. In such cases you should enter only sufficiently connected matroids to avoid error messages or, even worse, incorrect results. So read carefully the description of commands below.

Another major problem with using the Macek program is caused by an existence of inequivalent representations of matroids. Two matrices (with lines labeled by the matroid elements) are called *strongly equivalent* if one can be transformed to the other one using elementary matrix operations. Two matrices are, on the other hand, called *unlabeled equivalent* if they are strongly equivalent up to an isomorphism of the underlying matroid. (In other words, if they are strongly equivalent after forgetting the line labels.)

For example, the following two quaternary representations of the matroid $U24$ are not strongly equivalent, but they are unlabeled equivalent:

```
1    1                1    1
1    w                1    w+1
```

You have to thoroughly consider the **problems with inequivalence** of representations when dealing with matroid minors, equivalence, or when generating matrix extensions. The Macek program has, so far, no way to find out that two inequivalent matrices actually represent the same matroid. (There are, however, no such problems at all when working with binary or regular matroids only.) Read the next chapters for specific information. See Section 6.6 [Structural], page 37, See Section 6.8 [Generating], page 40.

4.4 Subframes

A *subframe* starts with the keyword 'SUBFRAME' on a separate line, and ends with the keyword 'EOFRAME'. All input between these keywords is read into a new son of the current frame. There may be arbitrarily many subframes given, and arbitrarily nested, forming thus a *rooted-tree structure*. One may also use shortcuts '{' and '}' for start and an end of a subframe.

If the shortcuts '{' and '}' are used, then more than one (or even all of them) may be written on the same line. Moreover, all other words on such a line are taken as a separate subframe each. For example, the following shortcut produces the next tree of frames.

```
bash$ ./macek -pGF4 '!prtree;{ U24 {{U24 U25} {U35}} }'
~944~   Printing the subtree of the frame 0x81551f8 [noname]:
 ~        (1.1)fr  [noname]    ""
 ~          (2.1)fr  [U24] m2x2  "the matroid U_2,4 uniform"
 ~            (3.1)fr  [U24-1]    ""
 ~              (4.1)fr  [U24] m2x2  "the matroid U_2,4 uniform"
 ~              (4.2)fr  [U25] m2x3  "the matroid U_2,5 uniform"
 ~              (4.3)fr  [U35] m3x2  "the matroid U_3,5 uniform"
```

When reading the program input, the first frame argument on the command line forms the root of the frame tree, and all possible other frame arguments are then arranged as its sons (in addition to its subframes). One may combine the concept of subframes and multiple frame arguments like in the following example.

```
bash$ ./macek -pGF4 '!prtree' '{ U24 U25 }' U35
~998~   Printing the subtree of the frame 0x8155b28 [noname]:
 ~        (1.1)fr  [noname]    ""
 ~          (2.1)fr  [noname-1]    ""
 ~            (3.1)fr  [U24] m2x2  "the matroid U_2,4 uniform"
 ~            (3.2)fr  [U25] m2x3  "the matroid U_2,5 uniform"
 ~          (2.2)fr  [U35] m3x2  "the matroid U_3,5 uniform"
```

4.5 Addressing the Frame-tree

So far, we have shown only default parameter addressing in frame commands. However, one often needs to address arbitrary frames in the subtree, not only the pre-defined ones. Notice that some commands require precisely one frame in the parameter, while most of them accept an arbitrarily long list of input frames. This section shows the syntax of the *parameter addressing*. It is possible to skip this section until you get to the chapter Chapter 6 [Commands], page 33.

In general, nodes in the frame tree are addressed using natural correspondence between rooted trees and *balanced bracketings*. In this interpretation, '()' means the current (root) frame — that one holding the command, and '(())' means the first son of the current one. You can see that the frame addressing is relatively rooted at the current frame. There is no way to address frames out of the current subtree. To actually point to a node in the tree, one must give a letter 't' or 'T' in the bracketing. Alternatively, a letter 's' or 'S' picks all sons of the pointed node instead. The difference between the letters 't,s' and 'T,S' is that the lower-case letters request to erase (destroy) the selected nodes after processing certain commands, while the upper-case letters do not. The current frame is never destroyed.

We now provide several sample addresses to illustrate the concept. The address '(T(T))' picks both the root and its first son. The address '(()()((T)))' picks the son of the son of the third son of the root. To save repetitions in the address, one may use numbers; '((2)((T)))' is equivalent to the previous example. The address '((3)(5T))' picks five sons of the root starting with the fourth one (the first three are skipped). Moreover, you may use, instead of the repetition number, a text '/name/' which skips sons up to the one named name. If the frame tree does not contain the requested nodes, an error message is printed.

Special rules concern non-positive repetition numbers. If '(N...)' is used, and $N <= 0$, then the repetition number actually used in $N + D$, where D is the number of remaining sons of the parent of the current node (inclusive). For example, '((0T))' has the same effect as '(S)', while '(()(-1T))' picks all sons of the root except the first and the last one.

In addition to the previous, you may concatenate more than one address to one with '+', like '((t))+((t))' picks the same first son twice. To save typing of closing brackets, you may end the address expression simply with '|', like '((()(T(s|'. As a special concept, you may write the (whole) address as '~1' to pick all resulting frames of the previous command, or '~N', $N = 1, 2, ..., 9$ to pick the resulting frames of the N-th previous command. Similarly '^N' picks the previous results and allows them for further erasing, depending on the command. (The same thing as 'T,S' versus 't,s'.) See Section 6.11 [Filtering], page 45. Be careful not to destroy these frames before using them again — in most cases such erased frames would be silently skipped, but an unexpected behavior may occur if another frame was created since with the same memory address.

Some commands also have output an parameter address which tells them where to store the resulting frames. The output address starts with '>' and continues with the same bracketed expression as described above. However, this time all nonexistent nodes from the address are automatically created (no error is reported). In particular, the S specification always creates a new tree node, and stores frames to its sons. (Hence the address '>(S)' is not valid since it refers to the current node which cannot be created again.) If one wants to store frames to (new) sons of an existing node, he has to use an address like '>(((0T)))'. Another exception concerns non-positive repetition numbers; if they are followed by a t,T,s,S letter, then they refer to the number of remaining output frames instead to the remaining sons in the tree. For example, '((-1t))' stores all but the last output frames as (new) sons of the root.

To learn the concept of parameter addressing well, it is best to play with the command '!move' which moves (copies, deletes) nodes across the frame tree. We provide several examples next. Make more examples yourself.

```
bash$ ./macek '!prtree;!move ((T)) >(()(t));!prtree' 'W3;{}{}'

~052~   Printing the subtree of the frame 0x8155048 [noname]:
   ~       (1.1)fr  [noname]    ""
   ~         (2.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spok"
   ~           (3.1)fr  [W3-1]    ""
   ~           (3.2)fr  [W3-2]    ""
~052~   ----------------------------------
~052~   Printing the subtree of the frame 0x8155048 [noname]:
   ~       (1.1)fr  [noname]    ""
   ~         (2.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spok"
   ~           (3.1)fr  [W3-1]    ""
   ~           (3.2)fr  [W3-2]    ""
   ~         (2.2)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spok"
~052~   ----------------------------------


bash$ ./macek '!prtree;!move ((S)) >(()(s));!prtree' 'W3;{}{}'

~153~   Printing the subtree of the frame 0x81553b8 [noname]:
   ~       (1.1)fr  [noname]    ""
   ~         (2.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spokes"
   ~           (3.1)fr  [W3-1]    ""
   ~           (3.2)fr  [W3-2]    ""
~153~   ----------------------------------
~153~   Printing the subtree of the frame 0x81553b8 [noname]:
   ~       (1.1)fr  [noname]    ""
   ~         (2.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spokes"
   ~           (3.1)fr  [W3-1]    ""
   ~           (3.2)fr  [W3-2]    ""
   ~         (2.2)fr  [W3-0]    ""
   ~           (3.1)fr  [W3-1]    "fr #1 got by '!move ((S))', to '(()"
   ~           (3.2)fr  [W3-2]    "fr #2 got by '!move ((S))', to '(()"
~153~   ----------------------------------


bash$ ./macek '!prtree;!move ((t));!prtree' 'W3;{}{}'

~298~   Printing the subtree of the frame 0x8155128 [noname]:
   ~       (1.1)fr  [noname]    ""
   ~         (2.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spokes"
   ~           (3.1)fr  [W3-1]    ""
   ~           (3.2)fr  [W3-2]    ""
~298~   ----------------------------------
~298~   Printing the subtree of the frame 0x8155128 [noname]:
   ~       (1.1)fr  [noname]    ""
~298~   ----------------------------------
```

```
bash$ ./macek '!prtree;!move (T) >((()(t)));!prtree'

~379~    Printing the subtree of the frame 0x8155050 [noname]:
    ~        (1.1)fr  [noname]    ""
~379~    ----------------------------------
~379~    Printing the subtree of the frame 0x8155050 [noname]:
    ~        (1.1)fr  [noname]    ""
    ~          (2.1)fr  [noname-0]    ""
    ~            (3.1)fr  [noname-0]    ""
    ~            (3.2)fr  [noname]    "fr #1 got by '!move (T)', to '(("
~379~    ----------------------------------


bash$ ./macek '!prtree;!move ((T|+((T| >(()((0t|;!prtree' 'W3' ''

~164~    Printing the subtree of the frame 0x8155208 [noname]:
    ~        (1.1)fr  [noname]    ""
    ~          (2.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spokes"
    ~          (2.2)fr  [noname-2]    ""
~164~    ----------------------------------
~164~    Printing the subtree of the frame 0x8155208 [noname]:
    ~        (1.1)fr  [noname]    ""
    ~          (2.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spokes"
    ~          (2.2)fr  [noname-2]    ""
    ~            (3.1)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spok"
    ~            (3.2)fr  [W3] m3x3  "the matroid W_3, wheel of 3 spok"
~164~    ----------------------------------
```

4.6 Macro Substitutions

The Macek program provides also simple text-based macro processing. When you write '$macro' or '${macro}', then this expression is replaced with the first word (use quotas for longer text with spaces) following the latest option '@sub-macro'; Section 5.3 [Substitution], page 30. If no such option is found, then the first word of '@subd-macro' is taken. If even this is not found, then the replacement text is empty. To input the character '$' itself, use '\$' or '$$'. Under normal circumstances, the macro processing is not recursive. This is a different concept than '~1,^1' shortcuts described in the previous section.

This kind of macro-processing is provided only for command-, option-, and include-lines. For replacements in matrix entries use '@replace' described above Section 4.2 [Matrix-Entry], page 20.

Again, it is best to illustrate the use of macros in several examples.

```
bash$ ./macek '@sub-mac ABCDefgh;!prtext $mac'
395~    ABCDefgh
```

```
bash$ ./macek '@sub-mac "ABCD efgh .. WXYZ";!prtext $mac'
457~    ABCD efgh .. WXYZ

bash$ ./macek '@sub-mac TEXT;@sub-mac "X-$mac-$mac-X";!prtext $mac'
519~    X-TEXT-TEXT-X

bash$ ./macek '@sub-mac TEXT;@sub-mac "X-$mac-$$mac-$mac-X";!prtext $mac'
519~    X-TEXT-$mac-TEXT-X
```

More involved and nonstandard examples are provided here. Be careful when using text macros in this nonstandard way since strange things may happen... (Like an error in the last example.)

```
bash$ ./macek -pGF4 '@sub-line "U24;{U35}";$line;!prtree'
273~ Printing the subtree of the frame 0x80f6f38 [U24]:
    ~    (1.1)fr 0x80f6f38 [U24] "the matroid U_2,4 uniform"
    ~        (2.1)fr 0x8116610 [U35] "the matroid U_3,5 uniform"

bash$ ./macek '@sub-line "!prtree";$line' R10
    ~    (1.1)fr 0x80f8918 [noname] ""
    ~        (2.1)fr 0x8103090 [R10] "the matroid R_10"

bash$ ./macek '@sub-a "$$b";@sub-b "$$a";$a'
... error happens ...
```

5 Frame Options

An *option* starts with the keyword '`option`' or the character '`@`' on a line. The next word is the option name, and option values continue after the name. Various options have various numbers of values, and an arbitrary number of option lines of the same name may appear. Quote the option values if they contain spaces or other special characters.

In general, an option holds arbitrary information that we want to store in the input frame. This information affects the input frame immediately from the line containing the option. Moreover, depending on particular situation, option values may be inherited by all subframes of the frame holding this option.

To learn more, read next about particular option groups recognized by the Macek program. The list of all recognized options is obtained by calling '`./macek -HHo`'. (Not all of them may be described in this manual.) If an unknown option or the wrong number of values are given, then an error message is reported.

5.1 Inheritance of Option Values

An option affects the frames it appears in, and, in most cases, it also affects all descendant frames. However, in some situations we want to explicitly request repeating an option in the descendants, like when writing a frame to a file, or when generating new subframes in the program. (This does not concern the `name` and `comment` options which are repeated automatically, see below.)

For all options having names that are listed as the values of some '`@finherit`' option, the last one option instance is copied to every descendant frame when it is written to a file. The option '`@finheritall`' works similarly, but it copies all option instances to the written file.

Learn more about inheritance from the following examples. (View the results in the output file '`sample.mck`'.)

```
bash$ ./macek '@replace X 1;!writeto sample' '@replace Y 2'
bash$ ./macek '@replace X 1;!writeto sample' '@finherit replace'
bash$ ./macek '@replace X 1;@finheritall replace;\
        !writeto sample' '@replace Y 2'
bash$ ./macek '@replace X 1;@finheritall replace finheritall;\
        !writeto sample' '@replace Y 2'
```

The options '`@extinherit`' and '`@extinheritall`' achieve similar effects when new subframes are generated by extending a matrix; See Section 6.8 [Generating], page 40.

5.2 Naming the Frames

Each frame has its name, and optionally a text comment (about one line of a text). The name for a frame may be explicitly set by giving the option '`@name "frame-name"`'. Similarly, one may give a comment to the frame by '`@comment "some comment..."`'.

The frame comments are there just for user information, while the names are used elsewhere in the program, such as when writing frames to files. See Section 6.3 [FWriting], page 35.

5.3 Options for Substitutions

The Macek program provides two ways of replacing text on the input. The first one, called *substitution*, is used for option/command values, include files, etc. The second one, called *replacement*, is used in matrix entries or in field expressions.

When '`$macro`' appears on the input, then it is substituted with the first value of the last instance of the option '`@sub-macro`'. (Based on the current input scanned so far.) When '`@sub-macro`' option is not found, then '`@subd-macro`' is tried instead.

When an option '`@replace X (expr)`' is given in a frame, then all following appearances of the symbol '`X`' in matrix-entry expressions is replaced with the text '`(expr)`'. We suggest to use capital letters for such symbols. Similarly, an option '`@repl-PF X (expr)`' replaces the symbol '`X`' only when in the specific partial field `PF`. This pfield-specific replacement has priority.

You may prevent a symbol from a recursive replacement '`@replace X (expr+_X)`' by prefixing it with the underscore '`_`', which is otherwise silently ignored on matrix lines. To ensure that the replaced values satisfy your algebraic requirements, you may use the option '`@require (expr) [01.]`'. See Section 4.2 [Matrix-Entry], page 20.

5.4 Adding and Erasing Options

All frame options are read and interpreted before command processing starts, but it is still possible to add more options to selected frames during command execution using the command '`!append "input" >dest`'. See Section 6.4 [FReading], page 36. The intention of this command is to append the additional input to the selected frame — so the command '`!append "@option value" >(T)`' adds the given option to the current frame at the time when this command is executed.

It is not possible to delete options that were already scanned to a frame, but special options '`@erase optname`' and '`@eraseall optname`' are provided to suppress the last or all previous occurrences of the selected option(s). This means that the suppressed option will not be interpreted further. (However, the option still remains in the option list of its frame.) You may also use a combination like '`!append "@eraseall optname" >(T)`' to erase the selected option values via a command.

Try to play with the next examples to learn more:

```
bash$ ./macek '@sub-x AB;@sub-x $x-C' '!prtext $x'
bash$ ./macek '@sub-x AB;@sub-x $x-C;@erase sub-x' '!prtext $x'
bash$ ./macek '@sub-x AB;@sub-x $x-C;@eraseall sub-x' '!prtext $x'
bash$ ./macek '@sub-x AB;@sub-x $x-C' '@erase sub-x;!prtext $x'

bash$ ./macek '@sub-x A;!append (T) "@sub-x $x-B;!prtext $$x"'
bash$ ./macek -pGF4 '!extend b;!append (T) \
      "@ext-forbid U25";!extend b >(()(s));!prtree' U24
```

5.5 Options for Generating Extensions

The program provides commands for generating extensions of matrices, See Section 6.8 [Generating], page 40. These commands need to keep additional information about the

matrices during the generation process, which is done using the various `ext-*` options described here.

The option '`@ext-bsize R C`' tells that the base minor of generating process occupies the first `R` rows and `C` columns of the matrix. The option '`@ext-signature`' keeps the signature of the elimination sequence of generating process. (The signature tells, in a bit-representation, whether rows 0 or columns 1 were extended at each step of the process.) These options are set during the generating process, and they should never be altered by hand.

Another option '`@ext-entries e1 e2 ...`' requests that only extensions which start with the prefix '`e1 e2 ...`' are generated. (That means the first entry of the added line must be $e1$, the second one $e2$, etc. . .) Reader should understand that this option is not provided for altering the generation process in any way, it just filters out the other extensions. So, in particular, the prefix entries should start with 1 as the first nonzero entry, which is requested for all extensions. The option has been added to version 1.2, and it should be used with care.

The option '`@ext-forbid min1 min2 ...`' lists matroids which are forbidden as minors when generating extensions. Each value of the option represents one forbidden minor, and these values are processed in the same way as the input frames in the program. Notice that this value-processing happens later, when the respective '`!extend`' command is executed. For example, the option '`@ext-forbid F7 "F7;!dual"`' means that the matroid $F7$ and its dual will be excluded in the next extension-generating command.

The option '`@ext-tight min1 min2 ...`' lists matroids which form the set defining a "tight-major" for generated extensions. This option works similarly as `ext-forbid`. The option '`@ext-nofan f`' tells that no fan of length f or longer may appear along an elimination sequence when generating extensions. Notice also the options '`@extinherit`' and '`@extinheritall`' that control inheritance of other options in the generated extensions Section 5.1 [Inheritance], page 29.

5.6 Other Options

The option '`@nopfcheck`' causes the program to skip the partial field test. If the partial field currently used in the program is not total, then not all matrices are valid Section 3.6 [P-Fields], page 16. So a randomized quick test is usually run to uncover most of undefined matrices. However, for a very long input you may want to skip even this quick test.

The option '`@transpose`' immediately transposes the matrix scanned so far. If more matrix lines appear after this option, then they continue the transposed matrix. Usually you would use this option to obtain a dual matroid from an existing one. This option is never inherited or written to files.

```
bash$ ./macek '!print (S)' 'grK5' 'grK5;@transpose'
```

The option '`@inputpf PF`' immediately switches the input partial fields to new `PF`. That means all subsequent matrices in the subtree of the current frame are read and stored in the new partial field. See Section 6.9 [Diff-fields], page 43. No other frames than the current one and its subframes are affected. The validity of a partial field change always ends at last with the end of the input string; that means it is not inherited from the first program argument to the next ones, even when they are logically subframes of the first argument. Do not mix different partial fields for one matrix.

```
bash$ ./macek -pGF2 '!print ((T));!pfield GF3;!print (()(T))' \
         U23 '@inputpf GF3;U24'
```

6 Frame Commands

A *command* starts with the keyword '`command`' or the character '`!`' on a line. The next word is the command name, and command parameters continue after the name. For example, we write a frame to a file with '`!writeto filename ((T))`'. It is possible to quote parameters with spaces '`"long parameter text"`'.

When reading the input, commands are scanned and immediately stored into their frames. However, they are executed (in order) later, after the whole input is scanned. If more than one frame of the input tree holds commands, then the frames are processed in the reversed depth-first order. In particular, descendants are processed before the root. Each command can access only the frame it is stored in, and its descendants. (Like if this frame was the root of the whole tree.) Usually, you should give all commands in the root frame.

6.1 Command Overview

When using commands and options in the Macek program, it is important to fully understand the order in which options and commands are scanned and applied, as written above. The later execution of commands, in particular, means that all matrix entries and all options from the input are already known when the command is executed, even if they appear after this command. It does not matter how you mix between commands and options on the input.

Yes, there is an exception to the previous rule – the `@sub-*` options vs. command parameters. Keep in mind that the input macro substitution applies when reading the command, not when executing it. Therefore, all `@sub-*` options must appear before the respective macros are used on the input.

Also notice that some options, like '`@transpose`' or '`@inputpf`', behave more like commands, but they still remain as options with their immediate application during input scanning. (You may consider using the corresponding commands '`!dual`' or '`!pfield`', respectively.)

Before proceeding further, be sure that you understand about frame-addressing in command parameters; See Section 4.5 [Addressing], page 23.

A command has several or no parameters. (Among them, the possible output address '`>xxx`' has special meaning and position.) If the required parameter is missing, then it is substituted by its default value. Determining the default value is a kind of a magic; it depends on a command context, current frame tree, etc. If you are not sure what the default value in the specific situation is, then read the program output where the parameters are printed, or add the command '`!verbose`'.

Run the program with `-g3` and see the substitution made in the debugging output.

The purpose of this "magic" parameter substitution is to save you typing the parameters over and over. In most easy situations you may just use the commands with no parameters at all, and the default values will do what you expect. (Unless your expectations are very unrealistic.)

Another note concerns frame names and comments. Some commands change the frame name to indicate their effect on the frame (matrix). They also may set the frame comment to a description of the command.

To learn more, read next about particular commands recognized by the Macek program. The list of all recognized commands is obtained by calling '`./macek -HHc`'. (Not all of them may be described in this manual.)

6.2 Printing Commands

We start with the printing commands in the program. The command '`!pr`' prints a simple description of the given frame(s). The command '`!print`' similarly prints the matrix(ces) of the given frame(s), and '`!prtext`' prints the text given as a parameter.

In general, one may control the amount of information printed out with the commands '`!verbose`' and '`!quiet`', that can be used with a number `+-N`.

The command '`!prmore (mat-list)`' prints various additional matroidal information about the given matroid(s). The printed information does not depend on particular representation or the partial field, only on abstract matroid properties. This may be used to better understand the structure, or to compare matroids over different pfields by hand (using a hash-value printed at the end). Currently, numbers of bases are printed out, and small flats and separations are listed; all depending on the current output verbosity by '`!verbose N`'.

From version 1.2, several other interesting matroid properties are printed out in '`!prmore`', like connectivity, girth, representability, and aut group orbits for elements. Be prepared for long computation when calling '`!prmore`' on high `!verbose` levels — parts of the computation are currently implemented inefficiently by brute force methods, and so they take long time even on matroids of moderate size.

To display the whole subtree of the given frame(s), use the command '`!prtree`'. Each descendant frame (up to certain depth) is printed on a separate line, in the depth-first order. From version 1.2, only short lists of descendants are printed all, while for long lists the middle part is skipped to keep the listing reasonably long. The full listing may be obtained by using '`!verbose 2`'.

Try the following examples:

```
bash$ ./macek -pGF4 '!pr (s)' U24 U25
bash$ ./macek -pGF4 '!quiet;!pr (s)' U24 U25
bash$ ./macek -pGF4 '!prtree' U24 '{U25 U35}'
bash$ ./macek -pGF4 '!prtree (s)' U24 '{U25 U35}'
bash$ ./macek -pGF4 '!print ((t)(s))' U24 '{U25 U35}'
bash$ ./macek -pGF4 '!verbose;!print' U24

bash$ ./macek -pREG '!prmore' R10
bash$ ./macek -pGF3 '!verbose 2;!prmore' F7-
bash$ ./macek -pGF2 '!verbose 3;!prmore' F7
```

In addition to structural information printed out in '`!prmore`' above, new commands '`!prcircuits (mat-list) [id [id..]]`' and '`!prbases (mat-list) [id [id..]]`' are provided from version 1.2. They list all circuits or bases, respectively, in the given matroids. Optionally, one may specify a matroid element (via its label, up to 3 elements at once) that all printed circuits or bases must contain. Hence, in particular, one may easily print out all circuits using the specified element.

```
bash$ ./macek -pREG '!prbases' R10
bash$ ./macek -pREG '!prbases "" -1 5' R10
bash$ ./macek -pREG '!prcircuits' grQ3
bash$ ./macek -pREG '!prcircuits "" 1' grQ3
```

6.3 Writing to Files

The commands described here are provided for saving (writing) frames to files. One may write either one frame (including its matrix), or the whole subtree of a frame. The format of the output file is a text described in the section Section 4.1 [Frame-syntax], page 19. (The syntax is likely more formal than what you would give to the program on a command line, but the general rules are the same.)

When writing (or reading) files, search paths are used, See Section 3.5 [Environment], page 15. Each file name is automatically appended with the extension '.mck' (if it is not given otherwise). If no file name is specifically given to the command, then the name of the frame is used.

The command '!write (frame)' writes the frame addressed by (frame) to a file named by the frame. More than one frame may be given. The command '!writetree (frame)' writes the whole subtree of the frame addressed by (frame) to a file named by the frame. The commands '!writeto fname (frame)' and '!writetreeto fname (frame)' do the same, but they use a name 'fname' for the saved file. If 'fname' ends with a slash '/', then it is used as a directory prefix for writing, and the frame name is used for the file itself.

Possible commands contained in a frame are never written to a file. On the other hand, all options except @transpose are written, and even some options inherited from ancestors may be written if requested Section 5.1 [Inheritance], page 29. The exception is the @name option which is not written for the root frame, so that this frame later gets its name from the file name.

Again, we provide few examples:

```
bash$ ./macek -pGF3 '!writeto sample1 ((T))' 'W3'
bash$ ./macek -pGF3 '!writetreeto sample2 (T)' '{U24 W3}'
bash$ ./macek -pREG '!extend c;!write ((S))' grK33
```

There is also a separate command '!writecom (frame)' that writes the given frame (not its subtree) to a file named by this frame with many interesting (human-readable) comments concerning structural properties of the matroid stored in this frame. An error is issued when there is no matrix in the frame.

The properties printed out to the file in '!writecom' include matrix size, number of bases, listing of small flats, connectivity and girth, orbits of the automorphism group, and representability of the matroid over several small fields. Since some of the above computations are very complex (orbits and representability), be prepared to wait quite *long time* for this command on large matrices. The best way to learn this command is to look at the file created by the following example:

```
bash$ ./macek -pGF2 '!writecom' S8
bash$ ./macek -pGF7 '!writecom' '<Wh5;@name Wh5-commented'
```

6.4 Reading Frames

Under normal circumstances, you do not need any command to read frames, since the input frames are scanned with the input. However, in some cases extra commands are necessary; like if you want to read a frame in another partial field than the current one, or if you want to add more options to frames after scanning the input.

The command '`!read input >(dest)`' reads a frame (sub)tree from the string `input`, and stores the tree as rooted at the position `(dest)` in the current tree. The string `input` is considered similarly as a command-line argument to the program, including use of the line shortcuts as described in [line-shortcuts], page 19. The related command '`!mread input >(dest)`' reads all matrices from given `input`, and stores them to the given destination as a list of new frames.

The command '`!append (fram) input`' reads the given text `input`, and appends it to the given frame `(fram)`. The appending works as if the given text continued the original input stream of the frame, but you must understand that many other things may have already happened from the original input scanning, which may result in unexpected effects. In general, we suggest to use this command only in situations when you want to add more options to some frame during command executions, or if you want to add additional commands to the current root frame. (However, if you add commands to descendant frames, they never get executed again unless `!restart` is used.)

6.5 Manipulating Frames and Matrices

We provide two commands for rearranging the frame tree in the program. The command '`!move (src) >(dest)`' moves (copies, or deletes) the given source frames addressed by `(src)` to the destination positions addressed by `(dest)`. In accordance with the addressing convention Section 4.5 [Addressing], page 23, the source frames are copied if they are selected with `T` or `S`, and they are moved if selected with `t` or `s`. If the destination parameter is not given, then the selected frames are deleted from the tree. When deleting a frame with descendants, the whole subtree is disposed of. The root frame (of this command) cannot be deleted.

The command '`!flatten (src) >(dest)`' collects all descendants of the frames in `(src)`, and stores them in the positions addressed by `(dest)`. The command '`!mmove (src) >(dest)`' is similar to `!move`, but it moves only the matrix, and no other frame attributes. The command '`!setname name (frame)`' sets a new name to the given frame(s). In the case of multiple frames named with '`!setname`', one may use `"%d"` format string inside the `name` to number the frames from 1.

Many examples of the '`!move`' command are presented in Section 4.5 [Addressing], page 23. We provide a few more relevant samples here:

```
bash$ ./macek '!move ((T))+((T)) >(()(S));!prtree' grK33
bash$ ./macek '!mmove ((T))+((T)) >(()(S));!prtree' grK33
bash$ ./macek '!flatten (T) >(()(s));!prtree' '{grK5 grK33}'
bash$ ./macek '!flatten (s) >((s));!prtree' '{grK5 grK33}'

bash$ ./macek '!mmove (S)+(S)+(S) >((S));!setname "grK33-%d" \
                 ;!prtree' grK33
bash$ ./macek -pGF4 '!extend;!setname x-%d-x;!prtree' P7
```

Besides the tree-manipulating commands described above, we provide a bunch of commands for manipulating matrices in the frames.

In particular, we provide a command '!msize' for selecting matrices of required size(s). See Section 6.11 [Filtering], page 45. The syntax of '!msize (frames) R C "cmp" [S]' is the following: R, C are the numbers of rows and columns of the (standard-form) matrix, and optional S is the number of elements (rows+columns). The string "cmp" contains up to three characters telling about how the actual matrix size r, c, s ($s = r + c$) is compared to given R, C, S, respectively; where < means that it should be $r < R$, etc, > means $r > R$, and = means $r = R$. A special character dot . is used for values which are *not* compared. Notice that there is no character for "less or equal" or for "greater or equal".

```
bash$ ./macek '!prtree;!msize ((S)) 5 5 "=="' '{R10 R12 grK5}'
bash$ ./macek '!prtree;!msize ((S)) 5 5 "<>"' '{R10 R12 grK5}'
bash$ ./macek '!prtree;!msize ((S)) 5 5 "..=" 12' '{R10 R12 grK5 grV8}'
bash$ ./macek '!filt-msize ((S)) 5 5 "..=" 12;!prtree' '{R10 R12 grV8}'
```

The command '!dual (mat)' transposes the matrix(ces) in the given frame(s) (mat). Its effect is similar to the option '@transpose' applied after the matrix, but the important difference follows from the fact that options are applied immediately while commands are executed later, Section 5.6 [Other options], page 31.

The command '!pivot row col (mat)' pivots the given matrix in (mat) on the entry at row times col. (Rows and columns are numbered in order from 1.) The pivoted entry must be nonzero. Pivoting switches the labels of the pivoted row and column. The resulting matrix replaces the previous one (in the same frame).

The command '!delete lab (mat)' deletes the element of the label lab from the matroid represented by the given matrix in (mat). Note that, unlike when pivoting, the elements are identified by their *labels*, not by their order! This allows to delete (as a matroid element) not only columns of the matrix, but also rows after (implicit) pivoting. The command '!contract lab (mat)' similarly contracts the given element. Notice that deleting a coloop means contracting it, and contracting a loop means deleting it. (So such operations may change the matrix dimensions in an unexpected way!)

The command '!deleach (mat) >(dest)' creates a list of new frames with matrices obtained by deleting each one of the elements of the matroid represented by the given matrix in (mat). The resulting new frames are stored according to the output address given in (dest). The command '!coneach (mat) >(dest)' works similarly for contractions. The command '!remeach (mat) >(dest)' produces all one-element deletions and contractions of the matrix (as the previous two together).

```
bash$ ./macek -pGF2 '!print (S)' grK5 'grK5;!dual'
bash$ ./macek -pGF5 '!print;!pivot 2 1;!print' P8
bash$ ./macek -pREG '!print;!delete 2;!print' grK5
bash$ ./macek -pGF2 '!coneach;!print ((S));!prtree' W3
bash$ ./macek -pGF4 '!remeach;!print ((S))' U24
```

6.6 Structural Matroid Functions

We start with the command '`!inpfield (mat)`' which checks whether the given matrix(ces) (`mat`) is proper over the current partial field Section 3.6 [P-Fields], page 16. The result is printed out. This command has no meaning for normal fields.

```
bash$ ./macek -pREG ’@nopfcheck;!print (S);!inpfield (S)’ \
      ’ 1 1; 1 0’ ’ 1 1; 1 -1’
```

Another command '`!mhash h-value (mat)`' is used to find matroids in the list (`mat`) which have the given matroid hash-value `h-value`. This is the same hash-value as computed and printed in '`!prmore`' Section 6.2 [Printing], page 34. The value is matroid-invariant, and so it may be used to informally compare distinct matroid representations, even over different partial fields. Non-equality guarantees that the matroids are not isomorphic. The next example works when using matroids hash-value ver 1.0. If the program is upgraded to a higher hash version, you have to adjust these examples first.

```
bash$ ./macek -pRoot6 ’!prmore;!pivot 1 4;!prmore’ O7
bash$ ./macek -pGF3 ’!mhash 13068150 (S)’ O7 P7
```

Next we describe a collection of minor-structural commands. (You should first understand problems concerning inequivalent matroid representation from Section 4.3 [Representations], page 22.) If you want to see more about the command result (like where the minor is displayed, etc.), use the command `!verbose` before Section 6.2 [Printing], page 34.

The command '`!minor (mat) (min)`' finds out whether the given matroid(s) in (`mat`) has a "minor" in the given list (`min`). Here by "M having a *minor* N" we mean that some strongly equivalent matrix representation of M displays a submatrix which is unlabeled equivalent to N. So when asking for a minor N in the matroidal sense, one has to give **all representations** of N up to unlabeled equivalence in the list (`min`). The minor (if found) can be displayed when '`!verbose`' printing was requested before. In such case a submatrix of an equivalent matrix of (`mat`) that is equal to (`min`) up to scale is printed out. With '`!verbose 2`', all displayed minors are printed out.

The command '`!minorusi (mat) (min) id`' similarly tests whether the given matroid(s) in (`mat`) has a "minor" equivalent to (`min`), such that the minor is using the element labeled `id` in (`mat`). This is available from version 1.2, and the implementation is still very inefficient and slow.

The command '`!equiv (mat1) (mat2)`' looks for unlabeled equivalent pairs of matroids in the given lists. (In general, equivalence testing is much faster than minor testing.) The command '`!eqpair (mat)`' is similar to the previous one – it looks for each matroid in the list, whether some other matroid farther in the list is equivalent to this one.

The command '`!tmajor (mat) (min)`' tests whether the matroid in (`mat`) is a "tight major" of the given list (`min`) of matroids. In theory, a matroid M is a tight major of a family F if no element of M can be both contracted and deleted keeping a minor in F. For our implementation, the same notes as for `!minor` apply here. A warning is printed if (`mat`) itself has no minor in (`min`).

```
bash$ ./macek -pREG ’!minor’ ’{W4 R10 R12}’ grK33
bash$ ./macek -pREG ’!print;!verbose;!minor’ R12 grK33
bash$ ./macek -pREG ’!verbose 2;!minor’ grK4 grK3
```

```
bash$ ./macek -pREG '!deleach;!equiv' R10 grK33
bash$ ./macek -pREG '!eqpair' 'R12;!coneach (T) >((0t))'
bash$ ./macek -pREG '!verbose;!eqpair' 'R12;!remeach (T) >((0t))'

bash$ ./macek -pGF2 '!tmajor' '{S8 R12}' '{F7 F7#}'
bash$ ./macek -pGF2 '!verbose 2;!tmajor' R12 grK33
bash$ ./macek -pGF2 '!extend bb;!tmajor ((TS))' F7 '{F7 F7#}'
```

Finally, we are left with several other structural commands. Use '`!bwidth3 (mat)`' to see whether the given 3-connected matroid(s) have branch-width at most 3, or higher. (It is yours responsibility to ensure that the tested matroid really is 3-connected!) Call '`!fan (mat)`' to print the longest fan found in the given connected matroid(s), and '`!hasfan f (mat)`' to see whether the matroid(s) has a fan of length at least `f`, Again, you may request printing the fan with '`!verbose`'.

```
bash$ ./macek -pREG '!remeach;!bwidth3 ((TS))' R10
bash$ ./macek -pREG '!extend r;!bwidth3 ((TS))' R12
bash$ ./macek -pREG '!verbose;!bwidth3 ((TS))' R12
bash$ ./macek -pGF2 '!fan' '{F7 W3 W4 R10 R12}'
bash$ ./macek -pGF2 '!verbose;!fan' '{F7 W3 W4 R10 R12}'
```

We provide next commands for determining matroid connectivity. The command '`!connectivity (mat)`' prints the connectivity of given matroids (2,3,4,...). The commands '`!isconn (mat) c`' and '`!isconn3 (mat)`' check the required connectivity of given matroids. The command '`!isconni4 (mat)`' checks whether given matroids are internally 4-connected. Notice that, unlike [Oxley], we do not consider the matroid $U23$ to be 3-connected. We define a matroid M to be n-connected, $n > 0$, iff M has at least $2n - 2$ elements, and M has no proper k-separation for $k = 1, ..., n - 1$.

For example, '`!isconn (mat) 4`' is passed by matroids that are at least 4-connected. Another example uses the 3-connectivity filter (See Section 6.11 [Filtering], page 45.) to prepare correct input to '`!bwidth3`' command.

```
bash$ ./macek -pREG '!remeach;!connectivity ((TS))' R10
bash$ ./macek -pREG '!remeach;!connectivity ((TS))' grK5
bash$ ./macek -pREG '!remeach;!filt-isconn3;!bwidth3 ((TS))' grV8
```

The command '`!girth (mats)`' prints out the girth (length of the shortest cycle) of each of the given matroids. Similarly '`!hasgirth g (mats)`' tests which of the given matroids have girth at least `g`. A modified command '`!ispaving (mats)`' tests which of the given matroids are "paving", i.e. which of them have girth at least equal to the rank.

```
bash$ ./macek '!girth (S)' R10 R12
bash$ ./macek '!hasgirth 4 (S)' R10 R12
```

6.7 Matroid Isomorphism and Representability

This section continues with other structural Macek functions which are related to "abstract matroids", like abstract matroid isomorphism (even over different pfields), or general matroid representability over supported fields. These functions here have been added in version 1.1.9 (officially from version 1.2), and their implementation is not yet optimal. That means

the functions are computed correctly, but they may take very long time to compute on large
or regularly-structured matroids.

We provide the command '!isomorph (mats) (mats2)' to test isomorphism of the ma-
troids in the given list (mat) against matroids in the second list (mat2) (each first one
against all in the second list). It is even possible to give the input matroids represented
over different partial fields, Section 6.9 [Diff-fields], page 43. As the result matroids from
(mat) having isomorphic mates (anywhere) in (mat2) are accepted.

The command '!isompair (mats)' tests pairwise isomorphism in the given one matroid
list (mat). Specifically, the first matroid of each abstract isomorphism class in (mat) is
accepted as unique up to isomorphism. So the command provides a quick way to find
out how many non-isomorphic matroids are in the list, and to remove isomorphic pairs,
Section 6.11 [Filtering], page 45. Basic examples are presented next, and few more handling
matroids with representations over different fields are in Section 6.9 [Diff-fields], page 43.

```
bash$ ./macek -pREG '!isomorph' R12 'R12;!pivot 5 6'
bash$ ./macek -pNREG '!extend r;!isompair' R12
bash$ ./macek -pGF3 '!verbose;!isompair' \
            '{ "@inputpf gf4;<U36" "@inputpf gf5;<U36" "<W3" }'
```

The command '!repres PF (mats)' tests which of the matroids in (mats) are repre-
sentable over the partial field PF. (No representation is actually constructed.) The matroids
in the list may be given represented over any other fields. On the other hand, the command
'!represgen (mats) [all[q]]' generates (one or all) representations of the given matroids
in (mats) over the current partial field. Again, the matroids may be given represented over
any other fields. The exact function depends on the second optional argument — only (at
most) one representation is generated per each matroid by default; all representations up
to labelled strong equivalence are generated for the argument all; and all representations
up to unlabelled strong equivalence are generated for the argument allq.

It follows from theory that binary matroids have only one representation over any field,
and ternary matroids have only one ternary representation, up to labelled equivalence. For
example, 3-connected quinternary matroids may have up to six inequivalent representations
over $GF(5)$. Examples of the use of these commands follow.

```
bash$ ./macek -pGF4 '!repres GF5' '{R10 S8 F7 U24 U25}'
bash$ ./macek -pGF4 '!filt-repres GF5;!prtree' '{R10 S8 F7 U24 U25}'

bash$ ./macek -pGF5 '!represgen ((T)) all;!prtree' U25
bash$ ./macek -pGF5 '!represgen ((T)) allq;!prtree' U25
bash$ ./macek -pGF5 '!represgen ((T)) all;!prtree' F7-
bash$ ./macek -pGF7 '!represgen ((T)) all;!prtree' '@inputpf GF3;AG23'
```

6.8 Generating Extensions

An important function of the Macek program is to generate 3-connected (co)extensions of a
matroid over the partial field. All matroids we speak about here **must be 3-connected**. We
refer also to the description of options used in the extension generating process Section 5.5
[Extensions], page 30.

The command '`!extend [rcb]+ (mat) >(dest)`' generates 3-connected (co)extensions to the matrix given in (`mat`) according to the first text parameter, and stores them in the given destination position (`dest`). A letter `r` in the first parameter means to do a row coextension, a letter `c` means to do a column extension, and a letter `b` means to do both of them. (You would mostly use `b` here unless you know really well what you are doing.)

It is possible to combine multiple letters in the first parameter of `!extend`. For example, '`!extend bbb (mat)`' generates three steps of one-element extensions, and all results of each of the steps are stored. It is also possible to give more than one matrix in the input list (`mat`). Then the extensions are generated for each of the matrices. However, in such case it is *not allowed* to do multiple extension steps, i.e. only one letter may be then given in the first argument.

Try the following few examples:

```
bash$ ./macek -pGF4 '!extend b;!prtree' F7
bash$ ./macek -pGF4 '!extend c (S);!prtree' F7 F7#
bash$ ./macek -pGF2 '!extend bbb;!prtree' F7
```

To extend the given one matroid to a specified size and rank, use the command '`!extendto r c (mat)`'. This command repeats the extension steps until all extended matrices of size `r` times `c` are produced. (Unlike for `!extend`, the intermediate constructed extensions are not stored here.) Similar commands '`!extendtor r c (mat)`', or '`!extendtoc r c (mat)`', generate all extension matrices with exactly `r` rows and at most `c` columns, or with exactly `c` columns and at most `r` rows. Finally, the command '`!extendupto r c (mat)`' generates all extension matrices of sizes up to `r` times `c` including (`mat`) itself.

```
bash$ ./macek -pGF2 '!extendto 5 5;!prtree' F7
bash$ ./macek -pGF2 '!extendupto 5 5;!prtree' grK4
bash$ ./macek -pGF2 '!extendtoc 5 4;!prtree' grK4
bash$ ./macek -pGF2 '!extendtor 5 6;!extend cc >((S));!prtree' grK33
```

Another set of examples shows the effects of additional generating attributes given by the `ext-` options Section 5.5 [Extensions], page 30.

```
bash$ ./macek -pGF5 '!extend;!prtree' U25
bash$ ./macek -pGF5 '@ext-forbid "U35";!extend;!prtree' U25
bash$ ./macek -pGF5 '!extend;@ext-forbid "U25;!dual";!prtree' U25
bash$ ./macek -pdyadic '!extend;!prtree' F7-
bash$ ./macek -pdyadic '!extend;@ext-nofan 4;!prtree' F7-
```

When generating extensions with '`@ext-forbid M`', only those extensions not containing an M-minor are created. This construction is equivalent to generating all extensions, and then filtering out those with an M-minor, but the above example is faster. On the other hand, the option '`@ext-nofan f`' is a *sequential* option – it restricts the appearance of the whole generating sequence, not only the resulting matroid. For example, a matroid N having no fan would not be generated with '`@ext-nofan 4`' if all sequences leading to N contain a 4-fan.

Warning!! The above described commands `!extend*` are very complex in their nature, and one may easily produce "false" results when (s)he does not fully understand all

the hidden details of the computation. That is why we provide here the following detailed explanation of the extension-generating algorithm in Macek. The theory behind our extension-generating algorithm is written in the paper [P. Hlineny: Equivalence-Free Exhaustive Generation of Matroid Representations, Discrete Appl. Math. 154 (2006), 1210-1222, http://dx.doi.org/10.1016/j.dam.2005.12.001]. However, you do not have to worry about most of the details if you are working only within "nice" partial fields with unique matroid representability like binary, regular, or ternary.

We assume that the reader is familiar with matroid representations and their (in)equivalence Section 4.3 [Representations], page 22. A matroid S is called a *stabilizer* for a given partial field if, for any 3-connected (or stable) matroid M with an S-minor, any two representations of M displaying the same subrepresentation of S are strongly equivalent. Moreover, S is called a *strong stabilizer* if every subrepresentation of S extends to whole M.

Consider that we want to generate all matroids having the given matroid (called further the *base minor*) S as a minor, subject to representability over the pfield and to other conditions (attributes). Then, by Seymour's splitter theorem, there is a sequence of single steps (extensions / co-extensions) building a matroid M from the base minor S keeping 3-connectivity; except the case when the base minor is a wheel or a whirl(!). Such a sequence of single steps, when viewed in the reverse order (i.e. as deletions / contractions), is called the *elimination sequence* for the (resulting) matroid M over the base minor S.

Formally, an elimination sequence consists of the base minor S in the given matrix representation, of the resulting matrix for the matroid M, of the order of lines of M as they are added to S, and of the *signature* of the sequence telling which lines of M are extended and which are coextended. (The sequence signature is taken separately from the order since the order naturally follows from the matrix representation of M, while the signature does not.) The base minor S is always displayed in the matrix representation of M in the upper-left corner.

We call two elimination sequences *equivalent* if their base minors are unlabeled identical, they have the same additional attributes, and their resulting matrices are unlabeled equivalent. To avoid generating equivalent sequences repeatedly, we require the generated elimination sequences to be minimal with respect to the following *canonical order*: We compare two sequences lexicographically first by their signatures (preferring the signature bits corresponding to lines closer to the base minor), and then by their lines as they are added in the sequence (again preferring the lines closer to the base minor).

One important note concerns the use of letters `r,c` as modifiers of the '`!extend [rcb]`' command: These letters allow you to choose which extensions (row/column) are done at each step of generating, but they do not modify in any way the canonical order of sequences. In particular, if you specify '`!extend cr`', then you get only(!) those canonical extensions that happen to add a column before adding a row, but not those that add a row before adding a column. Better use the '`!extendto*`' variant of generating to obtain specific size of matrices.

After all, unless you are using "nice" partial fields with unique representability like binary, regular, or ternary; we suggest to generate extensions from strong stabilizers, to guarantee extendability and to limit inequivalent representations. Notice, however, that even when the base minor S is a strong stabilizer for the current partial field, two nonequivalent

sequences may produce isomorphic matroids (to M) — this may happen if there are more (labeled) S-minors in M which display inequivalent representations of S.

If you would like to get only non-isomorphic extensions out of the generation process, you may use the command '`!filt-isompair`' afterwards, Section 6.7 [Isomorphism], page 39. Keep in mind that isomorphism-filtering may be used only after the generation has completed, since not all nonequivalent isomorphic extensions extend further in the same way. Read more in examples Chapter 7 [Practical], page 47.

Since the options `@ext-bsize` and `@ext-signature` describing the elimination sequence are stored with the generated extension matrices, one may continue the generating process in multiple steps while still keeping uniqueness of the generated sequences over the whole universe generated from S. This allows to continue the computation in parallel on many computers – make the first step(s), and then distribute each of the extensions to another computer.

However, never touch the `@ext-bsize` and `@ext-signature` options by hand, or(!) you twist the canonical order and lose extensions. Moreover, never change the matrix between generating steps for the same reason, and do not mix matroids of elimination sequences created in different major versions of the program. Do not even switch between partial fields during generating.

If you want to generate extensions of, say, two base minors $S1, S2$, then you have to manually exclude repetition of those extensions that contain both $S1, S2$. One easy way to achieve this is to generate the extensions of $S2$ with an additional option '`@ext-forbid S1`'. (Consider also the option '`@extinherit[all] ext-forbid`' to inherit the exclusion of $S1$ for the generated extensions.) You may learn more in the practical examples below Chapter 7 [Practical], page 47.

6.9 Working in Different Partial Fields

The Macek program allows to work in different partial fields than the given one by `-pPF` Section 3.1 [Command-line], page 13. The command '`!pfield NPF`' switches the program to temporarily use a new partial field `NPF`. The new partial field stays in effect until a new call to `!pfield`, or until the current frame execution is finished. Compare this command with the option '`@inputpf PF`', Section 5.6 [Other options], page 31.

However, calling `!pfield` only switches the program's internal arithmetic, but the matrices in the frame tree still remain represented over the original partial field. If you want to work with them over the new partial field, then you must first import them. (That concerns practically all commands working with the matrix or the matroid, except basic printing, abstract isomorphism test, and representability functions, Section 6.7 [Isomorphism], page 39.) For that purpose the '`!import transl (mat)`' command is provided. All entries of the matrices in the list (`mat`) are translated to the current partial field using the translation named `transl`. Read about partial-field translations in Section 3.6 [P-Fields], page 16.

After importing a matrix to a different partial field, this matrix may no longer be proper Section 3.6 [P-Fields], page 16. So, unless you are sure that this matrix is indeed proper, you should call the command `!inpfield` to test it.

```
bash$ ./macek -pGF4 '!print;!pfield GF2;!import Id0;!print' U24
bash$ ./macek -pNREG '!print;!pfield GF3;!import Nreg-tr;!print' P7
bash$ ./macek -pNreg '!extend r;!prtree;!pfield GF3;!import \
        Nreg-tr ((S));!print;!eqpair' P7
```

An interesting example of the use of different pfields in Macek computation is presented in connection with isomorphism testing. We generate extensions of the given matroid over different fields, and then we look for isomorphic pairs in the generated lists:

```
bash$ ./macek -pGF5 '!extend r;!pfield GF4;!import Id0 ((T)) \
        ;!extend r >((S));!prtree;!isomorph' grK33
```

6.10 Command-Flow Control

The Macek program provides simple command-flow control described here. The command '!restart' restarts command processing in the whole frame tree. (Commands already processed are deleted, so they are not executed again.) This command may be useful, for example, in connection with the command !append which adds new code to descendant frames. See Section 6.4 [FReading], page 36.

The command '!skip n' causes command processing to skip the next (up to) n commands in the current frame. Commands in other frames are not affected. The command '!exit r' immediately stops command execution, and returns the value r to the calling shell. See below for an example.

The command '!iflist len [<=>!] (fram)' is used to test whether the given frame-list (fram) contains number of frames comparable to the value len. One may use relations =, !=, >, >=, <, <=. If the relation is true, then the next command after !iflist is executed, otherwise the next command is skipped. If you want to skip more commands, use !iflist in combination with !skip.

Try the following simple examples.

```
bash$ ./macek '!iflist 0 < (S);!print (S)'
bash$ ./macek '!iflist 0 < (S);!print (S)' R10
bash$ ./macek '!iflist 0 = (S);!skip 3;!print;!extend;!prtree'
bash$ ./macek '!iflist 0 = (S);!skip 3;!print;!extend;!prtree' W3
bash$ ./macek '!iflist 0 = (S);!skip 2;!print;!extend;!prtree'
```

The next example is more involved. See that the command !extend is copied to all descendant subframes, and then it is executed in each one of these subframes after !restart.

```
bash$ ./macek '!append (S) "!extend c (T) >((0t))"\
        ;!restart;!prtree' W3 W4 R10
```

Another involved example is the procedure &splitlist (Section 6.12 [Procedures], page 46) distributed with the package:

```
# - use in macek as  '{<list>;&splitlist [length] [depth()]'
@subd-param1    10
@subd-param2    "(("
!move ${param2}(${param1}t)| >${param2})(s)|
!iflist ${param1} < ((S))
!append (T) "&splitlist $param1 $param2"
```

This procedure serves for breaking-up the given frame list into small pieces. Notice the command-flow in that procedure; first one small chunk of the list is moved to a new node, then the length of remaining list is tested, and, if longer than the given value, the whole procedure is appended again to the current frame. (The appended commands are automatically executed after `!append`.) An example of use is here:

```
bash$ ./macek '!prtree' '{<bw3-tern-exc};&splitlist 4'
```

Another useful command for "high-level" Macek programming is '`!ifshell "sh-com" [stat]`'. This command executes the string '`sh-com`' in the system shell ('`/bin/sh`'), and then it tests its return status against `stat` (or aginst `0 - Success` by default). If successful, then the next command is executed, and otherwise it is skipped.

So '`!ifshell`' is a general interface between Macek and the shell environment, allowing to execute *any system command or action* and to test the resulting status. One may create directories, copy or erase files, and do many other things in this way. (Be careful not to do nasty things to your loved computer. . .) One common use of this command would be testing existence of input files (to avoid error messages for missing input) using standard system utility `test`, like in the following examples. The first line checks whether there is a subdirectory `temp/` present in the current directory. The example in the second line looks whether there is a readable input file named '`infile`', and if so, then the matrices from '`infile`' are read into Macek.

```
bash$ ./macek '!ifshell "test -d temp" 0;!prtext YES'
bash$ ./macek '!ifshell "test -r infile" 0;!mread infile >((S))'
```

However, the above examples look for files in the current directory, ignoring internal Macek search paths. To find out whether there is a readable file `fname` somewhere on the input path, there is a specialized command '`!iffile fname`'. So the previous example would better be written as:

```
bash$ ./macek '!iffile infile;!mread infile >((S))'
```

Try also the next examples. Run the first one, and watch the result — if the answer is YES, then you may get into big troubles. The next two lines of examples show you that it is really possible to run any program via the '`!ifshell`' interface. (You may quit the vi editor by typing 'esc', ':', 'q', 'enter'.)

```
bash$ ./macek '!ifshell "test -w /etc/passwd" 0;!prtext YES'
bash$ ./macek '!ifshell "who"'
bash$ ./macek '!ifshell "vi i_love_macek.txt"'
```

The `!exit` command returns an integer value back to the calling shell. This is a mechanism commonly used to indicate a success/failure of program operations on unix systems. In the `bash` shell, one may retrieve the returned value as follows:

```
bash$ ./macek -g-2 '!exit 123' ; echo $?
123
```

6.11 Command-output Filtering

Some of the above described commands that usually print a "yes/no"-type answer, may be modified by a prefix to filter the input list of frames. The prefix '`filt-*`' causes the

command '*' to keep those frames for which the answer is "yes", and to delete the others. (Address the frames with s or t.) The prefix 'filx-*' has the exactly opposite meaning.

The prefixes 'rem-*' and 'rex-*' suppress both printing and filtering in the command. The only result of such a modified command is the resulting list to be remembered for subsequent '~N,^N' parameter addressing, Section 4.5 [Addressing], page 23.

The commands that can be modified by these prefixes include !minor, !tmajor, !inpfield, !isconn, !equiv, !hasfan, !bwidth3, !mhash, !msize, !hasgirth, !isomorph, !repres. Find out the current list of all modifiable commands by calling 'macek -Hc'.

Here are a few examples that illustrate these concepts:

```
bash$ ./macek -pREG '!deleach;!coneach;!minor' R12 grK33
bash$ ./macek -pREG '!deleach;!coneach;!filt-minor;!prtree' R12 grK33
bash$ ./macek -pREG '!deleach;!coneach;!rem-minor;!pr ~1' R12 grK33
```

A simple command '!mark (list)' is provided to mark all frames in the given list for subsequent '~N,^N' parameter addressing.

6.12 Procedures – Collecting Commands

Often, one needs to execute a whole sequence of commands repeatedly for different parameter values. For this purpose the program provides the concept of *procedures*.

A procedure line starts with the keyword 'procedure' or the character '&'. The procedure is written as '&proc p1 p2 ...'. When scanning input, such a procedure call is expanded into the following actions: Substitutions are created as '@sub-param1 p1', '@sub-param2 p2', etc. Then the file 'proc' is included into the place. It is assumed that this file contains a sequence of commands, using the parameter values as $param1, $param2, etc. The possible output parameter '>out' is accessed as $paramres. To give default values to the procedure parameters, use '@subd-param1 p1-default'.

See examples distributed with the program in 'Procedures/*'...

To simplify single command-line calls to some mostly used Macek functions, we provide few shortcuts that are implemented as include files. You may use simple calls like the following ones:

```
bash$ ./macek print R10
bash$ ./macek -pGF4 print U35
bash$ ./macek print grK33 grK5
bash$ ./macek -pREG prints grK5
bash$ ./macek -pREG connect R10 R12
bash$ ./macek -pREG minor R10 grK33
bash$ ./macek -pREG equiv R10 grK33
```

The above shortcuts use the feature of an automatic file-include for command line arguments to Macek. So 'print' is actually a file containing the print command, and similarly with others. See 'Procedures/shortcut/*'. User may easily prepare more such shortcuts. However, we suggest to use shortcuts only in those very simple situations like the above examples.

7 Practical Macek Computations

In this chapter we bring several practical examples of computations with the Macek program. They are intended both to demonstrate the power of our program in practice, and to indicate that the computation is correct and it can be verified here.

7.1 Simple Examples, ver. 1.0

We first present several rather simple examples showing computations in version 1.0 of Macek. Those computations are, of course, still correct in higher versions, but they can be made in different and more efficient ways now...

7.1.1 $R10$ as a Splitter for Regular Matroids

We start with a computer proof that the matroid $R10$ is a splitter in the class of regular matroids (see Seymour's decomposition theorem).

```
bash$ ./macek -pREG ’!extend b;!prtree’ R10
```

This program call works over the regular partial field. A representation of the matroid $R10$ (distributed with the program) is read from a file, Then the command '`!extend`' is called to get all 3-connected row- and column-extensions of the matrix of $R10$ in regular numbers, using the default parameter address (`(T)`). As you may immediately see, no extension is generated. (If there were some, they would be stored to `>(((0t)))`.) Thus, using Seymour's splitter theorem, $R10$ is a splitter for the class of all 3-connected regular matroids.

Similarly, we can show that $R10$ is a splitter for 3-connected near-regular matroids.

```
bash$ ./macek -pNREG ’!extend b;!prtree’ R10
```

7.1.2 Extending $F7$ in the Binary Field

In this case, we generate all binary extensions of the matrix of the Fano matroid $F7$.

```
bash$ ./macek -pGF2 ’!extend b;!prtree;!minor’ F7 F7#
```

Then we print the two generated extension in the tree (as sons of) $F7$, and finally we show that both the extensions have the dual of $F7$ as a minor. Hence $F7$ is a splitter for binary matroids with no $F7*$ minor.

Alternatively, one may achieve the same result with another call that excludes the $F7*$-minor immediately in the generating process. (Notice that commands are executed even inside option parameters.)

```
bash$ ./macek -pGF2 ’@ext-forbid "F7;!dual";!extend b’ F7
```

We continue in the direction of the previous example.

```
bash$ ./macek -pGF2 ’!extend r;!print ((s))’ F7
```

We compute the two binary row coextensions of the Fano matroid $F7$, and print them. See that the first one of them is the affine plane $AG(3,2)$, and the second one is known as $S8$. Both of these matroids are distributed with the program.

```
bash$ ./macek -pGF2 ’!extend c;@ext-forbid "AG32;!dual"’ S8
```

The latter call shows that there is only one column extension to $S8$ with no $AG(3,2)*$-minor. This extension is sometimes known as $P9$.

7.1.3 Extending $K5$ in the Binary Field

Here we generate all binary 3-connected extensions of the cycle matroid of the graph $K5$,

```
bash$ ./macek -pGF2 '!extend b;!minor' grK5 '{grK33,"grK33;!dual"}'
```

and then check which of them have the matroid of the graph $K33$ or the dual of it as a minor. We see that 5 out of 6 generated matroids have one of the minors. Can you verify why, by hand?

Now we compute the binary extensions to $K5$ in three steps, adding one element at each step. We immediately exclude those extensions with $K33$ or the dual as a minor.

```
bash$ ./macek -pGF2 \
        '@ext-forbid grK33 "grK33;!dual";!extend bbb;!prtree' grK5
```

Notice that this command should be called from one (logical) line of the shell. Since no new extension is generated at the third step, this computation proves that there are altogether only two binary 3-connected (row) extension to $K5$ without $K33$ or the dual as a minor. These two matroids are known as $T12$ and $T12/e$.

7.1.4 Ternary vs. Regular Extensions

In this example we show that generating regular extensions of a matroid gives the same results as generating the same extensions over the ternary field with a forbidden $U24$-minor. (The idea behind this is that ternary matroids without $U24$-minor are binary, and hence also regular.)

```
bash$ ./macek -pREG \
        '!extendsize 6 6;!prtree;!writetreeto ex-reg ((T))' grK33
bash$ ./macek -pGF3 '@ext-forbid U24;\
         !extendsize 6 6;!prtree;!writetreeto ex-tern ((T))' grK33
```

Again, these commands should be called from one (logical) line of the shell. The resulting lists of each of the extension commands are written to the files 'ex-reg.mck' and 'ex-tern.mck'. (Actually, the same list as ex-tern can be obtained by generating all ternary extensions, and then filtering out those with $U24$-minors.) Finally we look at the generated lists again, and show that they are the same with the following command. We do not need to switch between the regular and ternary partial fields here since the regular representations may be read anywhere.

```
bash$ ./macek -pGF3 '!equiv' ex-tern ex-reg
```

7.1.5 Extending $F7$ in the Quaternary Field

In this example we show all the quaternary 3-connected extensions of the Fano matroid $F7$ without $U25$-minors.

```
bash$ ./macek -pGF4 '!extend bb;@ext-forbid U25;!minor' F7 U24
```

Notice that all such generated extension have no $U24$-minor either, and so they are all binary. Since this computation would continue forever, it cannot serve as a rigorous proof, but it suggests that all quaternary extensions of $F7$ without $U25$-minor are binary. (Which is, indeed, known to be true.)

Try the same example with more extension steps, that is like:

```
bash$ ./macek -pGF4 '!extend bb..b;@ext-forbid U25;!minor' F7 U24
```

7.2 Notes on Extension Generation

In this section we mention two simple practical issues which a user running extension generating commands in Macek should understand. (The examples come from version 1.0, and more can be obtained from them using current means of Macek.)

7.2.1 Examining Near-Regular Extensions

The near-regular partial field is a well-known structure (analogous to the reguar one), where all those matroids can be represented which have representations over every field except possibly $GF(2)$.

   We show how to generate near-regular extensions of the matroid $P7$, and how to view the inequivalent ones (of the same matroid). Run the following two commands, and watch their output carefully.

```
bash$ ./macek -pNREG '!extend b;!quiet;!prmore' P7
bash$ ./macek -pGF3 '!extend b;!quiet;!prmore' P7
```

You see that a total of 8 extensions are generated in the near-regular case, but only 4 of them have distinct matroid hash-values. Distinct matroid hash-values always mean non-isomorphic matroids. This suggests that there are, in fact, only 4 non-isomorphic extensions. (Which can be verified by other means, try it... See Section 6.7 [Isomorphism], page 39, in version >1.1) On the other hand, there are 12 extensions generated in the ternary case, and you may find all those 4 near-regular hash-value classes as distinct matroids there (non-isomorphic since ternary representations are unique).

   This example is to show you that nonequivalent representations of the same matroid frequently occur when working over more complex (partial) fields, and present a possible way how to handle them. (See Section 6.7 [Isomorphism], page 39, for a better current way.)

   Another interesting point here is that two distinct matroids of the 12 ternary extensions have the same hash-value. (This is in version 1.0 hash-values which may change in the future!) To see that the matroids are really different, find out that one has two triangles while the other has only one. Run the same script without the suppressing command !quiet to see more about structure of these two (and others) matroids. Hence, even if two hash-values are the same, the matroids still may be non-isomorphic.

```
bash$ ./macek -pGF3 '!extend r;!prmore' P7
bash$ ./macek -pGF3 '!extend r;!verbose;!prmore' P7
```

7.2.2 Extending Whirls

Here we want to demonstrate the fact that matroid wheels and whirls are exceptions in Seymour's splitter theorem which is a base of our extension-generating algorithm, Section 6.8 [Generating], page 40. This is another potential problem, in addition to non-equivalent representations, that must be closely watched when using the extension generating functions of Macek.

```
bash$ ./macek -pGF4 '!extendsize 4 4;!writetreeto xu24 ((T))' U24
bash$ ./macek -pGF4 '!extendsize 4 4;!writetreeto xwh3 ((T))' Wh3
bash$ ./macek -pGF4 '!equiv' xwh3 xu24
```

The first call generates 8-element rank-4 extensions of $U24$ – the 2-whirl. The second call generates 8-element rank-4 extensions of $Wh3$ – the 3-whirl. On the third line you may

then see that not all matroids generated secondly appear in the first list, despite all of them having a $U24$-minor.

7.3 Ternary Excluded Minors for Near-Regular

It is known that if M is an excluded minor for near-regular representability, then M has at most 8 elements [Geelen, unpublished]. Moreover, it happens that all four excluded minors for GF(3) representability are also excluded minors for near-regular representability. Thus to find all excluded minors for near-regular representability, one just has to search through ternary non-binary matroids. Similarly as above, all such matroids must contain an extension of the 3-whirl.

We show how to find all 6 such excluded minors using the Macek program. (They are $F7-$, $AG(2,3) - e$, their duals, $P8$, and $AG(2,3) - e$ by $DeltaY$.)

The example comes from version 1.0, and it can be made much simpler with means of Macek 1.2. (Try to design a better procedure as an exercise, and compare the results.)

The whole script, named `excnreg`, is prepared as a procedure. Each call executes one step of the whole computation. (There are, actually, only two meaningful steps, since we need to get from 6-element Wh3 to 8 element matroids.) The procedure takes one argument which is then used to construct the filenames of three input lists - starting near-reg and ternary lists, and previous excluded minor list. (Possible second argument should be always 'b'...) The default value for the argument is `"nre6."`.

```
#
# Call this script as  macek -pNREG '&excnreg nre6.' .
@subd-param1 "nre6"
@subd-param2 "b"
@sub-extdesc $param2
@sub-nrein $param1
@sub-nretin $nrein-tern
@sub-nrexcl $nrein-excl
```

This part prepares the working subframes with their names and comments. (Not really necessary, but analogous to neat data declarations in usual programms.)

```
{
@comment "For extensions in Near-Reg :"
{
@comment "the starting list"
<$nrein
}{
@comment "the extensions"
}}
{
@comment "For extensions in GF(3) :"
@ext-forbid $nrexcl
{
@comment "the starting list"
# (must be read in ternary later!)
```

```
}{
@comment "the extensions"
}}
{
@comment "For filtering minors"
{}{}
}
```

Here we generate all next-step extensions in the near-regular partial field. Then we switch to the ternary field, and generate analogous ternary extensions. The forbidden list for ternary extensions — excluded minors known so far, is given by an option above. Notice that we can read the input ternary list only after switching to the ternary field, otherwise an error would occur. (Other possibility is to consider the option '@inputpf GF3' above...) We have to keep the ternary and near-regular lists separately (despite them containing the same matroids), because their extension signatures are different.

```
!quiet
!prtree
!prtext "Now we generate all next-step extensions in Near-reg."
!extend $extdesc (((S))) >((()((0t)|

!pfield GF3
!prtext "Now we generate extensions in GF3 with no known excl minors."
!read $nretin >((3)(t))
!move ((3)(s)) >(()(((0t)|
!extend $extdesc (()((S))) >(()(()((0t)|
```

Next we look at which of the ternary extensions have no minor among the near-regular extensions. These are the new excluded minors. Resulting lists are written to files for use in the next step.

```
!move ((()(S)| >((2)(((0t)|
!import nreg-tr ((2)((S)|
!rex-minor (()(()(S)| ((2)((S)|
!move ^1 >((2)(()((0t)|
!read "@comment \"For merging excluded minors :\";$nrexcl" >((3)(t)|
!move ((2)(()(S)| >((3)((0t)|
!prtree

!writetreeto "$nrein$extdesc-exc" ((2)(()(T)|
!writetreeto "$nrein$extdesc-excl" ((3)(T)|
!writetreeto "$nrein$extdesc-tern" (()(()(T)|
!pfield Nreg
!writetreeto "$nrein$extdesc" (((()(T)|
!prtext "One step of &excnreg finished. Call next with one more \"b\"."
```

Finally, we show how the above procedure is called to get the final result. The whole file defining the procedure should be included in the Macek distribution under the name 'excnreg'. You first have to create the starting lists of matroids 'nre6.' and 'nre6.-tern' looking like

```
    { Wh3 }
```
and an empty file 'nre6.-excl' for collecting the excluded minors. Then you call:

```
bash$ ./macek -pNREG '&excnreg nre6.'
bash$ ./macek -pNREG '&excnreg nre6.b'
...
```

7.4 New Simple Examples, ver. 1.2

Here we add few examples demonstrating the new powerful functions of Macek from version 1.2.

7.4.1 Extended Matroid Information

Run the following command to get many interesting informations about a matroid. (However, note that computing these informations may take quite long time.)

```
bash$ ./macek '!verbose 2;!prmore' R10
```

Among other, you can see the orbits of the automorphism group, listings of all small flats and separations, representability over other fields, etc.

Moreover, you may list all bases or all circuits of a matroid in the following way. The last line shows an invocation which lists only those bases containing the two elements labeled by 2 and by -1.

```
bash$ ./macek '!prbases' R10
bash$ ./macek '!prcircuits' R10
bash$ ./macek '!prbases "" 2 -1' R10
```

7.4.2 Inequivalent Representations

The issue of inequivalent representations of a matroid has already been discussed from the point of view of version 1.0. See Section 7.2 [Pr-gener], page 49. Now we show how new functions from version 1.2 can be used to get much more.

Matroids have unique representations over the fields $GF(2)$ and $GF(3)$. 3-connected matroids have at most 2 inequivalent representations over $GF(4)$, and at most 6 over $GF(5)$ [Whittle et al]. We may sample-verify these facts using computations like:

```
bash$ ./macek -pGF4 '!extend rrr;!verbose;!represgen "" all' U25
bash$ ./macek -pGF5 '!extend rr;!verbose;!represgen "" all' U26
```

On the other hand, the numbers of possible (labeled) inequivalent representations of matroids over larger fields is unlimited, and we may see rapidly growing numbers of representations here:

```
bash$ ./macek -pGF7 '!extend r;!verbose;!represgen "" all' U27
bash$ ./macek -pGF9 '!extend r;!verbose;!represgen "" all' U27
```

7.4.3 Isomorphism and Exhaustive Generation

Macek can be simply used to exhaustively generate all matroids over some fields. See details in [P. Hlineny: Equivalence-Free Exhaustive Generation of Matroid Representations, Discrete Appl. Math. 154 (2006), 1210-1222, http://dx.doi.org/10.1016/j.dam.2005.12.001]..........

7.5 Matroids of Branch-Width 3

7.5.1 Regular Excluded Minors for Branch-Width 3

In this example we show that the regular matroid $R10$ is an excluded minor for matroids of branch-width 3.

```
bash$ ./macek -pREG '!bwidth3 ((T));!deleach;!coneach;!bwidth3' R10
```

The first command `!bwidth3 ((T))` verifies that $R10$ itself has branch-width bigger than 3. Then the next two commands construct all one-element deletions and contractions of $R10$, which are all stored to the default address `>(((0t)))`. Finally, we see that all these deletions and contractions have branch-width 3 in `!bwidth3` (with the default frame address `((S))`).

One *important remark* here is that you cannot perform the same computation with an arbitrary matroid, since the command `!bwidth3` requires a 3-connected matroid on the input. Since the matroid $R10$ is 4-connected, all its one-element deletions and contractions are indeed 3-connected, but this may not be true for other matroids! Then you have to use the command '`!filt-isconn3`' to filter out the matroids which are not 3-connected.

```
bash$ ./macek -pREG '!remeach;!filt-isconn3;!bwidth3 ((TS))' grV8
```

(The graph $V8$ is another such an excluded minor.)

7.5.2 Generating the Excluded Minors for Branch-Width 3

It is known that if M is an excluded minor for branch-width 3, then M has at most 14 elements [Hall, Oxley, Semple, Whittle]. Moreover, Dharmatilake conjectured that all binary excluded minor for branch-width 3 belong to the following set: $\{grQ3, grO6, grK5, grK5*, grV8, grV8*, R10, ND11, ND14, ND23\}$. The first seven of them are regular matroids. We need to look only at remaining binary non-regular matroids on up to 14 elements.

We have proved this conjecture [P. Hlineny: On the Excluded Minors for Matroids of Branch-Width Three, Electr. J. of Combinatorics 9 (2002), R32, `http://www.combinatorics.org`], using a Macek script `bw3bin` which has been described in the older versions of the manual. (The script is still included in the distribution.) However, since the script is outdated in the current version of Macek, we describe here the new script usable for generating branch-width-three excluded minors over any given field.

New flexible script `bw3excg`...................

```
#
# This script is used to compute all (small) excluded minors for branch-width 3
# over a given field.
# Call the procedure as   ./macek '&bw3excg gfX filebase suffix [b]' .
# (The correct pfield -pgfX is set automatically.)
#
# Before starting the procedure, create the files "bw3-gfX-" with the starting
# list of matroids to generate from, and "bw3-gfX+exc" containing (possible)
# extra excluded minors for bwidth3 (over other fields, with '@inputpf gfZ'
# for each one).
# Then run a sequence of commands like these:
```

```
#  ./macek '&bw3excg gfX bw3 '
#  ./macek '&bw3excg gfX bw3 b'
#  ./macek '&bw3excg gfX bw3 bb'
#  ...
#  ./macek '&bw3excg gfX bw3 bbb...'
# These will generate the excluded minors step-by-step (each step adding one
# element to the matroids), storing the excluded minors to "bw3-gfX-b..b-exc.mck".
#
# A table of computed results:
#  GF2:  6 on 10, 4 on 12  (7 out of them known regular,
#  only 3 binary ones are actually generated!),
#  GF3:  plus 18 on 9, 31 on 10, and no more!
#  GF4:  plus 5 on 8, 90 on 9, 32 on 10, ...  (nonisomorphic!)
#  GF5:  plus 38 on 8, 444 on 9, 29 on 10 (nonisomorphic!)
#  GF7:  plus 2 on 7, 119 on 8, 344 on 9 (nonisomorphic!)
#  GF8:  plus 0 on 7, 5 on 8
#  GF9:  plus 0 on 7, 0 on 8
#

@subd-param1  "gf3"
@subd-param2  "bw3"
@subd-param3  ""
@subd-param4  "b"

!pfield $param1
#!verbose

# the file names to use: usefilen the base, usefilenb the starting list,
# and the excluded and generated lists...
@sub-usefilen  ${param2}-${param1}
@sub-usefilenb ${usefilen}-${param3}
@sub-excextra  ${usefilen}+exc
@sub-treeall   ${usefilenb}-all
@sub-listin   ${usefilenb}
@sub-list3out  ${usefilenb}${param4}
@sub-list4out  ${usefilenb}${param4}-4
@sub-list4outn  ${list4out}n
@sub-exclist   ${listin}-exc
@sub-exclistout ${list3out}-exc

@sub-excluded   "((((S)(S)|"
@sub-excludedin  "((((1)("
@sub-excludedout "((((2)("
{
@name "bw3excg-w"
@comment "bw3excg (over $param1) working subframe:"
{
```

```
@name "exc-known"
@comment "known bw3 excl minors - extra, smaller, and new (generated)"
{
@name $excextra
!quiet
!iffile "$excextra"
!skip 1
!skip 4
!read $excextra
!filx-isompair ((s))
!pfield $param1
!represgen "((s))" allq >((0t))
}{
!quiet
!pfield $param1
!iffile "$exclist"
!mread $exclist >((0t))
}{ }
}{
@name extens1
@comment "all new ${param4}-extensions of input [${listin}]..."
}{
@name e-bwidth4
@comment "those generated with bwidth 4 get here:"
}{
@name e-bwidth4n
@comment "those new excl-minors with bwidth 4 get here:"
}{
@name e-bwidth3
@comment "those next with bwidth 3 get here:"
}}
@sub-input "(()(S))"
{
@inputpf $param1
<$listin
@comment "this is the starting set of matroids $listin:"
}

@sub-gener3  "(((4)("
@sub-generall  "(((1)("
@sub-gener4bw  "(((2)("
@sub-gener4n  "(((3)("
@extinherit ext-forbid

!extend b $input  >$generall(0t)|
!move ${generall}S|  >${gener3}(0t)|
!rex-bwidth3 ${gener3}S|
```

```
!move ^1  >${gener4bw}(0t)|

!writetreeto ${list3out} ${gener3}T|
!iflist 0 "<" ${gener4bw}S|
!writetreeto ${list4out} ${gener4bw}T|
!move ${gener4bw}S|  >$gener4n(0t)|
!filx-minor ${gener4n}s| $excluded
!iflist 0 "<" ${gener4n}S|
!writetreeto ${list4outn} ${gener4n}T|

!move ${excludedin}S| >$excludedout(0t)|
!move ${gener4n}S| >$excludedout(0t)|
!iflist 0 "<" ${excludedout}S|
!writetreeto ${exclistout} ${excludedout}T|
!writetreeto ${treeall} (T)
!prtree
```

We have used the above described script to find all 49 ternary (non-binary) excluded
minors for branch-width 3, and to generate other more than 1100 such excluded minors over
larger fields: [P. Hlineny: Using Computer in Matroid Theory Research, Acta Math. Univ.
M.Belii 11 (2004), 27-44, http://actamath.savbb.sk] (Some detail are also included in
the script comments.)

7.6 Excluded Minors for Representability

The following complex script reprXexc can be used to generate excluded minors for repre-
sentability over various fields. See the included comments...............

```
#
# This MACEK script generates the small excluded minors for PFXX-representable
# matroids over any partial field, starting in steps from a user-defined
# given matroid list ({U25} suggested for bigger field than ternary).
# All inequivalent representations are produced, so it may be desirable
# to call the '!isompair' command afterward.
# Beware of possible inequivalent representations of the starting matroids
# (even for U25!) - all of them have to be considered.
#
# Use: ./macek -pGFX '&reprXexc filebase PFYY [rcb] [suffix] [verbosity]'
#  Example as  ./macek -pGF7 '&reprXexc gf7_gf5excl_5_ GF5 b bb'
#
# Before starting the procedure, create the files "gfX-gfYexcl-" with the starting
# list of matroids to generate from over GF(X),
# and "gfX-gfYexcl-eexc" containing (possible) extra excluded minors for
# representability (over other fields, with correct '@inputpf gfZ' for each one).
# Then run a sequence of commands like these:
#  ./macek -pGF4 '&reprXexc gf4-gf5excl- GF5 b '
#  ./macek -pGF4 '&reprXexc gf4-gf5excl- GF5 b b'
#  ./macek -pGF4 '&reprXexc gf4-gf5excl- GF5 b bb'
```

```
#  ...
#  ./macek -pGF4 '&reprXexc gf4-gf5excl- GF5 b bbb..'
# These will generate the excluded minors step-by-step (each step adding one
# element to the matroids), storing the excluded minors to "gf4-gf5excl-b..b-exc.mc
#
# We briefly explain the starting list "gfX-gfYexcl-", for example for GF(X)
# bigger than GF(3) - then all nonbinary nonternary representable matroids contain
# a U_2,5 or U_3,5 minor, and hence we start from the file (list):
#  { U25 }
# !represgen (s) allq >((0T))
# This file actually includes the default representation of U_2,5, but it also
# generates all nonequivalent representations for it (which exist over, say, GF(9))
# (Notice also that we miss all U_k,k+2 matroids in such a way!)
#
# A table of computed results:
#  over GF2 for GF5:  known excluded F7, F7#
#  over GF3 for GF5:  4 on 8 elem, 1 on 10 elem, 1 on 12 elem, ...
#  over GF4 for GF5:  additional 5 on 8 elem, 29 on 9 elem, ...
#  over GF7 for GF5:  additional 9+1 on 7 elem, 65? on 8 elem, ...
#  ...
#


@subd-param1 unknown
@subd-param2 unknown
@subd-param3 b
#@subd-param4
@subd-param5 1


# the file names for the lists of previously generated exclusions, of the starting
# exclusions (like those over different pfield), and of the new filename-base:
@sub-prevexcname "${param1}${param4}-exc"
@sub-basexcname "${param1}eexc"
@sub-newfname "${param1}${param4}${param3}"

@sub-givenm "((("
@sub-prevexc "((()("
@sub-otherexc "(((2)("
{
@name reprXexc-l
@comment "Given lists..."
{
<${param1}${param4}
@comment "starting list of matroids"
}
{
@comment "previous excluded minors"
}
```

```
{
@comment "other excluded minors"
}
}
@sub-work "(()("
{
@name reprXexc-w
@comment "Working subframe..."
{
@comment "generated extensions"
}
{
@comment "new excluded minors"
}
{
@comment "new matroid list"
}
}
@sub-temp "((2)("
{
@name reprXexc-t
@comment "Temporary subframe..."
}


# conditionally reading the list of previous excluded minors
!quiet
!prtext "Reading the previous (smaller) excluded minors for ${param2} representabil
!iffile "${prevexcname}"
!read "${prevexcname}" >${temp}(t)|
!iflist 0 < ${temp}s)|
!move ${temp}(s)| >$prevexc(0t)|
# reading also a possible list of other basic excluded minors, which are represente
#  in general over diff pfields, and so their current representations are generated
!iffile "${basexcname}"
!read "${basexcname}" >${temp}(t)|
!iflist 0 < ${temp}s)|
!represgen ${temp}(s)| allq >$otherexc(0t)|
!verbose


# generating the "param3"-extensions to the given list (one step!)
!prtext "Extending the previous list of current- and ${param2}-representable matroi
!extend $param3 ${givenm}S)| >${work}((0t)|
!move ${work}(S)| >${work}()((0t)|


# filtering representability over "param2" and testing previous excl minors
!prtext "Testing ${param2}-representability of the generated extensions..."
!verbose $param5
```

```
!rem-repres $param2 ${work}()(s)|
!move ^1 >${work}(2)((0t)|
!quiet -$param5
!quiet
# (skipping excl minor test if empty lists)
!prtext "Testing possible previous (smaller) excluded minors in the nonrepresentabl
!iflist 0 = ${prevexc}S)|+${otherexc}S)|
!skip 9
!iflist 0 = ${work}()(s)|
!skip 6
!verbose +2
!verbose $param5
!filx-minor ${work}()(s)| ${prevexc}S)|
!filx-minor ${work}()(s)| ${otherexc}S)|
!quiet -$param5
!quiet -2
!move ${prevexc}S)| >${work}()((0t)|
!verbose

!prtree
!writetreeto "${newfname}-exc" ${work}()(T)|
!writetreeto "${newfname}" ${work}(2)(T)|
!prtext "All done in this step, start next from the list ${newfname} ..."
```

We have used the above described script to look for the excluded minors for (say) $GF(5)$ representability. We have generated 4 such ternary excluded minors on 8 elements (already known), one new excluded minor on 10 elements, and one on 12 elements which is (somehow surprisingly) already known. Then we generated many more such excluded minors over larger fields, not attempting to find all of them. (Unfortunately, no explicit upper bound for their size is known, and there are huge numbers of generated matroids.)

7.7 How to Self-Test Macek Computation

There is, unfortunately, no large-scale computation data about matroid generation available in the literature nowadays (2004), and so we have little chance to compare our computing results with other reliable sources.

So, in this final section we present a simple way how Macek computation results can be self-tested in a nontrivial way. Imagine we generate all inequivalent representations of matroid extensions up to certain size over, say, $GF(5)$. Then we select those extensions which are also representable over, say, $GF(4)$, and finally we find all those non-isomorphic ones. That is simply run as:

```
macek -pgf5 '!extend bb;!filt-repres gf4;!isompair' grK33
```

In this specific case we get total 469 inequivalent matrices representing 329 non-isomorphic matroids.

In the second stage we run an analogous computation starting over $GF(4)$ instead:

```
macek.nodebug -pgf4 '!extend bb;!filt-repres gf5;!isompair' grK33
```

Since, in theoretic words, we generate in both cases all the matroids of certain size representable both over $GF(4)$ and $GF(5)$, the resulting numbers must agree. Indeed, now we get total 544 inequivalent matrices representing 329 non-isomorphic matroids. Good!

The reader is invited to check that the above two computations run in Macek in really different ways, and there is almost nothing in common between them. Hence what else (than correctness) could make the result the same? One may run countless different variants of the described test...

Moreover, in some specific cases like the above one, there is more to say and to test. It is known, for example, that the matroids representable over both $GF(4)$ and $GF(5)$ are the so called golden-mean matroids [Vertigan, unpublished?]. Since the golden-mean partial field is included in Macek, we may easily verify also that:

```
macek -pgmean '!extend bb;!isompair' grK33
```

Now we get total 889 inequivalent matrices representing 329 (again!) non-isomorphic matroids.

See more in [P. Hlineny: Equivalence-Free Exhaustive Generation of Matroid Representations, Discrete Appl. Math. 154 (2006), 1210-1222, http://dx.doi.org/10.1016/j.dam.2005.12.001].

8 Remarks

8.1 Program Reliability

In this chapter we discuss correctness and reliability of the results of Macek computations. When developing this project, we made every possible effort to produce a stable and very reliable software tool. Such an effort is, indeed, necessary if we want to use Macek computation results in research papers. However, if your view of computer programs is distorted by bad experiences with products of one unnamed Redmond company, then there are many ways how you can test our program, and ensure that you are getting correct answers.

Almost every algorithm in our program contains thorough internal checks. These include watching data consistency at critical steps, repeating algorithms on modified input data (an equivalent input like a matrix with swapped or pivoted lines, or a dual matrix, etc.), and using alternative or brute-force algorithms for the same question. User may find much more information about the internal checks directly in Macek source files.

To save time, time-consuming internal checks are randomized and executed only occasionally. Moreover, we provide an alternative executable 'macek.nodebug' which does not include the internal checks, and so it is significantly faster. We suggest to use 'macek.nodebug' carefully and only when you already know that your computation is (likely) correct.

Extensive debugging messages generated by the program with '-g3' show user exactly what the program does, and they may be easily followed in the program source files (which contain also detailed descriptions of the algorithms). Since randomized routines are used in the program, the course of program computation may vary from one run to another, but the final results should, of course, stay the same.

User may also test correctness of Macek computations by comparing the output with known theoretical results. Several such examples are presented in Chapter 7 [Practical], page 47. In particular, it is often the case that the computed results are closed under duality, which can be subsequently tested; or that one result may be obtained in different ways which are theoretically equivalent but require the program to do significantly different operations; etc.

Another way of checking the program output is to compare the results between different versions of the program. For example, from Macek version 0.8 many structural algorithms were replaced by new ones that are cleaner and faster. In particular, this means that comparing structural answers from version 0.8.2 with the same answers from version 1.0 provides a nontrivial proof of correctness. Of course, such testing is limited only to functions implemented already in earlier versions of Macek. Find out more about these by reading the corresponding source files and source documentation.

Some new computations with version 1.2 compared Macek with (partial) independent enumeration results found in the literature. See [P. Hlineny: Combinatorial Generation of Matroid Representations: Theory and Practice, Acta Math. Univ. M.Belii 12 (2005), 31-41, http://actamath.savbb.sk] for a comparison with results obtained by [Kingan et al.], or [P. Vymola: Computer-assisted enumeration of small matroids (in Czech only), MSc. Thesis, FI MU Brno, http://is.muni.cz/th/60663/fi_m/thesis-final.pdf] for

extensive enumerations compared with independent related results obtained in the theory of linear codes by [Wild].

Lastly, we want to note that we have designed our program to be an advanced tool for skilled users. (Here we mean mainly skilled in matroid theory.) The Macek program is not idiot-proof. So keep in mind that *"garbage in, garbage out"*. Watch out carefully whether the program is really computing the tasks you expect it to. And finally, (RTFM!) read this whole manual very carefully before starting with complicated computations.

8.2 Troubleshooting

The troubleshooting section is not written yet. If you have troubles with the Macek program, let me know so that I may include some solution tips here...

`http://www.mcs.vuw.ac.nz/research/macek/`, `hlineny@fi.muni.cz`.

Anyway, in case of computer troubles try to ask your system administrator first, (s)he may help even when (s)he knows nothing about matroids.

8.3 Adding Functions to Macek

This chapter of the Macek manual is provided for those who are not satisfied with the current functionality of this program; who want more functions in it, and who are able to contribute to Macek development. There are lots more things you can do if you really want!

According to the GPL license covering Macek, you may obtain and modify any source file of the program. However, we suggest you follow the guidelines provided next, so that your additions to Macek will be compatible with the future development. And, if you write a nice piece of code for Macek, let me know so that I can arrange it within the master distribution. `http://www.mcs.vuw.ac.nz/research/macek/`.

Before playing with new functions for Macek, read the relevant source files of the distribution since they contain a lot of technical description and comments. Then prepare you code separately in a separate directory. (You may use the provided subdirectory '`src/addons`'...)

There are three areas in which it is useful to add you code directly to the existing Macek code, and special (empty so far) files '`*.inc`' are provided there for you: additional partial field definitions and translations '`pfield/pfdef-more.inc,pftran-more.inc`', additional command handles and option descriptions '`frame/frcoms-more.inc,fropts-more.inc`', and extra control rules for extension generating '`gener/gener-more.inc`'. Read the comments in these files, and follow samples there.

Good luck with development!

Index

Here we list the concept index for this manual. (Go to the next index if you look for program commands / options.)

Here we list the index of all frame- commands and options described in this manual.

S

T

V

W